J. Plasma Fusion Res. SERIES, Vol. 3 (2000) 566–570

Plasma Confinement in a Magnetic Field of the Internal Ring Current

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(Received: 18 January 2000 / Accepted: 20 August 2000)

Abstract

Plasma confinement in compact region surrounding an internal ring current is considered. As the limiting case of large aspect ratio system the cylindrical tubular plasma is considered initially. Analysis of the cylindrical tubular plasma equilibrium and stability against the most dangerous flute (m = 0) and kink (m = 1) modes revealed the possibility of the MHD stable plasma confined by magnetic field of the internal rod current, with rather peaked plasma pressure and maximal local beta $\beta(r)$ = 0.4. In case of the toroidal internal ring system an additional external magnetic field creates the boundary separatrix witch limits the plasma volume. The dependence of the plasma pressure profiles, marginally stable with respect to the flute modes, from the shape of the external plasma boundary (separatrix) in such kind closed toroidal systems is investigated. The internal ring system with circular poloidal magnetic mirror, where the ring supports could be placed, is proposed.

Keywords:

Keywords: magnetic confinement, tubular pinch, internal ring system, dipole

1. Introduction

In levitating internal ring systems (dipole, quadrupole, octupole) plasma is confined mainly in a magnetic field of internal ring current. Among such system called in the review [1] by word "galateyas" (from the name of nymph of the quiescent sea) the simplest is the one-internal-ring system "dipole" (e.g. the LDX device [2]). The additional external poloidal magnetic field could change the simple dipole magnetic configuration creating an external separatrix of various shapes with the zero points of magnetic strength B [3]. At these points an integral $U = \int \mathrm{d}l/B$ diverges logarithmically. But the marginally stable against flute modes plasma pressure profile is $p(U) \propto U^{-\gamma}$ (adiabatic index $\gamma = 5/3$) [4], thus the plasma pressure tends to zero at separatrix. This allows to bound the size of the system. In addition, the external magnetic field can create the poloidal magnetic field mirror where one could try to place the supports of the internal ring in the experimental device.

For detailed investigations of plasma pressure profiles in considered system the tubular toroidal plasma equilibrium code TuTor was created [5]. Here we present the results of computation of marginally stable plasma pressure profiles at different shape of its external boundary. But initially we consider the equilibrium and stability of the cylindrical tubular plasma with longitudinal current as the limiting case of the large aspect ratio toroidal tubular system.

2. Equilibrium of the Cylindrical Tubular Plasma

To understand the peculiarity of the tubular plasma

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consider the radial force acting on the cylindrical tubular plasma

$$F_r = -\frac{\partial p}{\partial r} - j_z B_\theta . \quad (1)$$

The zero and the first momenta of this equation are ($dS = 2\pi r dr$)

$$\int_{r_1}^{r_2} F_r \mathrm{d}S = 2\pi\int_{r_1}^{r_2} p\,\mathrm{d}r - \mu_0 \int_{r_1}^{r_2} j_z\left[J_c + J_p(r)\right]\mathrm{d}r , \quad (2)$$

$$\int_{r_1}^{r_2} rF_r \mathrm{d}S = 2\int_{r_1}^{r_2} p\cdot 2\pi r\mathrm{d}r - 2\frac{\mu_0}{8\pi}\left(J_2^2 - J_1^2\right). \quad (3)$$

At equilibrium $F_r = 0$, and the last equation is reduced to Bennet condition for axisymmetric infinite tubular plasma in the external azimuthal magnetic field created by current of the central rod (Fig. 1). Here $J_1 = J(r_1) \equiv J_c > 0$ is the current in the internal rod and J_2 is a sum of the rod current J_c and the total current in plasma $J_p^* = J_p(r_2)$, where

$$J_p(r) = \int_{r_1}^{r} j_z\, 2\pi r\mathrm{d}r :$$

$$J_2 = J_c + J_p^* . \quad (4)$$

It is seen from (3) that for equilibrium the condition $J_2^2 > J_2^1$ should be satisfied. The Eq. (3) at $F_r = 0$ can be written in the form

$$\int p\mathrm{d}S = \langle p \rangle \pi\left(r_2^2 - r_1^2\right) = \frac{\mu_0}{8\pi}(2J_c + J_p^*)J_p^* . \quad (5)$$

Let us note that in the case of toroidal tubular plasma the integral $\int p dS$ divided by "large" radius R of torus characterized the R-force, $F_R = 1/R \int p dS$, along the major torus radius. At equilibrium this force is compensated by transverse magnetic field.

