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Velocity Distribution in the Edge Region of a Steady State Field-Reversed Configuration

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Abstract

Considering collisions and adiabaticity breaking processes at X-points of a field-reversed configuration (FRC), we derive the kinetic equation for the edge plasma. Velocity distributions of ions and electrons are obtained in a case that collisions alone are considered. We also propose the algorithm to determine the ambipolar electrostatic potential at mirror points.

Keywords:

field-reversed configuration, peripheral plasma, velocity distribution, end loss, ambipolarity

1. Introduction

Field-Reversed Configuration (FRC) is one of the most attractive alternative confinement plasmas to tokamaks, which has no toroidal field and consequent relatively high beta value. Main mechanism of particle transport between FRC bulk and edge region (outside the separatrix) has not been proved so far by both theoretical and experimental studies [1-5]. In terms of transport in the edge plasma, particles are trapped in the open mirror field, and those mirror loss rates are found recently to relate with the ambipolar electrostatic potential, which affects confinement of bulk plasmas [6,7]. Since FRCs have, however, field-null X-points on axis and separatrix, the particles suffer from collisionless pitch angle scatterings and resultant adiabaticity breaking processes [8,9], which enhance the mirror loss rate of ions. Thus, it is important to investigate physics of collisionless pitch angle scattering at X-points and to formulate the kinetic equation for edge plasmas.

In this paper, we concentrate on the kinetic properties of the edge plasma particles. Section 2 is devoted to explain the particle transport and loss processes, including in the kinetic equation. In Sec. 3, we present results and discussions, and finally we summarize this paper in Sec. 4.

2. Particle Transport in Edge Region

Plasma particles inside the separatrix are diffused out due to the cross-field transport, whose dominant mechanism for FRC, however, have been unproved as yet. After crossing the separatrix, particles move toward X-points because of $-\vec{\mu} \cdot \nabla B$ force. Therefore, particles in open-line field have reciprocating motion around X-points. If particles move adiabatically, those entering the loss cone are passing through the mirror points and the others are trapped in open field. Collisions and collisionless pitch angle scatterings at X-points, however, make particles move nonadiabatically. Thus particles with large μ initially suffer from the pitch angle scattering and consequently are lost away from the confinement system. This is discussed below.

2.1 Kinetic equation

Peripheral plasma particles have loss region in

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velocity space determined by the mirror ratio. So the velocity distribution becomes never isotropic Maxwellian. Thus we employ the kinetic theory to understand the particle transport in the edge region.

Almost all of plasma particles in the edge plasma are off-axis gyrating particles. In the absence of collisions and collisionless pitch angle scattering, the velocity distribution function averaged in time (enough long for gyration and enough short for axial bounce motion) is found for the axisymmetric plasma to be

$$f(H, J, P_\theta, \psi, \chi)=f(H, J, \bar{\psi}), \tag{1}$$

where H, J, P_θ, ψ, and χ denote the Hamiltonian, the radial action integral, the canonical angular momentum, the magnetic flux function, and the stream function, respectively. The quantity $\bar{\psi}$ indicates the time averaged flux function which is regarded as the guiding center. After this, we simply describe it ψ. The detailed derivation of the above form is in [10]. Velocity distribution $f(H, J, \psi)$ in Eq. (1) is the general solution to the zeroth order equation for perturbations. Explicit form of $f(H, J, \psi)$ is obtained by considering first order equation to collisions and collisionless pitch angle scattering. Steady state velocity distribution is obtained by

$$\left(\frac{\mathrm{d}f}{\mathrm{d}t}\right)=\left(\frac{\mathrm{d}f}{\mathrm{d}t}\right)_{\mathrm{col}}+\left(\frac{\mathrm{d}f}{\mathrm{d}t}\right)_{\mathrm{ABP}}, \tag{2}$$

where the first term on right-hand side designates the collision term and the second one represents the term concerned with adiabaticity breaking processes. Each term will be described in the following section.

