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Near-Wall Plasma in Magnetic Field and Bohm's Criterion

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Abstract

The near-wall plasma in a magnetic field and the Bohm criterion are discussed on the hydrodynamical model. A magnetic field is assumed to be in parallel to the solid surface.It is essentially that the methods used allow to consider the near-wall plasma without deviding it into separate sheaths.It has been found that in the case of a magnetic field directed in parallel to the surface the conditions for the Bohm criterion validity (and formation of monotonous electric potential) depend upon the plasma temperature, the magnetic field strength and the velocity of poloidal plasma rotation.It is marked that ion scattering processes play a very important role in formation of the near-wall plasma structure. For instance, the characteristic length of the quasineutral plasma where ions acquire a directed velocity equal to the ionic sound depends on the electronic ionization rate of atoms, the magnetic field strength and the velocity of poloidal plasma rotation.

Keywords:

near-wall plasma, electric potential profiles, particle scattering, the Bohm criterion

1. Introduction

The investigations of physical phenomena in a plasma near a solid surface attract considerable attention. Hundreds of experimental and theoretical works have been devoted to the subject. The great interest to the problem is explained in view of the fact that phenomena on the plasma boundary play an extremely important part in lots of plasma applications: electric discharge devices, thermoionic converters, plasma technologies in microelectronics, etc. Recently the problem has become of great interest because of the investigations of the near-wall plasma in tokamaks and stellarators [1,2].

In the majority of the suggested up-to-date theoretical models an approximation of "two" or "three scale theory" has been used. A brief survey of existing theoretical models describing the plasma boundary can be found in [1,2]. The main assumptions underlying these models are as follows:

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(1) As a rule particle collisions are ignored in the theoretical models used for the description of the near-wall plasma.
(2) The thickness of a region, in which the plasma neutrality is violated, is assumed to be comparable with the Debye radius.
(3) The fluxes of positively and negatively charged particles to the wall are exactly the same, i.e., no charge transfer takes place in the system.
(4) There are three spatial regions [3]:
   (a) the Debye sheath,
   (b) upstream of it, the Chodura magnetic pre-sheath.
   (c) upstream of that, the ordering pre-sheath.
(5) According to the pointed models, the monotonic potential distributions near the wall can be realized provided that the Bohm criterion is satisfied [4] (see also [5]): the velocity of the

directed motion at which ions penetrate into the area of the charged layer must exceed that of the ionic sound, $u_0 \geq s = \sqrt{T_e/m_i}$, where T_e is the electron temperature and m_i is the ion mass. In the area of the discharged layer ions gain an additional energy of the order of several electron temperatures.

In a number of works efforts were made to validate or revise the Bohm criterion; references to some of these works can be found in the reviews mentioned above [1,2].

As the Bohm criterion is widely used for physical interpretation of the phenomena close to the plasma boundary (in the theory of probe characteristics, in lots of numerical codes etc.) it seems expedient to discuss the limits for Bohm criterion validity. This statement has been considered in our papers [6,7] in the frame of a strict theory where it was shown that the above-said criterion was not universal. It was found that ion scattering processes play an important part in formation of electric potential profiles near the wall. In [7] a method of solving a kinetic equation for ions near the plasma boundary has been proposed and on its basis a numerical analysis of the problem has been done. It was shown that under conditions when a charge exchange frequency exceeds that of electronic ionization of atoms monotonous regimes of electric potential distribution take place regardless of the Bohm condition including the case when $u << s$. At the same time, at a predominant role of ionization even in the region of a quasi neutral plasma ions can gain the velocity equal to the ion-sound one (see also [6]). Thus, using approximation models with ion collisions being neglected can result in blunders. In particular, as follows from the results of the numerical experiment [7] the extension of the wall plasma (areas of basic potential drops), the wall potential value, the electric field on the surface and the ion energy close to the wall substantially differ from the results of the approximation models (see [1,2]). Below the results of our works [6,7] are generalized for the case of a magnetic field directed in parallel to the surface. In the case of a magnetic field directed almost perpendicular to the surface the near-wall plasma structure has been considered by Chodura [8] (see also [3]) in view of the approximation model. Below we consider a theoretical model which does not require deviding the near-wall plasma into different sheaths with essentially different characteristic scales. Thus in the frame of our theoretical description the understanding of "a sheath", "pre-sheath", etc is of no importance.

