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Sheaths, Double Layers and Dust Levitation

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Abstract

This paper contains an overview of certain sheath phenomena, namely the criterion for sheath formation, some examples of double layers and a description of some recent experiments on the suspension of dust particles in a sheath. A detailed review is not attempted and results obtained at Oxford are used to illustrate the last two topics.

Keywords:
sheaths, Bohm criterion, negative ions, double layers, dusty plasmas

Introduction

The first topic to be discussed is that of the space charge sheath at the boundary of a low pressure plasma. Relatively high electric fields exist in such a sheath, but the positive ions, on entering the sheath, have already travelled through weak but long-range electric fields in the plasma. The latter is *quasi-neutral*, the small imbalance between the electron and ion densities is important, being associated with a potential difference of the order of the electron temperature in electron volts. Early work on the subject was carried out by Tonks and Langmuir; in 1949, however, Bohm published a simple criterion for sheath formation. This stated that the velocity of the ions on entry to the sheath is given by $v > (kT_e/M)^{1/2}$, the theory being relevant to the case where the Debye distance is small compared with the plasma dimension. It can be demonstrated that an equality sign, rather than an inequality sign, is usually valid. The Bohm velocity is the same as that of an ion acoustic wave in a plasma, this fact has led to an analogy between the plasma boundary and the Mach surface of fluid mechanics. A generalized Bohm criterion that included the distribution of ion energies has been shown to be consistent with this concept. The Bohm criterion can be readily extended to several other cases including a plasma containing a mixture of positive ions, and a plasma containing negative ions.

The second topic to be discussed is that of double layers. Space charge regions are to be found not only adjacent to electrodes or discharge chamber walls, but also between two different plasmas. An example is given of a double layer, formerly termed a double sheath, at a constriction in a discharge tube. That is followed by a detailed description of experiments carried out to study a "free double layer", i.e. one formed between two plasmas in a tube without constrictions. It was found that the *double layer* was in fact a *triple layer* of charge, but the term *double layer* is still usually employed. The experiments showed that the layer was formed when the electron drift velocity approached the electron thermal velocity, a result in agreement with simulations found in the literature.

The third topic discussed here is that of dust levitation in a sheath. This subject is of much current interest following reports in 1994 that "plasma crystals", constituted of dust particles, had been produced in the laboratory. In these experiments, however, the dust particles are pulled out from the plasma by their weight and reside in the sheath beneath. The strong electric field in the sheath enables the weight of the particle to be balanced by an electric force. The dust particles

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behave rather like miniature Langmuir probes and attain a floating potential, and a certain charge, such that the electron and ion currents become equal in magnitude. One method of determining the charge acquired is to observe vertical oscillations of the particle about its equilibrium position. A description is given here of an experiment on the damped oscillations of particles introduced into the plasma. Dust charges of the order of 10,000 electrons have been determined in this way, for particle radii of the order of microns. A knowledge of the potential variation in the sheath is required in order to determine the charge. A number of sheath models have been examined and it was found that the potential profile is almost parabolic over much of the sheath in all the cases examined. This is of interest and considerable convenience in analysing the experimental data.

1. Sheaths

1.1 The Work of Langmuir

The first topic to be discussed is that of the space charge sheath that forms at the boundary of a low-pressure plasma. Such a sheath is to be found surrounding a probe immersed in a plasma; other situations are those of sheaths forming at the wall of the discharge vessel or adjacent to an electrode. Langmuir, the pioneer of this subject, first assumed that the sheath adjoined a perfectly neutral plasma. Figure 1 shows a diagram from one of Langmuir's early papers [1]. In this diagram a charged particle moves in a straight line until it enters the space charge region. Thereafter it is deflected by the electric field; the particle illustrated is deflected but not captured by the probe. Other particles would hit the probe and contribute to the probe current.

Later it was realized that this picture was simply not self consistent. The plasma is a *quasi-neutral* region that sustains an electric field due to the small but finite imbalance between the positive ion and electron densities. Langmuir never corrected his probe theory but later Tonks and Langmuir [2] derived a theory of the low-pressure positive column. In this case the space charge sheath forms at the tube wall and a radial electric field is set up in the quasi-neutral plasma. The potential distribution in the plasma is shown in Figure 2, together with the early probe measurements of Killian [3].

1.2 The Bohm Criterion for Sheath Formation

Bohm gave a very simple derivation of a sheath criterion, which I shall cite here. The case considered by Bohm [4] was that of a negatively charged wall (or probe) which acts as a sink for the charged particles that reach it. The geometry considered is plane and the wall is so negative that most of the electrons are reflected back into the plasma. Bohm made three assumptions: firstly, that ionization in the sheath is negligible, secondly that the electric field at the plasma boundary is very small and thirdly that the energy distribution of the positive ions may be neglected (i.e. mono-energetic ions are considered). In addition it is assumed that the electron distribution is Maxwellian so that their density can be described by the Boltzmann relation.

