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On Possible Role of Parallel Flow Profile in the Transition of Enhanced Reverse Shear Modes

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Abstract
The linear behaviour of the ITG drived perturbation with a parallel velocity shear is studied in a sheared slab geometry. Full analytic studies show that when the magnetic shear has the same sign as the second derivative of the parallel velocity with respect to the radial coordinate, the linear mode may become unstable. On the other hand, when the magnetic shear has opposite sign to the second derivative of the parallel velocity, the linear mode is completely stabilised.

Keywords:
ITG mode, ERS transition, transport barrier, magnetic shear, flow curvature

1. Introduction
Arguably the most remarkable story of fusion research over the past decade is the discovery of the enhanced reverse shear modes (ERS modes) in Tokamak Fusion Test Reactor (TFTR) [1] and the negative central magnetic shear modes (NCS modes) in DIII-D [2]. It is not often that a system self-organises to a high erenergy state with such a large reduction of turbulence and transport when an additional source of free energy is applied to it [3]. It is usually believed that the ERS or NCS configurations can provide the characteristics desirable for a fusion reactor [4].

The understanding of the formation of transport barriers in the ERS or NCS plasma configurations is therefore fundamental to the development of techniques to control such barriers for tailoring profiles and for improving operating regimes further. This is especially significant because it is now widely accepted that the negative magnetic shear is not the only factor needed for the transport reduction in the ERS or NCS modes. Some of the clearest evidence comes from the comparison of the RS (reverse shear) and ERS (enhanced reverse shear) transition data in TFTR [5]. It shows that the RS phase is not necessarily an ERS phase and some other factor is needed to explain the transition. Theoretical study also indicates that the reversal of magnetic shear alone might have a little effect on the ITG-type microinstabilities [6].

Most recently, the $\mathbf{E} \times \mathbf{B}$ shear stabilisation mechanism has been proposed to explain the core transport barriers formed in plasmas with negative or reverse magnetic shear regimes [3]. It is believed that the change in the radial electric field in the core is

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produced by a number of ways, for example, by toroidal flow ($v_{\phi i}$) [7] and/or by pressure gradient (∇p_i) [5] and more recently by poloidal flow ($v_{\theta i}$) [8]. However, while this **E** × **B** shear stabilisation mechanism *alone* can satisfactorily explain the confinement improvement in the edge, it may not be an obvious explanation for the core confinement improvement in the ERS and NCS plasma [9]. For example, the formation of the ERS mode in TFTR has been reported [5] at values of $\gamma_{E\times B}$ (**E** × **B** shearing rate), as much as a factor of 3 below $\gamma_{\max}$ (the maximum linear growth rate), while for the suppression of turbulence-induced transport the condition $\gamma_{E\times B} \geq \gamma_{\max}$ needs to be satisfied [3]. It is therefore natural that an explanation of these experimental results should be sought in the effects such optimized magnetic configurations have on micro-instabilities and on the consequent transport.

In this work, we identify another effect which might be playing a key role in the reverse shear transition. We show here when the magnetic shear has the same sign as the second derivative of the parallel flow with respect to the radial coordinate, the ion temperature gradient (ITG) mode may become unstable. On the other hand, when the magnetic shear has opposite sign to the second derivative of the parallel velocity, the ITG mode is completely stabilised. This result, therefore, shows that it is the relative sign of the second radial derivative of the equilibrium parallel flow with respect to the magnetic shear which may be the key factor for the enhanced reverse shear transition.

2. Stability Analysis

We will study the ITG-like perturbation. We adopt a two-fluid theory in a sheared slab geometry, $\mathbf{B} = B_0[\mathbf{z} + (x/L_s)\mathbf{y}]$, where L_s is the scale length of magnetic shear. The x, y and z directions in the sheared slab geometry are defined as the radial, poloidal and toroidal directions in the tokamak configuration. We assume a background plasma with all inhomogeneities only in the radial direction, where perturbations have the form $\phi(\mathbf{x}, t) = \phi(x)\exp[i(k_y y + k_z z - \omega t)]$. We ignore finite gyroradius effects by limiting consideration to the wavelength domain $k_\perp \rho_i \ll 1$, where ρ_i is the ion gyroradius. We then write down the equations of continuity, the parallel momentum equation and the equation of adiabatic pressure evolution respectively:

$$\frac{\partial \tilde{n}_i}{\partial t} + \nabla \cdot (n_0 + \tilde{n}_i)(v_0 + \tilde{v}_\perp + \tilde{v}_\parallel) = 0 \qquad (1)$$

