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The Evolution of Structurally Unstable 2-D Magnetic Configuration with Two Null Lines

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Abstract

We consider analytically and with MHD simulations the regimes of driven magnetic field line reconnection in two planar magnetic configurations with two null lines (X-lines), with boundary conditions that correspond to the excitation of the finite amplitude magnetoacoustic and Alfvén waves. We demonstrate that the change of the magnetic field topology due to the magnetoacoustic type perturbations is accompanied by the global redistribution of the electric current curried by the Alfvén waves.

Keywords:

noindent magnetic reconnection, structural stability, current sheet

The problem of magnetic field line reconnection, which is of great importance for space and laboratory plasmas [1] is closely related to the problem of the structural stability of vector fields [2,3]. As a result of magnetic reconnection the topology of the magnetic field changes. It is natural to expect that structurally unstable magnetic configurations must be transformed into structurally stable ones. We have investigated the process of transformation from structurally unstable to structurally stable configuration and distributions of the electric current density at the quasistationary stage.

We consider an initial planar two-dimensional plasma configuration. The initial magnetic filed lies in the $x - y$ plane, is current-free and vanishes at null lines (along z). The magnetic field is described by the vector potential $\boldsymbol{A_0} = A_0(x, y)\boldsymbol{e}_z$ with

$$A_0(x, y) = \frac{g}{3}(x^3 - 3xy^2) + \varepsilon_1 x + \varepsilon_2 y. \qquad (1)$$

In the configuration described by vector potential with $\varepsilon_2 = 0$ the separatrix connects two saddle X-lines. It is structurally unstable and is shown in Fig. 1. The configurations described by the vector potential with $\varepsilon_2 \neq 0$, $\varepsilon_1 \neq 0$ is structurally stable.

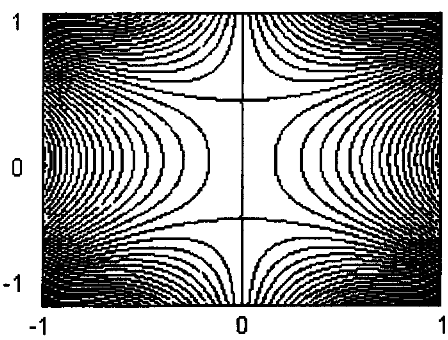

Fig. 1 Structurally unstable magnetic configuration with two X-lines.

The equilibrium of the configuration is violated under the action of magnetoacoustic and Alfvén waves. The magnetoacoustic and Alfvén waves are imposed at the boundary of the computational region.

The problem of the propagation of the small amplitude MHD waves in the planar potential magnetic field that depends on two coordinates was analyzed in [4] where it was shown that neglecting dissipation and thermal effects the magnetoacoustic wave is described by

$$\partial\partial_{tt}a - |f_0'|^2\Delta a = 0, \tag{2}$$

and for the Alfvén wave we have

$$\partial\partial_{tt}b - (\boldsymbol{B}_{0\perp}\nabla)^2 b = 0. \tag{3}$$

Here $a(x, y, t)$ and $b(x, y, t)$ are the perturbed z-components of the vector potential in the magnetoacoustic wave and the magnetic field in the Alfvén wave, respectively. The magnetic field associated with the MHD perturbations is $\boldsymbol{b}(x, y, t) = \partial_y a\boldsymbol{e}_x - \partial_x a\boldsymbol{e}_y + b\boldsymbol{e}_z$. The time is normalized to $(4\pi\rho_0)^{1/2}/gs^2$. In equation (2) $f_0' \equiv \mathrm{d}f_0/\mathrm{d}\zeta$, where $f_0 = F_0 - \mathrm{i}A_0$ is the complex potential of the magnetic field that depends on the complex variable $\zeta = x + \mathrm{i}y$. The unperturbed magnetic field, $\boldsymbol{B}_{0\perp}$, can be expressed via the scalar F_0 and vector potential A_0 as

$$B_{0x} = -\partial_x F_0 = \partial_y A_0, \quad B_{0y} = -\partial_y F_0 = -\partial_x A_0. \tag{4}$$

This is equivalent to $B_{0x} - \mathrm{i}B_{0y} = -\mathrm{d}f_0/\mathrm{d}\zeta$.

To solve equation (2) we perform following [4] the conformal mapping of the complex plane $\zeta = x + \mathrm{i}y$ to the plane $w = u + \mathrm{i}v$ given by

$$w - w_0 = \int_{\zeta_0}^{\zeta} \frac{\mathrm{d}\zeta}{f_0'(\zeta)}, \tag{5}$$

where w_0 and ζ_0 are constant. This mapping transforms equation (2) to the wave equation in the Cartesian coordinates u, v and t:

$$\partial\partial_{tt}a - \partial\partial_{uu}a - \partial\partial_{vv}a = 0, \tag{6}$$

with the Green function

$$E_2(w, w_0, t, t_0) = \frac{\chi((t-t_0) - ((u-u_0)^2 + (v-v_0)^2)^{1/2})}{2\pi((t-t_0)^2 - (u-u_0)^2 - (v-v_0)^2)^{1/2}} \left|\frac{\partial(u, v)}{\partial(x, y)}\right|_{w=w_0} \tag{7}$$

Here $\chi(x) = 1$ for $x > 0$ and $\chi(x) = 0$ for $x < 0$.