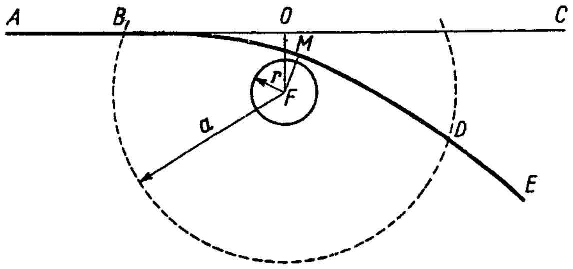

Fig. 1 Diagram of the path of a charged particle passing through a sheath surrounding a cylinder or sphere [1].

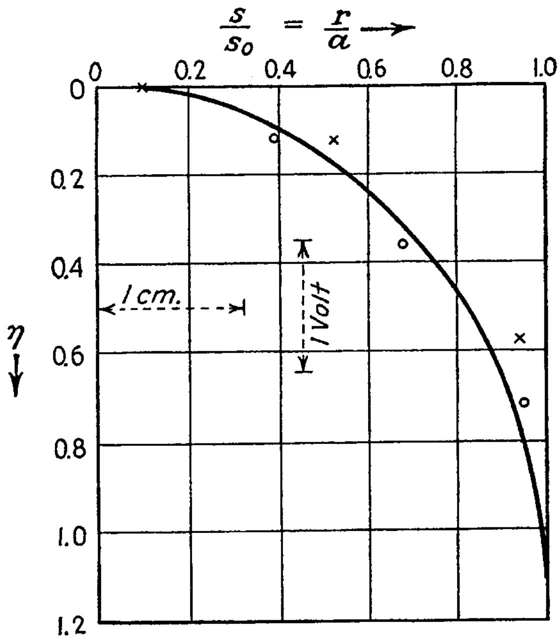

Fig. 2 Radial potential distribution for a low-pressure arc in a discharge tube. Experimental points are for a = 3.1 cm, T_e = 38,800 K, mercury vapour pressure = 0.2 × 10^{-3} mm Hg, ion free path = 31 cm [3].

The result found by Bohm is the following,

$$eV_0 \geq \frac{1}{2} kT_e$$

where eV_0 is the ion energy on entry to the sheath.
It is seen that the positive ions must have an energy of $\frac{1}{2} kT_e$ or more before a space charge sheath can form. This result is known as the Bohm criterion for sheath formation. This kind of result was first found in 1929 by Tonks and Langmuir, but not in such a concise form. Indeed Langmuir stated in 1923 [5] that the potential at the sheath edge is - 1 volt if the electrons have an energy of 1 volt!

The above criterion contains an inequality sign but it can be shown, by performing a plasma calculation rather than a sheath calculation, that an equality sign is valid in many cases.

1.3 The Work of Caruso and Cavaliere

By using the technique of the stretching of variables Caruso and Cavaliere [6] extended the theory of Tonks and Langmuir (for the plane case) by considering both the plasma region and the sheath region of space. The theory applies when the ratio of the Debye distance to the size of the plasma is vanishingly small. We are dealing here with a mathematical model, but one that is very useful and sufficiently accurate for many purposes.

Figures 3 and 4 show the potential distribution in the plasma and in the sheath, respectively. In the first case we can interpret the result as showing that the electric field at the plasma boundary is large compared with the average field in the plasma. The second diagram, figure 4, illustrates that the electric field at the plasma boundary is small compared with the average field in the sheath. In these calculations, as in those of Tonks and Langmuir, the ions have a distribution of energies corresponding to their different points of origin. In the plane case it turns out that the potential at the plasma boundary, $-0.854\ kT_e/e$, is independent of the ionization process, e.g. single stage or multi-stage ionization.

1.4 The Plasma Boundary as a Mach Surface

If one considers cold ions, a quasi-neutral plasma can be formed in a converging system in the absence of local ionizing processes. This differs from the case in which ions are produced locally [2,6]. Assuming that the electrons have a Maxwellian distribution, and that the electric field is conservative, one can readily obtain the following equation

$$\nabla^2 \phi = \frac{1}{2a^2} \nabla\phi \cdot \nabla(\nabla\phi \cdot \nabla\phi)$$

This equation is mathematically equivalent to the fundamental equation for compressible flow with a being the sound speed (Bohm speed). Thus we can adopt results from the theory of fluid mechanics and apply them to the problem of the plasma boundary. In particular we can utilize the concept of the Mach surface. In a region of supersonic flow the Mach surface is such that the fluid velocity component normal to the surface is equal to the sound speed a. Following this line of reasoning Stangeby and Allen [7] identified the plasma sheath boundary as a closed Mach surface. An application of this idea was to the flow of plasma around a Langmuir probe; some of the plasma ions are absorbed by the probe, after travelling through the