where,

$$v_0 = v_{\parallel 0}(x)\hat{e}_\parallel$$

$$\tilde{v}_\perp = v_E + v_{Di} + v_P$$

$$v_E = \frac{c}{B}\hat{b} \times \nabla_\perp \phi$$

$$v_{Di} = \frac{c}{eBn_i}\hat{b} \times \nabla_\perp P_i, \quad \hat{b} = \vec{B}/|B|,$$

$$P_i = P_{i0}(x) + \tilde{p}_i$$

$$v_P = -\frac{c^2 m_i}{eB^2}\left(\frac{\partial}{\partial t} + (v_0 + v_E + v_{Di}) \cdot \nabla\right)\nabla_\perp \phi$$

$$\Rightarrow \left(\frac{\partial}{\partial t} + v_0 \cdot \nabla\right)(1 - \nabla_\perp^2)\tilde{\phi} + v_D\left[1 + \left(\frac{1+\eta_i}{\tau}\right)\nabla_\perp^2\right]\nabla_y \tilde{\phi} - \hat{b} \times \nabla\tilde{\phi} \cdot \nabla(\nabla_\perp^2 \tilde{\phi}) + \nabla_\parallel \tilde{v}_\parallel = 0 \qquad (1a)$$

Here, in equation (1a) we rescaled the quantities as stated below $\tilde{\phi} \equiv e\phi/T_e$, $\tilde{v}_\parallel \equiv \tilde{v}_{\parallel i}/c_s$, $\tilde{p} \equiv [\tilde{p}_i/< P_{i0} >](T_i/T_e)$, $P_i = < P_{i0}(x) > + \tilde{p}_i$, $n_i = n_0 + \tilde{n}_i$, $v_D \equiv -\frac{cT_e}{eB}\frac{d(\ln n_0)}{dx}$, $\tau \equiv \frac{T_e}{T_i}$, $Y \equiv \frac{\Gamma}{\tau}$, $\mu \equiv \frac{\mu_\parallel \omega_{ci}}{c_s^2}$, $n_i = n_0 + \tilde{n}_i$. Here, Γ is the ration of specific heats, and $\mu_\parallel$ is the parallel viscocity (due to either Landau damping or collisional viscosity, for detail see reference [10]) required for saturation of the turbulence.

$$\frac{\partial \tilde{v}_{\parallel i}}{\partial t} + (v_E + \tilde{v}_{\parallel 0}(x)) \cdot \nabla(\tilde{v}_{\parallel 0}(x) + \tilde{v}_{\parallel i}) - \frac{e}{m_i}\nabla_\parallel \tilde{\phi} - \frac{1}{m_i n_i}\nabla_\parallel P_i + \mu_\parallel \nabla_\parallel^2 (v_0 + \tilde{v}_{\parallel i}) \qquad (2)$$

$$\Rightarrow \left(\frac{\partial}{\partial t} + v_0 \cdot \nabla\right)\tilde{v}_\parallel - \frac{v_0}{L_v}\nabla_y \tilde{\phi} + \hat{b} \times \nabla\tilde{\phi} \cdot \tilde{v}_\parallel = -\nabla_\parallel \tilde{\phi} - \nabla_\parallel \tilde{p} + \mu \nabla_\parallel^2 \tilde{v}_\parallel \qquad (2a)$$

$$\frac{\partial \tilde{p}_i}{\partial t} + v_E \cdot \nabla P_{i0} + v_E \cdot \nabla \tilde{p}_i + v_0 \cdot \nabla \tilde{p}_i + \Gamma P_{i0} \nabla_\parallel \tilde{v}_{\parallel i} = 0 \qquad (3)$$

$$\Rightarrow \left(\frac{\partial}{\partial t} + v_0 \cdot \nabla\right)\tilde{p} + T_e v_D\left(\frac{1+\eta_i}{\tau}\right)\nabla_y \tilde{\phi} + \hat{b} \times \nabla_\perp \tilde{\phi} \cdot \nabla\tilde{p} + Y \nabla_\parallel \tilde{v}_\parallel = 0 \qquad (3a)$$

Here x is the distance from the mode rational surface defined by $\mathbf{k} \cdot \mathbf{B_0} = \mathbf{0}$, and $V_{\parallel 0}$ is the equilibrium parallel velocity. All other symbols are assumed to have the usual meaning unless otherwise stated explicitly. It is important to mention at this stage that in this work we make no attempt to speculate about source of these flows although a strongly peaked ion velocity parallel to the magnetic field is observed to coexist in tokamaks in the region where the plasma confinement is improved [2,8,11]. Parallel flow, $V_{\|0}$, has therefore two effects. First, it introduces a Doppler shift, $k_\| V_{\|0}(x)$, in all time derivatives and second, an extra term, $\mathbf{V}_{\mathrm{E}} \cdot \nabla V_{\|0}(x)$, representing radial convection of ion momentum. It is the second term which makes the effect of parallel flow shear completely different from that of the perpendicular flow shear [12]. We eliminate the Doppler shift by performing Galilean transformations in the $\hat{e}_\|$ direction.