2.1.1 Collision

We employ, for simplicity, linearized Landau form for the collision term. According to Ref. [10], gyration time averaged collision terms are given as

$$\left(\frac{\mathrm{d}f^i}{\mathrm{d}t}\right)_{\mathrm{col}}=\frac{q^2}{8\pi\varepsilon_0^2}\frac{\lambda}{2W\sqrt{2WM}}\Bigg\{2n_i q^2 TW\frac{\partial}{\partial W}\left[\eta(x)\left(\frac{\partial}{\partial W}+\frac{1}{T}\right)\right] + n_i q^2\left[(1-1/2x)\eta(x)+\eta'(x)\right]\frac{1}{\cos\theta_p}\frac{\partial}{\partial\theta_p}\cos\theta_p\frac{\partial}{\partial\theta_p} + \frac{2n_e e^2}{3}\sqrt{\frac{mW}{\pi MT}}\left(\frac{4}{\cos^2\theta_p}-1\right)\frac{r^2MW\cos^2\theta_p}{q^2}\frac{\partial^2}{\partial\psi^2}\Bigg\}\times f^i(W,\theta_p,\psi) \quad \text{: for ions} \tag{3}$$

$$\left(\frac{\mathrm{d}f^e}{\mathrm{d}t}\right)_{\mathrm{col}}=\frac{e^2}{8\pi\varepsilon_0^2}\frac{\lambda}{2W\sqrt{2Wm}}\Bigg\{2n_e e^2 TW\frac{\partial}{\partial W}\left[\eta(x)\left(\frac{\partial}{\partial W}+\frac{1}{T}\right)\right] + n_e e^2\left[(1-1/2x)\eta(x)+\eta'(x)\right]\frac{1}{\cos\theta_p}\frac{\partial}{\partial\theta_p}\cos\theta_p\frac{\partial}{\partial\theta_p} + n_i q^2\left(\frac{2}{\cos^2\theta_p}-1\right)\frac{r^2mW\cos^2\theta_p}{e^2}\frac{\partial^2}{\partial\psi^2}\Bigg\}\times f^e(W,\theta_p,\psi) \quad \text{: for electrons} \tag{4}$$

where W and θ_p denote the kinetic energy of particle concerned and the pitch angle at the midplane, and where q, e, M, m, T, n_i, n_e, and λ are the ion and electron charge, the ion and electron mass, the plasma temperature, the ion and electron density, and Coulomb logarithm, respectively. The pitch angle θ_p is related with the radial action integral J in the form

$$J=\frac{H-q\phi(\psi)}{\Omega}\cos^2\theta_p$$

where Ω denote the gyro-frequency at midplane, and $\phi(\psi)$ is the electrostatic potential at the midplane. Note that Eqs. (3) and (4) describe how the velocity distribution near the midplane is deformed due to collisions. Though the pitch angle θ_p varies in order to keep the adiabatic invariance J in slowly varying magnetic field, the values of θ_p in Eqs. (3) and (4) are mapped on the velocity space at the midplane and are kept to be constant without collisions. Quantities x and η are defined as

$$x\equiv\frac{M_sW}{M_jT_s}, \quad \eta(x)\equiv\frac{2}{\sqrt{\pi}}\int_0^x t^{1/2}\exp(-t)\,dt.$$

Subscripts j and s indicate the particles concerned and the background particles. The important point of Eqs. (3) and (4) is the fact that collisional change in velocity is transformed into the shift of the guiding center ψ by the time average.

2.1.2 Adiabaticity breaking processes

Compared with an ion, an electron has a small gyro-radius, and thus the electron is hard to suffer the adiabaticity breaking processes. We assume now only ions are subject to this effect of X-points. The term concerned with adiabaticity breaking processes is given with use of Boltzmann collision integral form:

$$\left(\frac{\mathrm{d}f}{\mathrm{d}t}\right)_{\mathrm{ABP}} = \int_0^{J_{\max}} \frac{P^f(H,J,\psi\,|\,J'\to J)f(H,J',\psi)}{\Delta\tau_b^f(H,J',\psi)}dJ' + \int_{J_c}^{J_{\max}} \frac{P^o(H,J,\psi\,|\,J'\to J)f(H,J',\psi)}{\Delta\tau_b^o(H,J',\psi)}dJ' - \int_{J_c}^{J_{\max}} \frac{P^f(H,J,\psi\,|\,J\to J')f(H,J,\psi)}{\Delta\tau_b^f(H,J,\psi)}dJ' - \int_0^{J_{\max}} \frac{P^o(H,J,\psi\,|\,J\to J')f(H,J,\psi)}{\Delta\tau_b^o(H,J,\psi)}dJ' \quad (5)$$

