Analysis of Influence of the Radial Electric Field on Turbulent Transport in a Tandem Mirror Plasma

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Abstract
The model of anomalous transport in cylindrical non-uniform steady state plasma in uniform magnetic field under the influence of many mode drift wave oscillations is suggested. The effect of anomalous transport suppression due to radial electric field is studied, and physical picture of H mode in plasma of GAMMA-10 tandem mirror device is considered. Presented theoretical and numerical results agree with the experimental data obtained on GAMMA-10.

Keywords:
turbulent transport, anomalous diffusion, tandem mirror, radial electric field, stochastic motion

1. Introduction
Since the discovery of an enhanced confinement regime known as H mode by Wagner et al. [1] on ASDEX tokamak a great deal of research has been conducted on physics of this regime [2-6, etc.]. The enhanced confinement has been observed in many other devices worldwide, including tokamaks, stellarators, heliotrons, a tandem mirror. Therefore one can conclude this phenomenon is common for the magnetized plasma of any configurations.

The basic idea underlying many theories concerning the H mode is that a large non-uniform radial electric field reduces the amplitude of the turbulent fluctuations. Corresponding results of linear theory show that taking into account of non-uniform electric field perpendicular to the vector k of drift wave leads to decreasing growth rate of this drift wave [7-12]. A such qualitative behavior was predicted in Ref. [7]. Experiments [13,14] showed that amplitudes of plasma oscillations decrease if non-uniform electric field increases.

So, at present the main and sole argument of observed influence of non-uniform radial electric field on plasma confinement time is decreasing of plasma fluctuations. But as a rule picture of plasma transport is not discussed. Only authors of Ref. [15] consider this problem.

In this paper we consider the possible model of transport processes based on experimental data [13]. It is shown this model agrees qualitatively with observed dependence of confinement time on the gradient of radial electric field.

2. Physical Model
The most detailed and complete experimental investigations of this phenomenon were carried out in GAMMA-10 device [13]. In this study modes of low-frequency drift wave were observed. The main feature of experiments [13] is controlled distribution of the radial electric field \( E_r(r) \). Consequently, authors of Ref. [13] were able to investigate correctly dependencies of confinement time, and fluctuation level on profile \( E_r(r) \). It was shown fluctuation level, and confinement time

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dependent sensitively on radial profile of plasma potential.

Following results are especially important for justification of the physical model of low-frequency drift excitation of the non-uniform low-$\beta$ magnetized plasma. Firstly, phase velocity of the observed waves is $V_p = \left| V_d - V_e \right|$ ($V_d$ is the electron diamagnetic drift velocity, $V_e = E_e/B$ is rotation velocity, $B$ is the magnetic field). Secondly, in Ref. [13] it was discovered availability of the wide spectra of drift waves. It indicates many modes of drift waves exist simultaneously inside the plasma. Thirdly, in experiments [13] electrostatic oscillations have been observed, and beta value is about $<\beta^2 > \approx 2 \times 10^{-3}$, that corresponds theoretical consideration showing the electrostatic oscillations realized for $\beta < m_e/m_i$. The first and the second features are agree if we suppose that different modes of drift waves are excited on different distances from plasma axis. These modes correspond different values of plasma density and density gradient [16]. Therefore model of the plasma transport induced by low-frequency drift waves should take into account simultaneously existence of many drift modes connected with local plasma parameters. Every mode can propagate in the vicinity of the resonant radius.

Taking into account the presented experimental data we proceed from following picture of transport processes for GAMMA-10. We consider non-uniform cylindrical plasma with $\beta < m_e/m_i$. It corresponds to electrostatic plasma oscillations [16]. The magnetic field is homogenous. Radial profiles of electrostatic field and plasma density correspond to experimental data [13].

We assume $N$ modes of electrostatic drift waves are excited in plasma. Every mode corresponds to fixed local value of plasma density $n(r)$ and density gradient $dn/dr$. Therefore different waves are excited on different distances from plasma axis. Number of modes and their amplitudes are given and can be changed for different series of calculations. Besides we should give radial profiles of wave amplitude for every mode. Here we consider the simplest variant of these profiles: uniform amplitude for every mode in whole plasma $0 \leq r \leq a$. This assumption contradicts to experimental data of Ref. [13], where it is shown the maximal density fluctuations are localized in the region of maximal density gradient. But maximal and minimal amplitude values are differed approximately two times (Fig. 8 in Ref. [13]). The phase velocity is $V_p = \left| V_d - V_e \right|$, and it depends on radius so that angular velocity is constant, and it is independent of radius.