To model the equilibrium parallel velocity we assume a simple general case of variation with the radial distance x.

$$v_0(x) = v_{00} + \frac{v_{00}}{L_{\mathrm{v1}}} x + \frac{v_{00}}{2L_{\mathrm{v2}}} x^2$$

In the subsequent analysis, for simplicity of calculation, we keep only the curvature contribution as it is well-known that the role of a parallel velocity shear is always destabilising on microinstabilities. It is also justified in our problem as our main purpose is to show the role of the relative sign of the second radial derivative of the equilibrium parallel flow with respect to the magnetic shear.

Linearising equations (1a, 2a, 3a), and neglecting $\mathcal{Y}$ (which gives corrections of order $k_\|/k_\perp)^4$), we write down the eigenvalue equation as

$$\frac{\partial^2 \tilde{\phi}}{\partial x^2} + [\Lambda + Px^2]\tilde{\phi} = 0 \tag{4}$$

where,

$$\Lambda = -k_y^2 + \frac{1-\Omega}{\Omega + K}, \quad P = -\frac{J_2 S}{\Omega(\Omega+K)} + \frac{S^2}{\Omega^2},$$

$$\Omega = \frac{\tilde{\omega}}{k_y v_{\mathrm{D}}}, \quad K = \frac{1+\eta_{\mathrm{i}}}{\tau}, \quad S = \frac{L_{\mathrm{n}}}{L_{\mathrm{s}}}, \quad J_2 = \left(\frac{v_{00} L_{\mathrm{n}}}{L_{\mathrm{v2}}}\right).$$

Equation (4) is a simple Weber equation. Depending on the sign of P, we have two types of solution. If $P < 0$, i.e.,

$$\frac{J_2^{\frac{1}{2}} S}{\Omega(\Omega+K)} > \frac{S^2}{\Omega^2}$$

the solution which satisfies the physical boundary condition, i.e., $\phi \to 0$ at $x = \pm\infty$ is given by

$$\phi(x) = \phi_0 \exp[-\sqrt{|P|} x^2] \tag{5}$$

The wave therefore does not propagate and is intrinsically undamped.

On the other hand, if $P > 0$, Equation (4) has the solution

$$\phi(x) = \phi_0 \exp[-i\sqrt{|P|} x^2] \tag{6}$$

Thus, we have now a non-localised mode carrying energy outward. Because of the convective wave energy leakage the perturbation will decay in time in the absence of any energy source feeding the wave. The wave is therefore damped.

The overall stability of the mode may also be obtained from the dispersion relation

$$\Lambda = i\sqrt{|P|}$$

From the above discussion it is clear that it is the parallel flow curvature which actually plays the key role in the stability of the mode. When the magnetic shear has the same sign as the parallel flow curvature, i.e., for positive magnetic shear ($L_{\mathrm{s}} > 0$), parallel flow curvature acts to destabilise the mode. On the other hand, for the negative magnetic shear configuration ($L_{\mathrm{s}} < 0$), i.e., when the magnetic shear has the opposite sign to the second derivative of the parallel flow with respect to the radial coordinate x, the parallel flow curvature acts to stabilise the mode. Flow curvature now forms an additional antiwell which pushes the wave function away from the mode rational surface, thereby enhancing stabilisation.

3. Conclusion

In conclusion, we have identified the relative sign of the second radial derivative of the equilibrium parallel flow with respect to the magnetic shear as the key factor for the enhanced reverse shear transition. Our full analytic studies show that when the magnetic shear has the same sign to the second derivative of the parallel velocity with respect to the radial coordinate, the ITG mode may become unstable and turbulent momentum transport increases. On the other hand, when the magnetic shear has opposite sign as the second derivative of the parallel velocity, the ITG mode is completely stabilised and turbulent momentum transport reduces. It is shown that a large reduction in the momentum transport is possible by suitably tailoring the parallel flow profile. On the experimental front, recent results from the JET have shown that the reduction of small-scale turbulence in optimized magnetic shear regimes is directly related to the existence of a strongly sheared toroidal velocity in the area of the internal transport barrier [13].

Before concluding we would like to mention a couple of points. First, while considering the effect of flow profile on the ITG-type instabilities we assume that the equilibrium is stable, in other words, the system is unstable only due to the ITG-type perturbations. Second, in this work we have discussed only the slab branch of the ITG modes, extension of this work is planned to see the effect of the toroidicity in the stability analysis.

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