where $P(H, J, \psi|J' \to J)$ and $\Delta\tau_b(H, J, \psi)$ indicate the transition probability of the radial action integral and the bounce time of reciprocating motion, respectively. Superscripts o and f represent the bounce motion from a mirror point to an X-point and the one from the midplane to the X-point, respectively. The values J_{max} and J_c are defined as follows:

$$J_{\max} = \frac{H - q\phi(\psi)}{\Omega}, \qquad J_c = \frac{H - q\phi_m(\psi)}{\Omega_m}$$

where Ω_m denote the gyro-frequency at the mirror points, and $\phi_m(\psi)$ is the electrostatic potential at the mirror points. The probabilities are normalized to obey

$$\int_0^{J_{\max}} P(H,J,\psi\,|\,J'\to J)\,dJ' = 1\,,$$

$$\int_0^{J_{\max}} P(H,J,\psi\,|\,J\to J')\,dJ' = 1\,. \quad (6)$$

Then last two terms on right-hand side in Eq. (5) become

$$-\alpha(H,J,\psi)\frac{f(H,J,\psi)}{\Delta\tau_b^f(H,J,\psi)} - \frac{f(H,J,\psi)}{\Delta\tau_b^o(H,J,\psi)}\,. \quad (7)$$

We define $\alpha(H, J, \psi)$ as the trapping probability:

$$\alpha(H,J,\psi) \equiv \int_{J_c}^{J_{\max}} P(H,J,\psi\,|\,J\to J')\,dJ'\,. \quad (8)$$

In order to obtain the explicit Boltzmann collision integral form in Eq. (5), the transition probability of the adiabatic invariant should be given. Unfortunately, no data on collisionless scattering is available except from our previous work [8] where the transition probabilities have not been calculated. We calculate them numerically and present them in Sec. 3.

2.2 Ambipolarity

Electrons are lost rapidly into the loss region because of the higher collision frequency. Thus negative electrostatic potential is created at the mirror point in a case that the one is set to be zero at the midplane, in order to equalize the loss rate of ions and electrons. If there exists electrostatic potential, the boundaries of loss region in velocity space become a hyperbolic curves. Moreover, plasmas must satisfy the quasineutrality condition. So, the ambipolar potential is obtained from two conditions: 1) Plasma must satisfy the quasineutrality. 2) Loss rates of ions and electrons are equal.

Let us suppose that edge electrons are lost away due to collisions, on the other hand, dominant ion loss mechanism is the adiabaticity breaking processes. The first ambipolar condition is

$$N^i(\psi) = N^e(\psi)\,. \quad (9)$$

Quantities $N^i(\psi)$ and $N^e(\psi)$ are defined by

$$N^i(\psi) = \int_{H_0}^{\infty}\int_{J_c}^{J_{\max}} f^i(H,J,\psi)D\,\mathrm{d}H\mathrm{d}J\,,$$

$$N^e(\psi) = \int_{q\phi(\psi)}^{\infty}\int_{J_c}^{J_{\max}} f^e(H,J,\psi)D\,\mathrm{d}H\mathrm{d}J\,,$$

$$H_0 \equiv q\phi(\psi) + \frac{q\phi(\psi) - q\phi_m(\psi)}{R-1}\,,$$

$$D = \frac{M_j}{2B_w^2 r_s^2\sqrt{J}\sqrt{J_{\max}-J}}\,, \quad (10)$$

where R, r_s and B_w denote the mirror ratio, the separatrix radius and the magnetic field on midplane, respectively. The next condition is given:

$$\Gamma^i(\psi) = \Gamma^e(\psi)\,. \quad (11)$$

The ion loss rate $\Gamma^i(\psi)$ is calculated by

$$\Gamma^i(\psi) = \int_{H_0}^{\infty}\int_0^{J_c} -(1-\alpha(H,J,\psi))\frac{f(H,J,\psi)}{\Delta\tau_b^f(H,J,\psi)}D\,\mathrm{d}H\mathrm{d}J\,. \quad (12)$$

The electron loss rate is shown in Ref. [10]. To follow the ambipolar condition, $\phi(\psi)$ and $\phi_m(\psi)$ are controlled to satisfy Eqs. (9) and (11).

3. Preliminary Results and Discussions

At the beginning, we calculate the transition probability numerically to evaluate Eq. (5). Using particle tracing routine with monte-carlo method, we calculate an example of transition probability in Fig. 1.