2. What is the Essence of the Problem? The Problem Statement

The problem is reduced as it is known to the investigation of the Poisson equation solutions (we consider a one-dimentional case)

$$d^2\varphi/dx^2 = -4\pi \Sigma_{e,i} e_\alpha \int_{-\infty}^{+\infty} f_\alpha(v,x) d^3 v \,, \qquad (1)$$

where f_α is the function of distribution in velocities of particles α with a charge e_α. Significant difficulties arise at calculating charge densities in the right part of eq.(1). In some works (see, e.g., [1,2]) an approximate approach was used for deducing the Bohm criterion based on two or three scale theory. In order to make clear the notes given above let us use the mechanical analogy [6]. It follows from the Leuville theorem that in the absence of particles scattering processes their distribution function depends only on mechanical invariants of the type

$$\varepsilon_\alpha = m v_\alpha^2/2 + e_\alpha \varphi = \text{const}, \qquad (2)$$

i.e. we have $f_\alpha = f_\alpha(v_x, \varphi)$. Thus, the right part of eq.(1) will depend only on φ, i.e. equation (1) can be given in the form:

$$\frac{d^2\varphi}{dx^2} = -\frac{dU(\varphi)}{d\varphi} \qquad (3)$$

where

$$U = \int_0^\varphi F(\varphi) d\varphi \qquad (4)$$

is "the potential" and $F(\varphi) = 4\pi(n_i - n_e)$.

So we deal with a "conservative problem" where the law of conservation of energy is satisfied:

$$1/2 \left| d\varphi/dx \right|^2 + U(\varphi) = C \qquad (5)$$

If there is no magnetic field and ion scattering, for charge densities n_e and n_i we can use expressions (see for instance [6]):

$$n_e = n_{e0} \exp(e\varphi/T_e) \qquad (6)$$

$$n_i = n_{i0} u_0 / \sqrt{(u_0^2 - 2e\varphi/m_i)} \qquad (7)$$

where m_i is the ion mass, u_0 is the directed velocity and T_e is the electron temperature. Let us choose the origin x_0 at some distance from the surface , setting $\varphi(x_0) = 0$. At the same time it is assumed that $d\varphi/dx \neq 0$ at $x = x_0$, since the field other than zero is necessary so that ions could gain a velocity. The case, where $d\varphi/dx = 0$ in the center of the plasma layer symmetry, has been

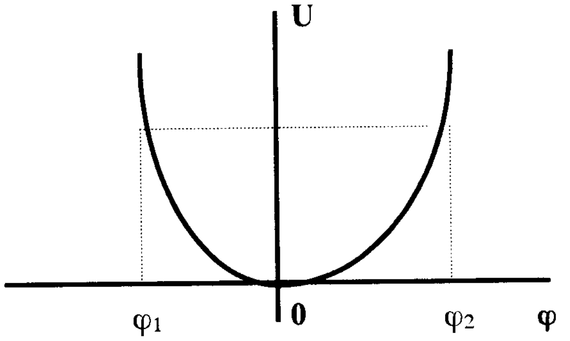

Fig. 1 Potential curve U = U(φ).

considered in [5].

Figure 1 shows an exemplary plat of function $U = U(\varphi)$ at $u_0 < s$ with charge densities of eqs.(6) and (7) taken into account. The constant in the right part of eq. (5) is expressed in terms of a derivative of $\mathrm{d}\varphi/\mathrm{d}x(x_0)$:

$$C = \left| \frac{\mathrm{d}\varphi}{\mathrm{d}x}(x_0) \right|^2 \tag{8}$$

As it is seen from the figure, no solutions exist for $u_0 < s$, which could correspond to an "infinite motion of a particle" (a monotonous potential profiles). Only oscillation type solutions can be realized with a oscillation amplitude between φ_1 and φ_2; however these solutions do not meet the boundary condition on the surface. Only at $u_0 \geq s$ possible are particles paths corresponding with a monotonous profile. Thus to form monotonous potential distributions near the surface the Bohm criterion must be satisfied. However, to gain the ion velocity, $u \geq s$, an electric field in plasma must be of a significant value. At the same time there are no electric fields without collisions.

