J. Plasma Fusion Res. SERIES, Vol. 3 (2000) 601–604

Normal Modes and Their High-Frequency Instabilities in Unbounded and Bounded Systems with Magnetized Electron Beam

WATANABE Osamu and OGURA Kazuo

Graduate School of Science and Technology, Niigata University, Niigata, 950-2181 Japan

(Received: 18 January 2000 / Accepted: 4 April 2000)

Abstract

Normal mode and their high-frequency instabilities are analyzed self-consistently considering electron beams propagating along a magnetic field in unbounded and bounded systems. For unbounded dielectric system, the beam couples with electromagnetic waves corresponding to X and O modes, resulting in the slow cyclotron and Cherenkov instabilities, respectively. For cylindrically bounded system, a self-consistent field theory is developed. Axisymmetric and nonaxisymmetric normal modes obtained are hybrid modes. Cherenkov instability for the EH mode and slow cyclotron instability for EH and HE modes are confirmed in a cylindrical slow wave structure system.

Keywords:

slow cyclotron instability, cherenkov instability, hybrid mode, self-consistent analysis

1. Introduction

Plasma heating and production utilizing microwave power will play a key role in steady state operation of large-scale fusion devices such as large helical device. For these purposes, high-power slow wave devices have been studied extensively, because they can be driven by an axially injected electron beam and are particularly suited to operation with an intense electron beam. Most previous studies on the slow wave devices have aimed at the Cherenkov instability and are based on interaction between the electromagnetic normal modes in vacuum system and electron beam guided by an infinitely strong magnetic field. Recently, slow cyclotron maser has been demonstrated as a candidate for a useful slow wave device. In order to analyze the slow wave devices including slow cyclotron maser, a finite strength field has to be considered self-consistently. In this paper, we develop a self-consistent field theory and analyze normal modes and their instabilities for systems with magnetized electron plasma or electron beam.

Corresponding author's e-mail: wata@cocoa.ocn.ne.jp

2. Normal Modes and High Frequency Instabilities in Unbounded System

We assumed that uniform cold beam neutralized by a fixed ion background is propagating with a velocity v_0 along a constant magnetic field B_0 in an unbounded dielectric system with dielectric constant ε_r.

The self-consistent dispersion relation has been derived and discussed by W.B. Case *et al.* [1]. It is sixth degree in ω and gives six normal frequencies. Two are electromagnetic modes and remaining four are electrostatic modes on the beam, those are fast space charge mode (FSCM), slow space charge mode (SSCM), fast cyclotron mode (FCM) and slow cyclotron mode (SCM).

The dispersion relation derived in reference [1] can be rewritten as the second order equation of $k_{B\perp}^2$,

$$a_4 k_{B\perp}^4 + a_2 k_{B\perp}^2 + a_0 = 0 \tag{1}$$

where, $k_{B\perp}$ is wave number of the perpendicular direction to B_0. Then, there are two normal modes having the following $k_{B\perp}$,

$$k_{B\perp}^2 = -\frac{a_2}{2a_4} \pm \frac{\sqrt{a_2^2 - 4a_0 a_4}}{2\left| a_4 \right|}. \qquad (2)$$

We designate these modes as O and X modes, which corresponds respectively to O and X modes in the limit of $v_0 \to 0$ [3].

The dispersion curves given by eq. (1) are shown in Fig. 1. The SSCM couples with O mode and results in Cherenkov instability. The SCM couples with X mode and results in slow cyclotron instability. When the guiding magnetic field is relatively weak, the slow cyclotron instability merges into Cherenkov instability [3]. At zero magnetic field, Cherenkov mode and slow cyclotron mode are degenerated.

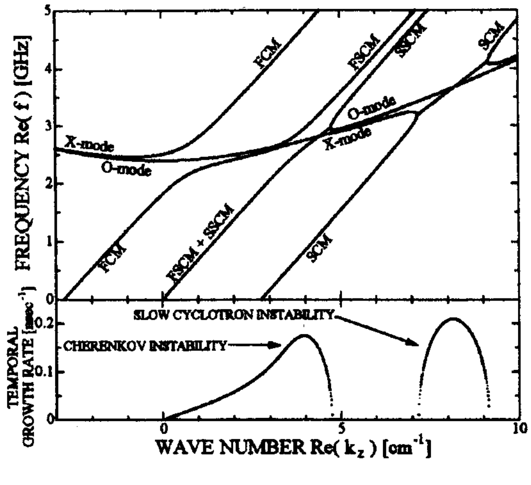

Fig. 1 The dispersion relation for unbounded system with electron beam.