The particles start with the same J initially and are put in different gyro-phases. Then those suffer the change in J after passing through X-points. We calculate the histogram of the transition probability of the scattered J, which corresponds to $P(H, J, \psi|J \rightarrow J')$. Preliminary calculation shows the small J given initially is converted into the larger one due to adiabaticity breaking processes, which implys that particles moving from the midplane bounce axially near the X-points. Some steep peaks are observed in Fig. 1. According to the trace of orbit and the calculation of Lyapunov's exponents, the peaks result from the fact that neighboring particles in initial gyro-phase gather round until they reach X-point. In order to describe the particle transport due to adiabaticity breaking processes, we have to obtain three dimensional dependences of $P(H, J, \psi|J \rightarrow J')$, thus this is the hard task and remains for the future study.

If we suppose collisions alone as the transport mechanisms, we are able to obtain the velocity distributions by solving Eqs. (3) and (4) under the given boundary conditions. We assume $f = 0$ in the loss region in velocity space, then the boundary conditions are given as the the hyperbolic curves. The solutions to Eqs. (3) and (4) are presented in Figs. 2 and 3. In figures, v_{th} denote the thermal velocity, i.e., $v_{th} \equiv \sqrt{2T/m_j}$. Self-consistent velocity distributions and ambipolar potential that satisfy Eqs. (9) and (11) will be presented in a subsequent paper.

For FRC, velocity distributions in the edge region become non-Maxwellian, which cause loss-cone instability. We are interested in its effect on transport and stability.

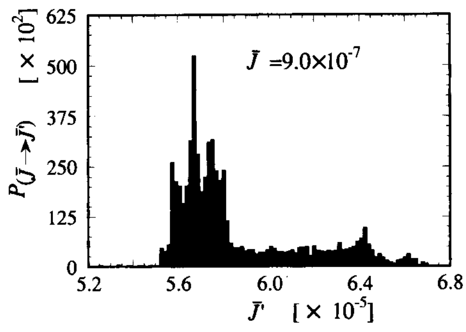

Fig. 1 Transition probability of radial action integral J, which is normalized by $qr_s^2B_w$, i.e. $\bar{J} = J/(qr_s^2B_w)$. The initial J is set in a case that $\bar{J} = 9.0 \times 10^{-7}$.

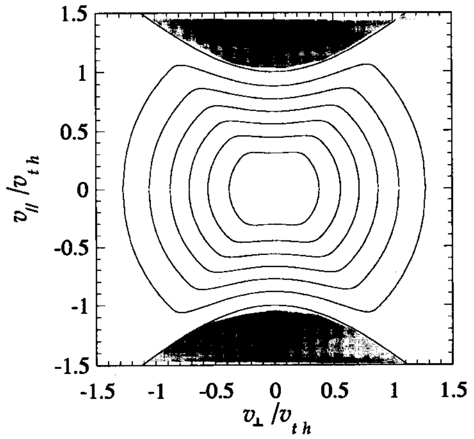

Fig. 2 Contours of electron distribution in the edge region for $|q\phi_m| = T$ in a case of the mirror ratio $R = 2$ [11].

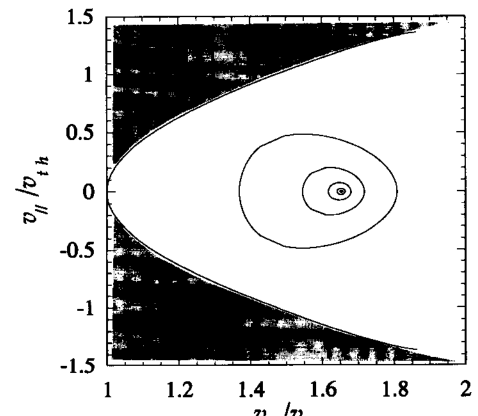

Fig. 3 Contours of ion distribution in the edge region for $|q\phi_m| = T$ in a case of the mirror ratio $R = 2$ [11].

4. Summary

We derive the kinetic equation averaged over gyration time for edge plasma particles in an FRC. This equation includes the collision term and the effect of adiabaticity breaking processes at X-points. The latter term is described with use of transition probability of the radial action integral, which is partially obtained with numerical calculation. It appears that transition probabilities form steep peaked shapes. Regarding collisions alone as the transport mechanisms, velocity distributions of ions and electrons are shown in this paper. Self-consistent ambipolar potential is remained as the subject for a future study.

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