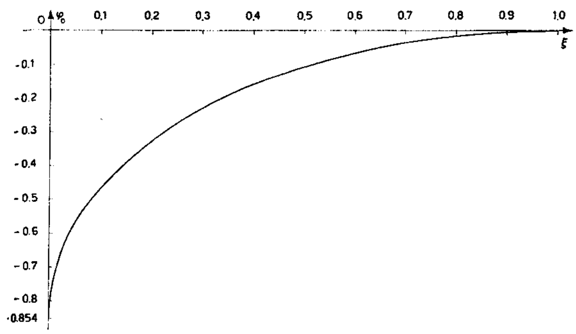

Fig. 3 The plasma potential distribution for the Langmuir model [6].

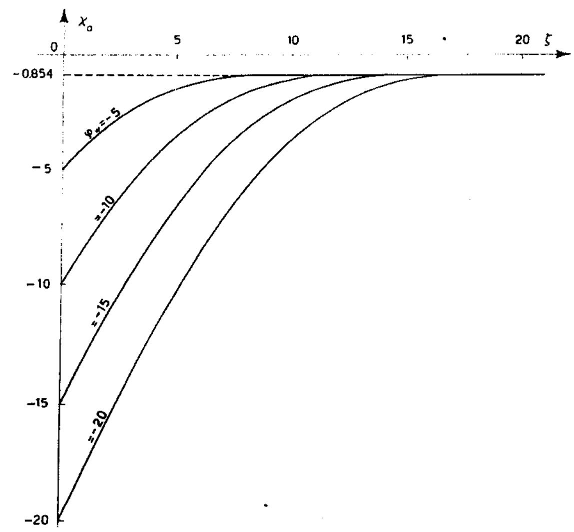

Fig. 4 The sheath potential distribution [6].

sheath, and some of them are deflected but not captured by the probe.

1.5 The Generalized Sheath Criterion

One can generalize the sheath criterion to allow for a distribution of ion energies. The first attempt at such a derivation was made by Boyd [8] and a later study was made by Harrison and Thompson [9]. The result is the following

$$\frac{1}{2} M \left\langle v^{-2} \right\rangle^{-1} = \frac{1}{2} kT_e$$

where we have assumed the equality sign since, as briefly discussed above, this can be justified by approaching the plasma-sheath boundary from the plasma side. It is clear that the criterion is *not* one simply concerned with energy; it is concerned instead with the ion velocity distribution function. The distribution function is not specified by the sheath criterion, but it must be of a form that satisfies the criterion.

By considering the propagation of longitudinal waves in a plasma Allen [10] was able to show that the above criterion corresponded to a quasi-neutral wave with a zero phase velocity in the laboratory frame of reference, i.e. an ion acoustic wave is unable to travel back into the plasma. Thus once again we arrive at the concept of *sonic flow* at the plasma boundary. It was verified that the criterion held for all of the different cases studied by Tonks and Langmuir [2], and for the spherical probe theory of Bohm et al [11].

1.6 Extension of the Bohm Criterion to a Mixture of Different Ions.

The extension of the Bohm criterion to a mixture of different ions is straightforward and was derived by Cooke in a D. Phil. thesis [12]. A convenient method to obtain the result is to consider once again the propagation of longitudinal waves (of long wavelength) in a plasma, and simply allow for different ion populations. In such waves the variation of the total ion space charge follows the variations in electron density, thereby maintaining quasi-neutrality in the plasma. The result is that, at the sheath edge,

$$\frac{n_1 Z_1^2}{n_e v_1^2 M_1} + \frac{n_2 Z_2^2}{n_e v_2^2 M_2} = \frac{1}{kT_e}$$

where the ion and electron densities are the local ones.

In some cases

$$\frac{1}{2} M_1 v_1^2 = \frac{1}{2} Z_1 kT_e$$

and

$$\frac{1}{2} M_2 v_2^2 = \frac{1}{2} Z_2 kT_e$$

but these latter results are *not* general. The first criterion above appears to be less useful than that for a single ion species. When any particular plasma model is developed the criterion is found to hold at the plasma boundary, but one cannot use it in a completely independent sheath model. One must carry out a plasma calculation first, in order to determine the ion densities and ion velocities appearing on the LHS of the equation.