In fact, in the frames of the considered statement the problem is not closed since the particle flux from plasma must be replaced, for example, through injection. If the injection proceeds in the center of the plasma layer symmetry at $x = 0$, then the central plasma area acquires a positive charge (electrons tend to quit this area). This results in a potential difference between the plasma center and the wall which equalizes electron and ion fluxes. As a result, the solution complies with boundary conditions. In tokamaks (stellarators) usually observed velocities of plasma expansions $u \sim 1 \div 10$ m/s at a plasma density $n \sim 10^{13} \div 10^{14}$ cm^{-3}. So, in order to keep the plasma flux density and have the ion velocity comparable with that of the ionic sound it would be necessary to reduce the plasma density by about 4–5 orders of magnitude which is not consistent with the observed values. Nevertheless, with account of volume ionization processes the ion velocity can attain $u \geq s$ [6], if the atom density in the boundary area is rather high (recycling [1]).

3. The Influence of Scattering Processes and Magnetic Field Effects

Thus, profile character for electric potentials is defined by the velocity of directed ion motion. In its turn this velocity depends on scattering processes in plasma. So together with monotonous profiles solutions of the type of spatial potential oscillations prove to be possible; a numerical study of the oscillations is not an easy task because in this case it is necessary to allow for charged particles, electrons and ions, captured in the potential maximum (minima). A similar task can be solved using numerical technique.

3.1 Ion Formation Rate Effect

Now turn to monotonous potential distributions. For simplicity we use the gas dynamic description for ions (it is well-known the hydrodynamic description is effective also in investigating rarefied plasma).

$$u_\mathrm{i}\frac{\mathrm{d}u_\mathrm{i}}{\mathrm{d}x} = \frac{e}{m_\mathrm{i}}E - s_\mathrm{i}^2\frac{\nabla_x n_\mathrm{i}}{n_\mathrm{i}} - \nu_\mathrm{i}u_\mathrm{i} \tag{9}$$

where u_i is the velocity of directed ion motion and ν_i is the frequency of ion scattering. In contrast to reference [6], here the ion gas pressure is taken into account (as known, in a boundary area of tokamaks electron and ion temperatures are usually comparable in their magnitude $T_\mathrm{i} \sim T_\mathrm{e}$). Above $s_\mathrm{i} = \sqrt{\gamma T_\mathrm{i}/m_\mathrm{i}}$ where γ is the adiabatic index. Equation (9) should be completed by the equation of continuity for ions

$$\frac{\mathrm{d}}{\mathrm{d}x} n_\mathrm{i}u_\mathrm{i} = \alpha_\mathrm{e} n_\mathrm{e}(T_\mathrm{e}), \tag{10}$$

where α_e is the electron ionization rate of neutral atoms. In the limit of a quasi neutral plasma, when $n_\mathrm{e} = n_\mathrm{i}$ and $\mathrm{d}n_\mathrm{e}/\mathrm{d}x = \mathrm{d}n_\mathrm{i}/\mathrm{d}x$ from eqs.(9) and (10) together with eq. (6) we have ($\nu_\mathrm{i} = 0$):

$$(u_\mathrm{i}^2 - s_\mathrm{ef}^2)\frac{\mathrm{d}u}{\mathrm{d}x} + \alpha_\mathrm{e} = 0, \tag{11}$$

where $s_\mathrm{ef}^2 = s^2 + s_\mathrm{i}^2$. Hence, s_ef comes into the Bohm criterion with the value of ion temperature taken into account. From eq. (11) we find that the ion velocity equal to s_ef is achieved at the distance $x_m = 2/3\ s_\mathrm{ef}/\alpha_\mathrm{e}$. Thus with due regard to ionization processes ions can gain a necessary velocity $u \sim s_{ef}$. The estimation is quoted. At a density of neutral atoms in the near wall plasma $n_a \sim 10^{14}$ cm^{-3} and $T_e \sim 50$ eV we have $\alpha_e \sim 10^7$ s^{-1}, so for the wall plasma extension where the main potential drop occurs we obtain $x_m \sim 1$ cm. Thus the processes in a fairly extended plasma area (e.g. compared to the Debye radius) have an effect on shaping the potential profile structure.