3. Derivation of Dispersion Relation in Cylindrically Bounded System

We consider a cylindrical system shown in Fig. 2. A finite strength magnetic field B_0 is applied uniformly in the axial direction. From the linerized relativistic equation of electron motion under small signal conditions, the perturbed current $\boldsymbol{J}_1 = en_1 v_0 - en_0 v_1$ can be derived in the cylindrical system. Here, v_1 and n_1 are the perturbed electron velocity and density, respectively. The volume charge density $\rho_1 = -en_1$ is related to the perturbed current $\boldsymbol{J}_1$ by the continuity equation. Using the Maxwell equations, $\boldsymbol{J}_1$ and ρ_1 can be expressed by the axial components of the field, E_{1z} and B_{1z}.

We assume the axial field components as $E_{1z} = A_{EZ} J_m(k_{B\perp} r)$ and $B_{1z} = A_{BZ} J_m(k_{B\perp} r)$. By Maxwell equations with the source terms, we obtained the wave equation,

$$0 = \frac{k_{B\perp}^2}{\Delta} \frac{\omega_p^2}{\gamma c^2} \frac{1}{\omega''^2} \frac{\Omega c}{\gamma} \left(k_z - \frac{v_0 \omega}{c^2} \right) A_{EZ} - \left[\frac{k_{B\perp}^2}{\Delta} \left(\frac{\omega^2}{c^2} - k_z^2 \right) - \frac{k_{B\perp}^2}{\Delta} \frac{\omega_p^2}{\gamma c^2} \frac{\omega'^2}{\omega''^2} - 1 \right] A_{BZ} \qquad (3)$$

$$0 = \left[\frac{k_{B\perp}^2}{\Delta} \left(\frac{\omega^2}{c^2} - k_z^2 \right) \left(1 - \frac{\omega_p^2}{\gamma^3 \omega''^2} \right) - \frac{k_{B\perp}^2}{\Delta} \frac{\omega_p^2}{\gamma c^2} \frac{1}{\omega''^2} \left(\omega'^2 - \frac{\omega_p^2}{\gamma^3} \right) - \left(1 - \frac{\omega_p^2}{\gamma^3} \frac{1}{\omega'^2} \right) \right] A_{EZ} - \frac{\omega_p^2}{\gamma \omega''^2} \frac{k_{B\perp}^2}{\Delta} \frac{\Omega c}{c \gamma} \left(k_z - \frac{\omega v_0}{c^2} \right) A_{BZ} \qquad (4)$$

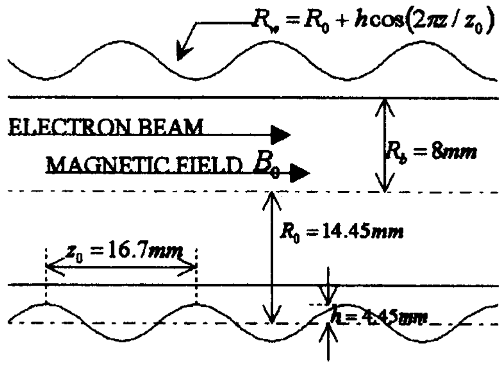

Fig. 2 The parameters of cylindrically bounded system.

where, $\omega' = \omega - k_z v_0$, $\omega''^2 = \omega'^2 - (\Omega/\gamma)^2$ and

$$\Delta = \left(\frac{\omega^2}{c^2} - k_z^2 - \frac{\omega_p^2}{\gamma c^2} \frac{\omega'^2}{\omega''^2} \right)^2 - \left(\frac{\omega_p^2}{\gamma c^2} \right)^2 \left(\frac{\omega'}{\omega''^2} \frac{\Omega}{\gamma} \right)^2. \qquad (5)$$

Here, ω and k_z are angular frequency and wave number in z-direction, respectively. And, cyclotron frequency Ω, speed of light c, plasma frequency ω_p, and relativistic factor γ are used.