1.7 The Bohm Criterion in the Presence of Negative Ions

Many plasmas used in plasma processing contain electronegative gases; for this reason the subject of plasmas that have a mixture of positive and negative ions is of considerable interest. Since the plasma is quasi-neutral there is a coupling between the different ion species; this results in a changed Bohm criterion. Early work on the subject was carried out by Boyd and Thompson [13], but they did not discuss the multiple solutions that are found in the mathematical model. Braithwaite and Allen considered the problem, being unaware of the earlier work, and obtained the results shown in Figure 5 [14]. The resulting ion energy required for sheath formation is decreased in the

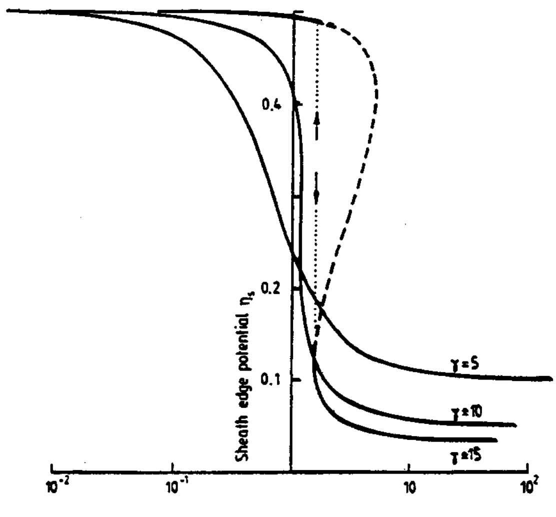

Fig. 5 Plot of the normalized sheath edge potential against the ratio of the negative-ion density to electron density in the bulk plasma for three values of gamma, the ratio of electron temperature to negative-ion temperature. The arrowed dotted line indicates the necessary jump in the solution [14].

presence of negative ions. When the population of the latter is very high the temperature of the negative ions replaces that of the electrons in the Bohm criterion. In general the required ion energy is between the electron temperature and the negative ion temperature, measured in eV. When the ratio of the electron temperature to the negative ion temperature exceeds a certain value multiple solutions are found. Braithwaite and Allen proposed that, in this case, the lower value should be employed, as shown in Figure 5. The subject is one of further investigation at the present time.

1.8 The Bohm Criterion in the Presence of Radio Frequency Fields

It has been shown that the criterion remains valid in the case of capacitively coupled RF plasmas, the *longitudinal* RF fields being largely screened out of the plasma [15,16]. The criterion is a very general one, for low-pressure plasmas, the only exception known to the present writer being that of the thermally produced Caesium plasma [17].

2. Double Layers

Space charge layers form not only at solid surfaces, but can form between one plasma and another. The term *double sheaths* is to be found in early papers on the subject, but the terminology changed and the term *double layers* took its place. One example of a double layer separating two plasmas is shown in Figure 6; in this case the double layer effectively increases the collecting area for electrons, directing them towards the constriction in the discharge tube. Calculations of potential and space charge within such a double layer are shown in Figure 7 [18]. Once again the ions enter the space charge region with a certain energy that could be described by a modified Bohm condition.

A discharge tube used in Oxford to study double layers is shown in Figure 8. In this case the vapour pressure was carefully controlled and sharp constrictions in the tube were avoided. The double layer was then observed in the middle of the discharge tube, separating two different plasma regions. Probe measurements showed that in fact a *triple layer* was formed, some typical results are given in Figure 9 [19]. The term *double layer* has been retained, in common usage, to include multiple layers of space charge, a potential difference existing across the assembly of layers.

A diagram illustrating the motion of ions and electrons in such a triple layer is shown in Figure 10. It is seen that five groups of particles are involved, the negative dip reflecting some of the electrons coming from the LHS. Figure 11 is an example of a numerically simulated profile based on this picture [20].

Figure 12 is a schematic diagram showing the position of an ion energy analyser placed in the wall of the discharge tube. Measurements showed that ion energies of tens of volts were acquired on the anode side

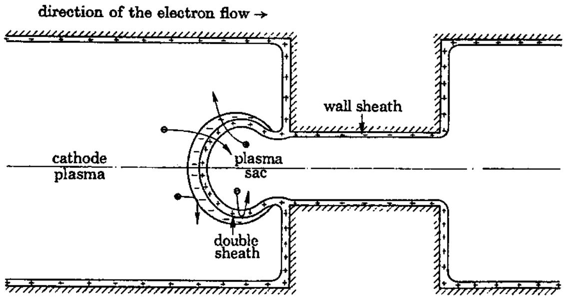

Fig. 6 Double sheath at a constriction in a gas-discharge tube [18].