Consider the following situation in cylindrical case:

I. The negative plasma density, $j_z(r) < 0$, thus $J_p(r) < 0$.

a) According to the Eq. (5) the condition $\int p dS > 0$ is not fulfilled at equilibrium if $|J_p^*| < 2J_c$. In this case the longitudinal magnetic field should be added for equilibrium (as in hard core pinch [6] and levitron [7]).

b) At sufficiently high ratio $|J_p^*| / 2J_c$ the rigid rod current does not prevent compression of tubular plasma to the axis $r = 0$ (the pinch case).

II. The positive total plasma current $J_p^* > 0$.

a) At $j_z(r) > 0$ the electromagnetic part of the radial force is always negative (the pinch case).

b) Let us consider now the "magnetic trap case" when there is no externally excited current in plasma. If plasma is created and heated (i.e. $\nabla p \neq 0$) in the region $r_1 < r < r_2$, the consistent diamagnetic current with density $j = -p'/B$ appears. On the internal part of the plasma tube the plasma pressure increases, $p' > 0$, thus the current density here is negative. On the external part of plasma where $p' < 0$ the current density is positive. Thus the total diamagnetic current J_p consist of the internal negative and external positive parts:

$$J_p = J_- + J_+ . \quad (6)$$

If the positive part is larger than negative one, $J_+ > J_-$, the equilibrium is possible without compression of plasma to the z-axis (the magnetic trap case).

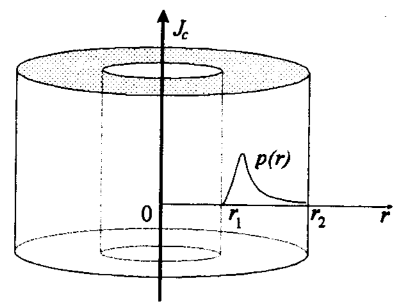

Fig. 1 Cylindrical tubular plasma confined in a magnetic field of the central current J_c.

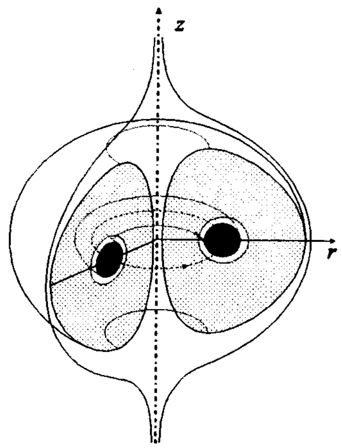

Fig. 2 Toroidal tubular plasma (dots) with the internal ring current J_c. The separatrix, formed by the additional ring currents (not shown), creates the external boundary of plasma.

3. MHD Stability of the Tubular Cylindrical Plasma

Kink instability (m = 1). The potential energy of the most dangerous kink mode is expressed through the radial component of displacement $\xi = rX(r)\exp i(\phi + kz)$. As it follows from [8,9] it has appearance

$$W = \frac{\pi}{2\mu_0}\int\left\{\frac{B^2 r^2}{1+k^2 r^2}\left(\frac{\mathrm{d}X}{\mathrm{d}r}\right)^2 + \left[B^2 + 2\mu_0 r p'(r)\right]X^2\right\} r\mathrm{d}r \quad (7)$$

As it is seen the sufficient condition of stability can be written as follows

$$-\frac{rp'}{p} < \frac{1}{\beta}. \quad (8)$$

(See Fig.3).

Flute instability (m = 0). Taking from the condition $p(U) \propto U^{-\gamma}$, $U \propto r/B(r)$ the logarithmic derivative and excluding $B'(r)$ with help of the equilibrium equation one can obtain the condition of the m = 0 mode stability [9,10,5]

$$-\frac{rp'}{p} < \frac{4\gamma}{2+\gamma\beta}. \quad (9)$$

The graphs of the stability boundary of both m = 1 and m = 0 modes versus parameter are shown in Fig. 3. At $\beta < 2/3\gamma = 2/5$ the m = 0 mode limits the value of $-rp'/p$, while at $\beta > 2/3\gamma$ the m = 1 mode is limiting one. On the Fig. 4 the corresponding Kadomtsev curve $p = p_K(r/a)$ as well as the $\beta(r/a)$ are plotted basing on the parametric dependencies from [5]. Introducing dimensionless $Y = (p_i/p)^{1/5}$ one could write:

$$\frac{p}{p_i} = \frac{1}{Y^5};\ \beta = \frac{0.8}{Y^2-1};\ \left(\frac{r}{a}\right)^2 = 1.25Y(Y^2-1) \quad (10)$$

Here $p_i = p(0)$, r/a = 1 at $p/p_i \approx 3$ (see Fig. 4). The sufficient stability conditions are satisfied at $\beta \le 0.4$ (see Fig. 3) when $r/a \ge 2$ (Fig. 4).