3.2 Magnetic Field Effect

The results of the proceeding analysis hold true for the case of the plasma in the magnetic field direction which is almost normal to the surface. Now we consider another case where the magnetic field is parallel to the surface. As before the electron distribution in the potential field is described by the Bolzmann formula (eq. (6). The equation of continuity for ions is kept also the same (eq. (10)). The plasma parameters are assumed to depend only on the x-coordinate. At the same time the ion velocity has two components (u_x, u_y) which are described by a set of two equations:

$$u_{ix}\frac{du_{ix}}{dx} = -e\frac{s_{ef}^2}{s^2}\frac{d\varphi}{dx} + \omega_i u_{iy}; \qquad (12)$$

$$u_{ix}\frac{du_{iy}}{dx} = -\omega_i u_{ix}, \qquad (13)$$

where ω_i is the ion cyclotron frequency. In the y-coordinate direction the plasma is considered uniform, and y-component of electric field is absent. Besides, there occurs plasma rotation perpendicular to the magnetic field $u_y \neq 0$ (with reference to tokamaks this is poloidal plasma rotation). The magnetic field is directed along the z-axis.

From eq. (13) for the y-velocity component we have:

$$u_{iy} = u_{iy}^0 - \omega_i x, \qquad (14)$$

where u_{iy}^0 is the velocity of plasma rotation at $x = 0$.

As it is seen from eq. (14) the velocity of plasma rotation reduces at approaching to the surface. We restricted the consideration to an approximation of non-viscous liquid. In the near wall plasma there are electric fields of fairly high magnitude which play a dominant part.

Equation (12) integrates, recording eq. (14) to the following expression

$$\frac{u_{ix}^2}{2} = -\frac{e}{m_i}\frac{s_{ef}^2}{s^2}\varphi + \omega_i(u_{iy}^0 x - \omega_i\frac{x^2}{2}) \qquad (15)$$

In this case the boundary conditions $u_x = \varphi = 0$ at $x = 0$ were used. As it follows from eq. (15) the ion velocity depends on, beside φ, the electron and ion temperatures, the velocity of poloidal plasma rotation and the magnetic field magnitude. Depending on the direction of rotation the latter can favour acceleration or deceleration of ions. If rotation is caused by a radial electric field, then ion deceleration takes place. Using a set of equations (6), (10) and (12)–(13) one can find the following equation for determining the velocity of the directed ion motion u, which the ions can gain not leaving the area of a quasi neutral plasma:

$$(u^2 - s_{ef}^2)\frac{du}{dx} + \left[\alpha_e s_{ef}^2 + u\,\omega_i(\omega_i x - u_{iy}^0)\right] = 0 \quad (16)$$

As in the tokamak plasma the Larmour radius of ions usually exceeds the thickness of the area of spatial charge separation, the influence of a magnetic field on ion motion can be regarded as insignificant. So the criterion $u > s_{ef}$ will be again a condition of formation monotonous profiles of potential. To obtain analytical data we substitute the second member in square brackets in eq. (16) by a model member

$$s_{ef}\,\omega_i(\omega_i x - u_{iy}^0) \qquad (17)$$

The solution of equation (16) with a model member (eq. (17)) is of the form:

$$\frac{u_{ix}^3}{3} - s_{ef}^2 u_{ix} + \int_0^x F(x')dx' = 0 \qquad (18)$$

A maximum velocity is gained by ions in the plasma of thickness x_m which is determined by the following equation:

$$\int_0^{x_m} F(x')dx' = \frac{2}{3}s_{ef}^3 \qquad (19)$$

At $\omega_i = 0$ we come to the result obtained in [6]. If the last member in the left part of eq. (19) predominates (strong magnetic fields) we have $x_m \sim s_{ef}/\omega_i$. In condition with a dominant effect of plasma rotation we have $x_m \sim s_{ef}^2/u_y^0\omega_i$. In the second case, assuming $s_{ef} \sim 10^7$ cm/s, $\omega_i \sim 10^8$ s^{-1} we have $x_m \sim 0.1$ cm.

4. Basic Conclusions

1. Processes of ion formation play an exclusive role in shaping electric potential profiles in the near wall plasma.
2. Monotonous distributions of electric potential are possible only at a fairly high ionization rate of neutral atoms, when ions can acquire the velocity of directed

motion equal ionic sound In the absence of any scattering processes only the oscillation-type solutions can exist.

3. In the magnetic field directed parallel to the surface, conditions for realization of monotonous potential distributions depend upon the magnetic field strength and the velocity of poloidal plasma rotation as well as on electron and ion temperatures.

4. In the paper the theoretical model was used which does not require deviding the near-wall plasma into separate sheaths and consequently the understanding of "a sheath", "a pre-sheath", etc is of no importance.

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