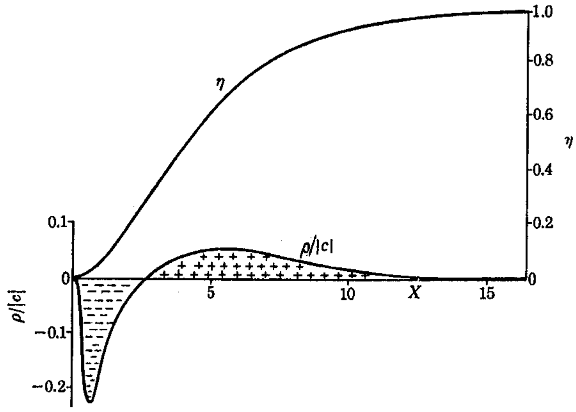

Fig. 7 Spatial variation of normalized charge density and normalized potential, for a certain set of parameters [18].

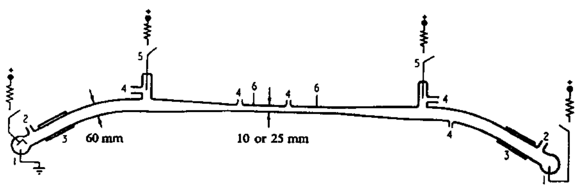

Fig. 8 Discharge tube: 1, Hg pool electrode; 2, port to diffusion pump; 3, cooling jacket; 4, ionization (pressure) gauge; auxiliary electrode; 6, Langmuir probe. The overall length is 5 m [19].

of the layer (the double layer conveniently drifted past the analyser in these experiments). The energies here were associated with the velocity component normal to the tube wall. I shall not attempt to explain the results in the present paper. There is no mystery about the source of the energy, however, it derives from that gained by the electrons that cross the layer from the cathode side to the anode side.

Figure 13 shows the ratio of electron drift velocity to thermal velocity as the pressure is lowered (moving

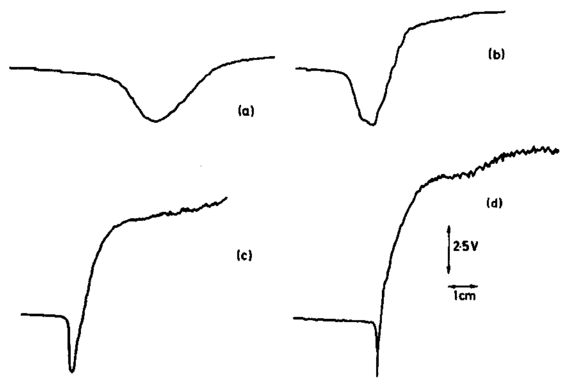

Fig. 9 The axial distribution of floating potential throughout the triple layer at different distances from the wall: (a) 0 mm (at the wall); (b) 2 mm; (c) 10 mm; (d) 13.5 mm (centre); pressure 1.2 × 10^{-4} torr. The triple layer slowly drifted past the probe [19].

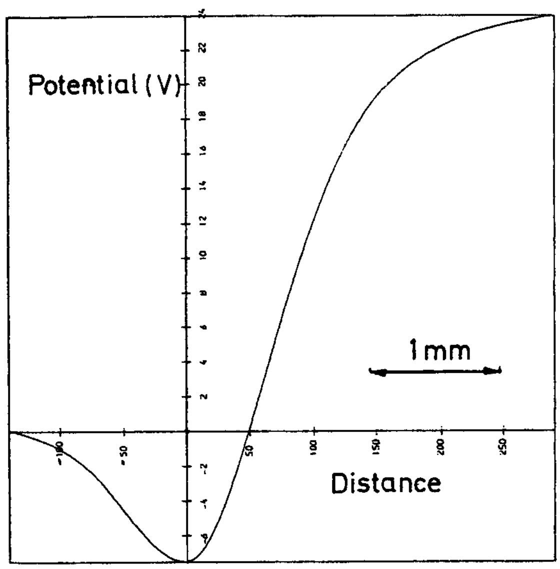

Fig. 11 Numerically simulated potential profile [20].

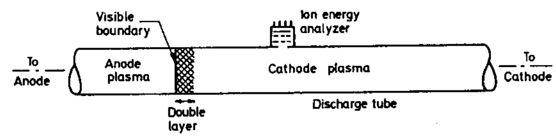

Fig. 12 Schematic diagram showing the ion energy analyser. The double layer slowly drifts from left to right [19].

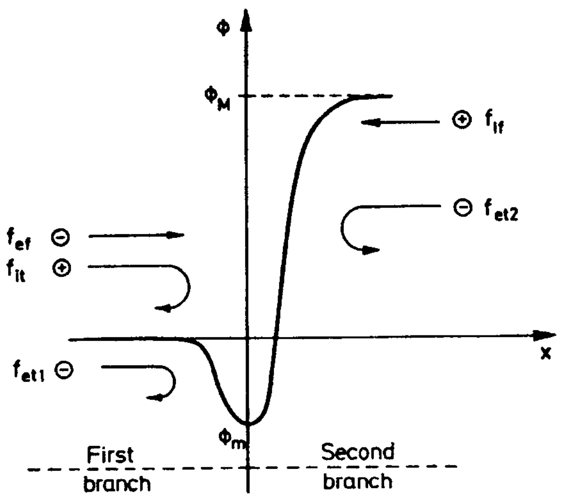

Fig. 10 Schematic plot showing five groups of particles. Subscripts refer to electron (e) or ion (i), free (f) or trapped (t), in the first (1) or second branch (2) [20].