4. Marginal Plasma Pressure Profiles in Toroidal Case

The sufficient stability condition for perturbation, differ from the flute one, $\mathbf{B}\cdot\nabla\xi \neq 0$, could be obtained from estimation of potential energy $W > W_1$, where

$$W_1 = \frac{1}{2\mu_0}\int\left\{(\mathbf{B}\cdot\nabla\xi)^2 - 2\mu_0(\mathbf{k}\cdot\nabla p)\,\xi^2\right\}\frac{dr}{|\nabla a|^2} \quad (11)$$

([11]).

Here $\mathbf{k}$ is the curvature of the magnetic field line.

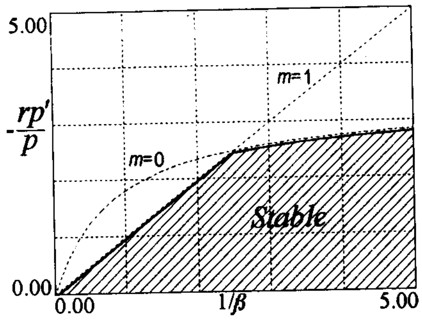

Fig. 3 Stability boundaries for cylindrical tubular plasma of the most dengerous modes m = 0, m = 1.

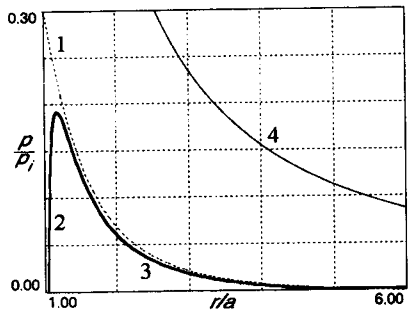

Fig. 4 Plasma pressure profile for cylindrical tubular plasma. *1* - Kadomtsev's m = 0 marginally stable profile; *2* - MHD stable ($\nabla p\cdot\nabla B < 0$) region; *3* - flute-stable pressure profile; *4* - Kadomtsev's marginal local $\beta(r)$.

Estimations $\mathbf{B}\cdot\nabla \approx B/r$, $\mathbf{k}\cdot\nabla p \approx kp/r$, led to rough estimation of type $\beta \sim 1/kr$. The more detailed calculations are needed to establish the limitation imposed by nonflute modes with minimal $(\mathbf{B}\cdot\nabla\xi)^2 \neq 0$. The local ballooning modes are stable [12].

The equilibrium marginal for the flute mode is described by the equation (in cylindrical coordinates)

$$\frac{\partial^2\psi}{\partial r^2} - \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{\partial^2\psi}{\partial z^2} = -4\pi^2\mu_0 r^2 p'(\psi), \quad (12)$$

where according to condition $p(U) \propto U^{-\gamma}$,

$$p(\psi) = p_0\left(\oint\frac{\mathrm{d}l}{B}\bigg|_{p=p_0}\right)^{\gamma}\Bigg/\left(\oint\frac{\mathrm{d}l}{B}\right)^{\gamma}. \quad (13)$$

Here ψ is the poloidal magnetic flux external relative to the current magnetic surface. The boundary conditions for dimensionless function $u = \psi/\psi_i$ are $u|_{S_i} = 1$, $u|_{S_e} = 0$, where S_i is the internal and S_e is the external boundaries of the tubular plasma. The tubular toroidal plasma equilibrium code "TuTor" described in [5] was used for investigation of the possibility to decrease the

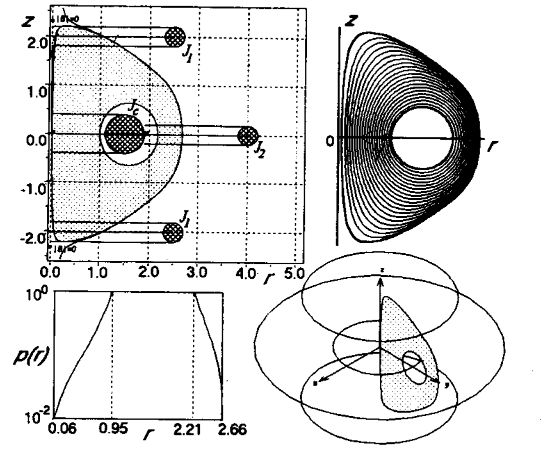

Fig. 5 Compact internal ring system with separatrix. The internal and external rings are shown. *1* - vacuum magnetic configuration; *2* - magnetic surfaces at low β; *3* - marginaly flute-stable plasma profiles at midplane (logariphmic scale). $J_1/J_c = -0.13$; $J_2/J_c = -0.33$.