This model is used to investigate the particles motion. In this approach anomalous transport is considered as a result of stochastization of particle motion under influence of many mode perturbation. Beyond we present both numerical and analytical results.

### 3. Qualitative Consideration

We consider the dynamics of particles in the magnetized plasma of slab geometry under simultaneous influence of the following electric fields: a) longitudinal electrostatic waves $E_p(y,t)$ propagating along axis $y$, and b) permanent electric field $E_p(x)$ directed along $x$-axis. The magnetic field $B$ is directed along axis $z$. The density gradient and oscillations of particles due to wave influence are directed along $x$.

The drift equations for the particle under influence of single mode perturbation are

\[
\frac{dx}{dt} = \frac{E_{yi}}{B} \sin (\omega_{yi} t - k_{yi} y),
\]

\[
\frac{dy}{dt} = V_y(x) - V_y(x_0) = \frac{E(x) - E(x_0)}{B},
\]

where $x_0$ is the coordinate of the resonant plate of number $i$.

Let's consider linear approximation

\[
E_p(x) = E_p(x_0) + (x - x_0) \frac{dE_p}{dx} (x_0).
\]

In this case Eqs. (1)-(3) are reduced to

\[
\frac{d^2 u}{dt^2} = -\frac{E_{yi}}{B} \frac{dE_p}{dx} \sin u,
\]

where $u = \omega_{yi} t - k_{yi} y$. Eq. (4) is the non-linear pendulum equation [17-19]. As it noticed above, we consider simultaneous influence of $N$ modes. This approach enables to consider the anomalous transport in non-uniform plasma as a result of the stochastization of particles motion. The diffusion coefficient for such process is defined as [20-22]

\[
D_1 = \frac{1}{\pi N} \sum_{i=1}^{N} \frac{1}{w_{i+1} - w_i} (\Delta x_i)^2,
\]

where $w_i$, $w_{i+1}$ are frequencies of the neighboring perturbations, $\Delta x_i$ is the maximal oscillation along axis $x$ under the influence of the mode of number $i$.

Using (1), (2) one can obtain
Therefore increasing $dE_r/dx$ leads to decreasing of $D_{1r}$. This result agrees with the experimental data [13].

Using Eq. (5) for cylindrical plasma we can obtain radial confinement time under the influence of drift waves in stochastic regime

$$\tau_1 = \frac{\epsilon B a^2}{2\epsilon^2 K T_e},$$

(7)

where $\epsilon$ is the charge of the electron, $K$ is the Boltzmann constant, $a$ is the radius of the plasma, $T_e$ is the electron temperature, $\epsilon = \frac{\epsilon_n \delta n_e}{K T_e}$ is the relative level of wave potential amplitude $\varphi_0$, $n_e$ is the electron density, and $\delta n_e$ is its fluctuation. Here we assume $\epsilon \ll 1$, $\delta n_e \ll n_e$. In Fig. 1 the dependence of $\tau_1$ is plotted for conditions of experiments on GAMMA-10 device [13] ($B = 0.4$ T, $a = 0.2$ m, $T_e = 60..120$ eV). One can see satisfactory agreement between experimental and theoretical data.

4. Some Numerical Results

Calculations were carried out for cylindrical plasma with parameters corresponding experimental conditions of Ref. [13]. In Fig. 2 examples of oscillations of the particles in the vicinity of resonant surface under the influence of single mode perturbation are presented for different values of applied radial electric potential $U_B$. In plasma without external electric field ($U_B = 0$) the own radial electric field exists. Corresponding averaged diffusion coefficient calculated in accordance with Eq. 5 using numerically obtained values of amplitudes of particles oscillations is plotted in Fig. 3.
5. Conclusions

Considered many mode model can explain the main features of anomalous transport in GAMMA-10 plasma. The idea to present the anomalous transport as the result of the stochastization of the motion of particles approaches reality.

It is shown that the permanent non-uniform electrostatic field directed along density gradient influence on transport processes, not only growth rates of the plasma instabilities. So, generally self-consistent consideration of both the plasma oscillations and the transport is necessary.

References