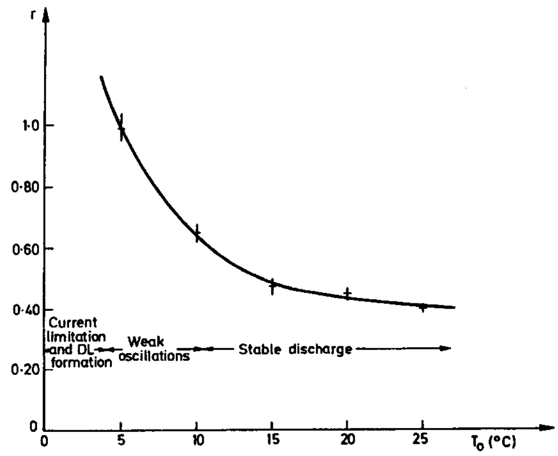

Fig. 13 Variation of the ratio between the electron drift velocity and the thermal velocity as the pressure is reduced by lowering the temperature of the cooling jackets. The discharge current is held at 5.6 A [19].

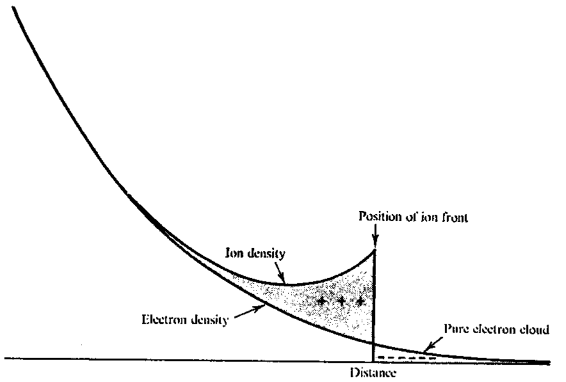

Fig. 14 Ion and electron densities at the front of a plasma expanding into a vacuum [22].

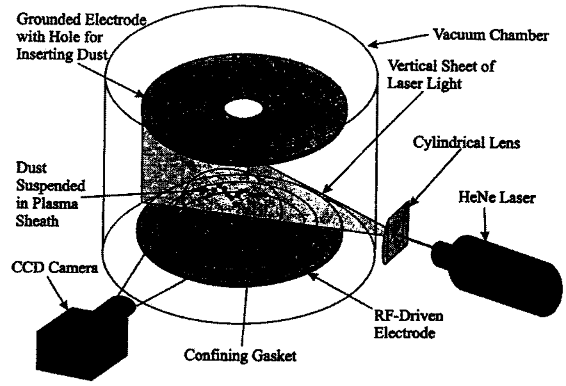

Fig. 15 Experimental set-up to study damped oscillations of dust particles in a sheath [25].

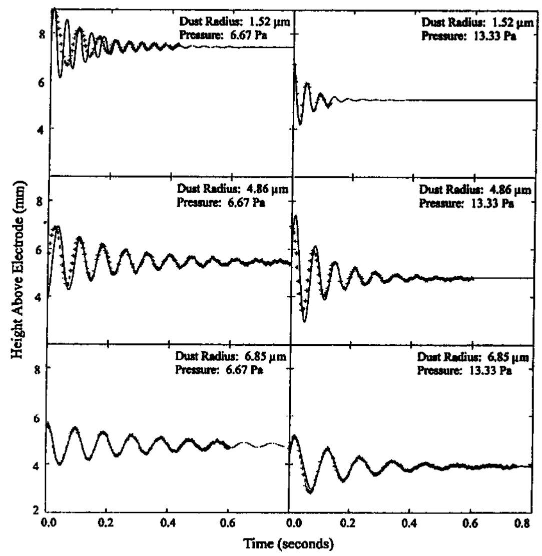

Fig. 16 Dust oscillation data and harmonic theory; the full lines are theoretical damped harmonic curves. Each column of plots is for the same pressure, while each row of plots is for the same particle. Typical position errors are of the order of 10 microns, not visible on this scale [25].

from right to left) at a fixed current. The "double layer" appears when this ratio approaches unity; this result is in agreement with simulations performed by De Groot et al [21].

Another example of a double layer in space, not adjacent to a solid surface, is shown in Figure 14. This diagram illustrates the electron and ion densities at the boundary of a plasma expanding into a vacuum [22]; quasi-neutrality is again approached behind the "front". The field existing in the double layer accelerates a small number of ions to high energies.