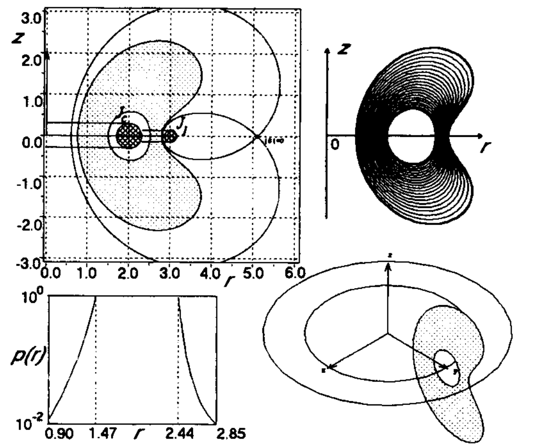

Fig. 6 Internal ring system with concave boundary and smooth flute-stable pressure profiles. $J_1/J_c = -0.33$.

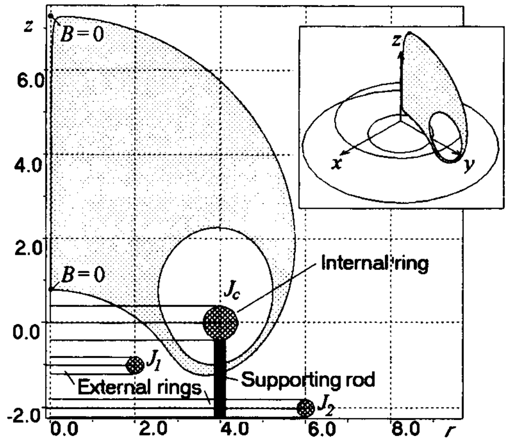

Fig. 7 Vacuum magnetic configuration with poloidal mirror and vertical supporting rod. $J_1/J_c = -0.60$; $J_2/J_c = -0.90$.

plasma pressure gradient near separatrix. Our computations show that in the case of system proposed in paper [3] the plasma pressure drops up to zero very steeply in very small region close to separatrix (Fig. 5). Increasing the number of the x-points on separatrix makes the pressure drop a little slower. If the boundary has the parts with concave magnetic field line, the integral $\int dl/B$ increases more uniformly close to boundary and the edge plasma pressure distribution becomes smoother (Fig. 6). Surely, the addition external rings are needed for shaping the plasma boundary.

Using such current rings with negative currents it is possible to arrange the circular poloidal magnetic mirror between close situated the internal and the additional external ring. One could try to use such magnetic mirror for placing the rigid rods for supporting the internal rings, thus escaping on the necessity of its levitation (Fig. 7). The magnetic mirror could weaken the flux of energetic particles on the supporting rods. At least they could simplify the first probe experiments on stationary plasma confinement in the magnetic field of the "internal" ring current.

Summary

Besides the pinch-type tubular plasma created by the gas discharge there is possibility to create such plasma by heating without "external" current. In such a case the plasma pressure is the primary while the diamagnetic current is the secondary. That is instead of habitual pinch effect we have here the magnetic trap. Investigation of the MHD stability has shown that the plasma could be stable at local β up to ~25%. (see Fig. 3).

The computation of plasma equilibrium in the magnetic field of the internal ring at presence of additional external poloidal magnetic field has shown possibility to restrict sufficiently the plasma volume with sufficiently small plasma pressure on the boundary. The difficulties with levitating internal ring are proposed to overcome, at least at first experiments by using the rods as supports. To preserve them from the energetic plasma particles the toroidally uniform poloidal magnetic mirror could be arranged at their position.

Acknowledgements

The authors wish to express our gratitude to Professor V.A. Kurnaev for interest to this paper and supporting two of us (P.P. and S.M.) during study in the Moscow State Engineering Physics Institute.

This work was partly supported by Russian Foundation for Fundamental Research, Projects No 97-02-17695.

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