From the condition that A_{EZ} and A_{BZ} are not zero simultaneously, we obtain the same second order equation of $k_{B\perp}^2$ as eq. (1). Hence, two possible normal modes in the cylindrical system are cylindrical O and X modes defined previously. The amplitudes of A_{EZ} and A_{BZ} are correlated to each other. The normal modes in magnetized electron beam have all field components and are hybrid modes. These modes are characterized by $A_{EZ}^{\pm}$ or $A_{BZ}^{\pm}$. Here, the sign of the superscripts corresponding to the sign of eq. (2).

The wave equation should be solved subject to the given boundary condition. At the beam surface, $r = R_b$, we obtain the following four independent equations,

$$E_{1z}^{out} - E_{1z}^{in} = 0 \tag{6}$$

$$B_{1z}^{out} - B_{1z}^{in} = -\mu_0 \kappa_{1\theta} \tag{7}$$

$$E_{1r}^{out} - E_{1r}^{in} = \sigma_1 / \varepsilon_0 \tag{8}$$

$$E_{1\theta}^{out} - E_{1\theta}^{in} = 0 . \tag{9}$$

Here, $\sigma_1 = -en_0 r_1$ is the surface charge density, r_1 is radial displacement of beam surface, and $\kappa_{1\theta}$ is the surface current density in the θ direction, the superscripts "out" and "in" mean outside and inside on the beam surface, respectively. The electromagnetic field out of the beam can be represented by $A_{EZ}^{\pm}$ or $A_{BZ}^{\pm}$ using eqs. (6)-(9).

At the waveguide wall, two electric fields, one is E_{1t} tangential to the wall in the r-z plane and the another is $E_{1\theta}$ in the θ direction, should be zero. In the periodic system, the normal mode has spatial harmonic waves with axial wave number $k_p = k_z + pk_0$, where p is harmonic number and $k_0 = 2\pi/z_0$. Then, the boundary conditions can be written as,

$$E_{1t} = \propto \sum_p E_{1z}(k_p R_w) + E_{1r}(k_p R_w) \frac{dR_w}{dz} \tag{10}$$

$$E_{1\theta} = \propto \sum_p E_{1\theta}(k_p R_w) . \tag{11}$$

These conditions can be represented by $A_{EZ}^{\pm}$ or $A_{BZ}^{\pm}$. From the condition that $A_{EZ}^{\pm}$ or $A_{BZ}^{\pm}$ are not zero, we can obtain dispersion relation for cylindrically bounded system [2-4].

4. Normal Modes and High Frequency Instabilities in Bounded System

Figure 3(A) shows the dispersion relation of axi-symmetric mode in straight cylindrical waveguide (h = 0) with magnetized plasma ($v_0 = 0$), and Fig. 3(B) is the non-axisymmetric one. Due to the effect of the plasma density, the waveguide mode's frequencies are up-shifted a little.

For a waveguide partially filled with an unmagnetized dielectric, it is well known that normal modes are TM and TE modes for axi-symmetric cases and become hybrid for non-axisymmetric cases. The hybrid modes are designated as EH and HE, in order to imply the hybrid nature consisting of TM and TE modes. Qualitatively, E_z is dominant in EH mode and H_z is dominant in HE mode. In Fig. 3, wave-guide and plasma modes become hybrid modes even in the

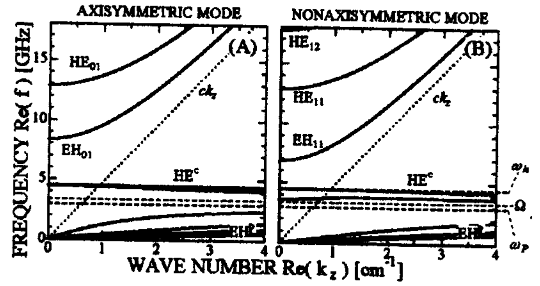

Fig. 3 The dispersion relations with straight cylindrical waveguide and zero beam velocity. The plasma frequency ω_p, cyclotron frequency Ω and upper hybrid frequency ω_h are plotted by broken lines. The dotted line ck_z is light line. The parameters are shown in Fig. 2.