3. Dust Levitation

Another field of research in which the sheath is of great importance is that of dust levitation. The subject of "dusty plasmas" has attracted a great deal of interest in the last few years; one reason for the surge of interest being the discovery that "plasma crystals", formed of charged dust particles, could readily be formed in laboratory experiments [23,24]. In many experiments, however, dust particles of a few microns in diameter are pulled out of the plasma by their weight, and subsequently reside in the space charge sheath situated below the plasma. In the sheath the strong electric field produces a force acting on the dust particle that can balance its weight. Needless to say, many of the theoretical papers on dusty plasmas consider a neutral plasma and not a space charge sheath!

Figure 15 illustrates an experimental set-up that has been used to determine the charge residing on a dust particle [25]. Dust particles are dropped into an RF plasma and then oscillate around their equilibrium positions in the sheath. The particles are illuminated by a HeNe laser and their trajectories are recorded using a CCD camera. Some results are shown in Figure 16 for particles of different sizes. Much of the trajectory can be fitted to the curves describing simple damped harmonic oscillations, as shown in the diagram. This means that the electric potential in the sheath must be almost parabolic over much of the sheath. The total potential including the gravitational potential is illustrated in Figure 17. The curve is, of course, a parabola, the minimum being where the equilibrium position of the particle is situated. A number of different sheath models have been examined, that shown in Figure 18 being only

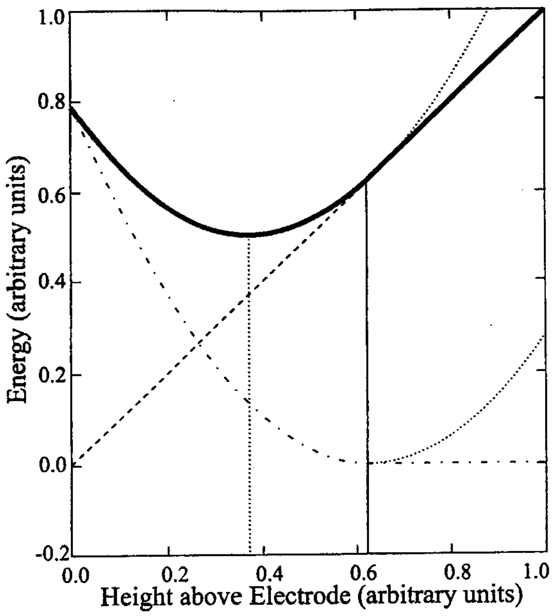

Fig. 17 Schematic diagram showing the parabolic total potential resulting from the addition of a linear gravitational potential and a parabolic electrical potential. The location of the sheath edge is shown by the vertical full line and the equilibrium position of the dust is indicated by the vertical dotted line. In this model the electric potential (dot-dash curve) is constant to the right of the sheath edge [25].

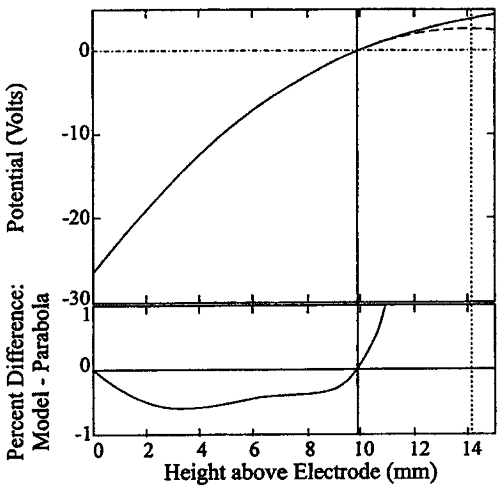

Fig. 18 Diagram showing the close fit between a parabola and a model of the sheath [25]. The horizontal (dot-dash) line shows the potential at the plasma boundary, allowing for a finite field at the boundary. The lower curve shows the difference between the model and the fitted parabola, i.e. less than 1%.

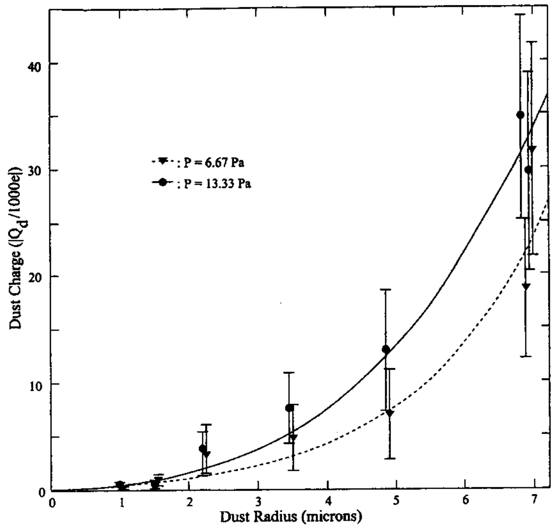

Fig. 19 Values of the dust charge as a function of particle radius. The data are for melamine formaldehyde particles suspended in an argon plasma-sheath [25].

one example [26]. It is most remarkable that, over much of the sheath, a parabolic fit is an excellent approximation. In the example shown the fit is to within 1%. We must not deduce that the ion density is uniform, perhaps we can introduce the idea of *quasi-uniformity* of charge density as an analogy of quasi-neutrality in the plasma.

Figure 19 shows some results obtained for melamine formaldehyde particles in an argon plasma [25]. It is seen that the charge on a dust particle is of the order of 10,000 electronic charges, for radii of the order of microns, and that it is a non-linear function of the particle radius. The latter fact illustrates that the floating potential is greater for the larger particles, assuming that the effective capacitance of the dust particle is that of a sphere in a vacuum. A comparison of the experimental results with existing theories is shown in Figure 20, the curves showing the ratio of the experimental value to the theoretical value. The theories employed here are the ABR (radial motion) theory [27] and the OML (orbital motion limited) theory [28]. The agreement is better with the ABR theory. It should be remembered, however, that these theories were developed for probes in a neutral plasma; what is now required is a viable theory of charging in the sheath. The problem was first considered by Nitter [29].

An alternative method of determining the charge residing on a dust particle is that of simply using the balance between the electric and gravitational forces [26]. The data used are the dust mass and the observed suspension height; the same model of the sheath is then employed to determine the electric field at the position of the dust. Some results are shown in Figure 21, together with those obtained using the previous method. It is seen that there is good agreement with the damped oscillation method of determining the charge. It is perhaps surprising that this simple method of determining the charge on a dust particle, in a space charge sheath, does not appear to have been used before.

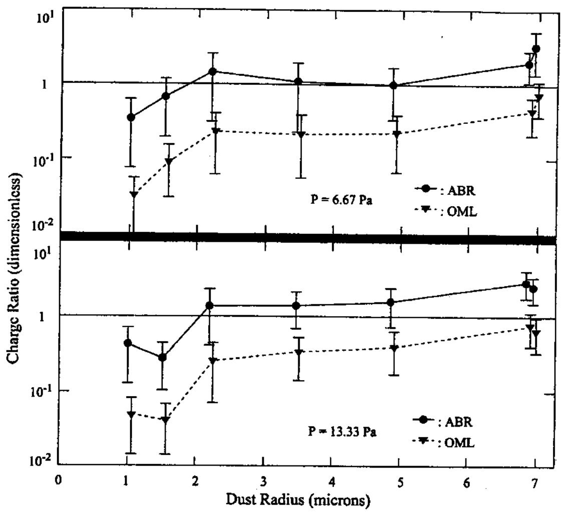

Fig. 20 Comparison of the measured dust charge with that obtained from the ABR and OML (plasma) theories. The plots show the ratio of the experimental value to the theoretical value [25].

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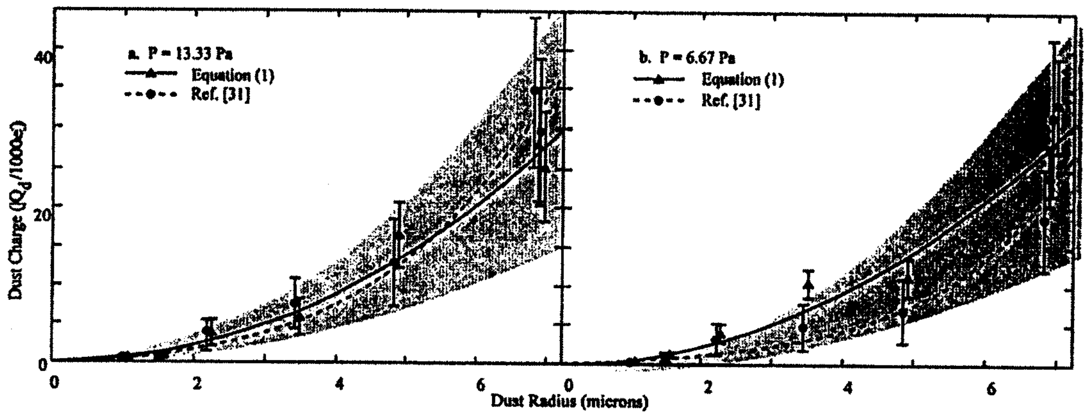

Fig. 21 Values of the dust charge determined by two different methods. The points labelled [31] refer to the method of damped ocillations, whereas those labelled (1) were obtained from the simple force balance between the gravitational and electric forces [26].

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