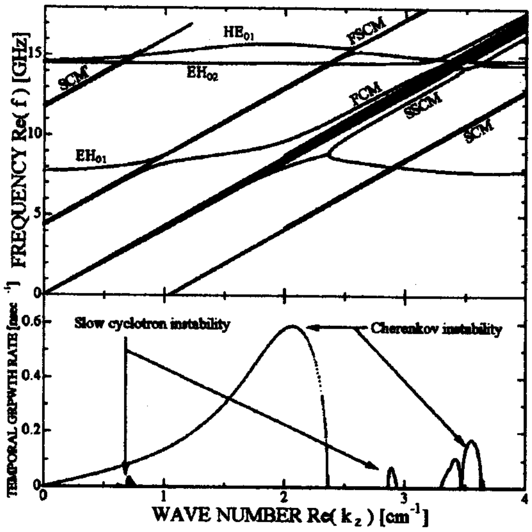

Fig. 4 The dispersion relation of axisymmetric mode for corrugated waveguide with electron beam energy 660 keV and B_0 = 0.35 T. The other parameters are the same as Fig. 2.

axisymmetric case. The hybrid cyclotron and plasma modes are designated as HE^c and EH^p modes, respectively.

For periodically corrugated waveguide, the dispersion relation is periodic in k_z-space with period k_0. Figure 4 shows this dispersion relation with magnetized beam in one period, k_z = 0 to k_0. Although there are many beam modes corresponding to Trivelpiece Gould modes, one of the SSCM coupled the EH_{01} and EH_{02} modes, and the SCM couples with EH and HE modes. The temporal growth rate due to SCM is much smaller than that of SSCM.

Magnetic field dependence of the temporal growth

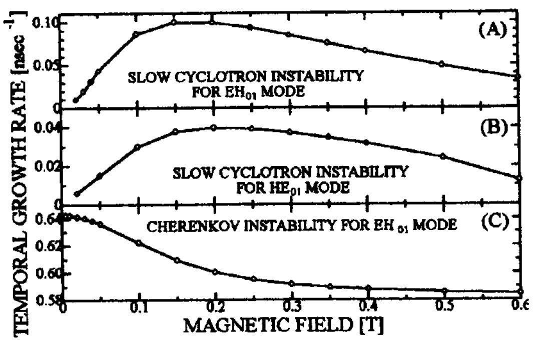

Fig. 5 Magnetic field dependence of temporal growth rates. The parameters other than B_0 are same as fig. 4.

rates is plotted in Fig. 5. The growth rate of slow cyclotron instability for EH_{01} mode, Fig. 5(A), has the maximum value 0.1 $nsec^{-1}$ at nearly 0.15 T and that for HE_{01} mode, Fig. 5(B), has the maximum value 0.04 $nsec^{-1}$ at nearly 0.2 T. These growth rates go to zero at B_0 = 0. The growth rate of Cherenkov instability for EH_{01} mode, Fig. 5(C), is nearly constant far from $B_0 = 0$, and increases to 0.64 $nsec^{-1}$ with decreasing the guiding magnetic field to zero.

5. Conclusion

Normal modes and their high frequency instabilities for systems with magnetized electron beam are analyzed self-consistently. In unbounded dielectric system, normal electromagnetic modes are O and X modes, which couple to SSCM and SCM, resulting in Cherenkov and slow cyclotron instabilities, respectively. In cylindrically bounded system, the normal modes are hybrid, EH and HE, modes. Cherenkov instability for the EH mode and slow cyclotron instability for EH and HE mode are confirmed. The temporal growth rates of the slow cyclotron instability depend on guiding magnetic field and have the maximum value at non-zero magnetic field. Decreasing the magnetic field to zero, the growth rate goes to zero. The temporal growth rate of the Cherenkov instability becomes maximum at B_0 = 0, and approaches constant value far from $B_0 = 0$.

Acknowledgements

The authors would like to acknowledge profoundly the helpful discussions with Dr. K. Okubo and Dr. T. Watanabe. The computation was supported by the Computer Center, National Institute for Fusion Science.

References

[1] W.B. Cace *et al.*, J. Appl. Phys. **55**, 2651 (1984).
[2] K. Ogura *et al.*, J. Phys. Soc. Jpn. **67**, 3462 (1998).
[3] K. Ogura *et al.*, Submitted to Journal of the Physical Society of Japan.
[4] S.T. Ivanov and E.G. ALEXOV, J. Plasma. Phys. **43**, 51 (1990).
[5] S. Pasquiers, J. Appl. Phys. **67**, 7246 (1990).