Equation (1) corresponds to the complex potential of the magnetic field of the form

$$f_0(\zeta) = -\mathrm{i}\,(\zeta^3/3 - \varepsilon\zeta), \tag{8}$$

where $\varepsilon = -\varepsilon_1 + \mathrm{i}\varepsilon_2$. The conformal mapping (5) is given by the function

$$w - w_0 = \frac{1}{2\sqrt{\varepsilon}} \ln\left(\frac{(\zeta - \sqrt{\varepsilon})(\zeta_0 + \sqrt{\varepsilon})}{(\zeta + \sqrt{\varepsilon})(\zeta_0 - \sqrt{\varepsilon})}\right). \tag{9}$$

If $|\varepsilon| \ll 1$ at large distance from the origin, $|\zeta| \gg \sqrt{\varepsilon}$, the magnetoacoustic wave can be cylindrical with $w - w_0 \approx 1/\zeta$. Assuming the wave to be excited at the circle with unite radius $|\zeta| = 1$, with the intensity $q_0(t)$, we find that near the front of the wave converging toward the origin, the wave amplitude behaves as

$$a(r, t) \approx r^{1/2} \int_0^{t-1/r} \frac{q_0(\eta)\mathrm{d}\eta}{\sqrt{2(\tau - \eta)}}, \tag{10}$$

where $r = \sqrt{x^2 + y^2}$. We see that the azimuthal component of the magnetic field associated with the magnetoacoustic wave increases as $r^{-3/2}$ when $r \to 0$. Close to the X-lines, where $\zeta \approx \pm\sqrt{\varepsilon}$, we have

$$a(s, t) \approx \sqrt{\frac{\ln|\varepsilon|}{8|\varepsilon|}} \int_0^{t-\ln s} \frac{q_0(\eta)\mathrm{d}\eta}{\sqrt{\tau - \eta}}, \tag{11}$$

where $s = |\zeta \mp \sqrt{\varepsilon}|$. Here the perturbations of the azimuthal component of the magnetic field grow as s^{-1} when $s \to 0$ [5,6].

Equation (3) has a solution $b(x, y, t)$ that is constant on characteristics $A_0(x, y) = A_0$ and

$$C_0 = t \pm \int \frac{\mathrm{d}x}{\varepsilon_2 \mp \sqrt{\varepsilon_2^2 - 4A_0 x + 4\varepsilon_1 x^2 + 4x^2/3}}. \tag{12}$$

If $\varepsilon_2 = 0$ we have the case with the X-X-separatrix, which is determined by the condition $A_0 = 0$. Expression (12) for the characteristic on the separatrix takes the form

$$C_0 = t \pm \frac{1}{4\sqrt{\varepsilon_1}} \ln\left(\frac{\sqrt{3\varepsilon_1 + x^2} - \sqrt{3\varepsilon_1}}{\sqrt{3\varepsilon_1 + x^2} + \sqrt{3\varepsilon_1}}\right). \tag{13}$$

The amplitude of the Alfvén wave remains constant while the wave approaches the X-X-separatrix $x = 0$ as $x \propto 1/t$, but the electric current density associated with the wave tends to infinity as x^{-2} for $x \to 0$.

We consider non-planar two-dimensional perturbations where all quantities depend on x, y and t only, but the perturbed magnetic and velocity vector fields have three components, which allow both magnetoacoustic and Alfvén-type perturbations, and solve the system of the MHD equations numerically. We use the MHD equations in dimensionless form [3]. To

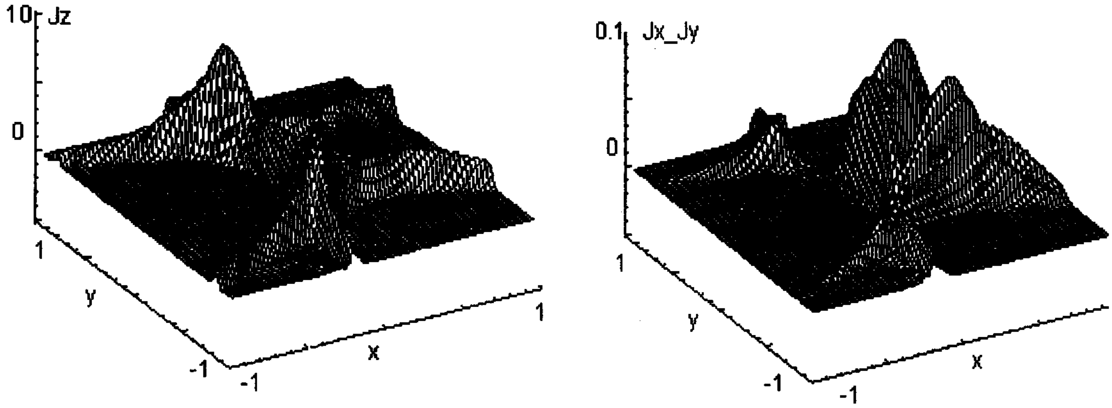

Fig. 2 Distributions of the electric current density for magnetoacoustic (a) and Alfvén (b) perturbation.

model magnetoacoustic waves we choose the vector potential at the boundaries $x = \pm 1$ and $y = \pm 1$ of the form $A(x, y, t) = A_0(x, y) + r^{1/2}\, \mathcal{F}(t - 1/r + 1)$, where $A_0(x, y)$ is given by (1), and $r^2 = x^2 + y^2$, and $\mathcal{F}(\xi) = -E_1(\xi - 1)^2/\xi$ for $\xi > 1$, 0 for $\xi < 1$. Alfvén waves are described by the boundary condition for the z-component of the magnetic field $B_z(x = \pm 1, y, t) = \pm B_1 \min\{1, t/t_{sw}\}$. All the results of the numerical simulations presented are obtained for dimensionless electric field $E_1 = 0.06$ and magnetic field $B_1 = 0.1$. We consider the boundary conditions related to perturbation of the magnetoacoustic and Alfvén type.

The first series of numerical calculations was performed for the structurally unstable magnetic configuration. The results of the simulations are shown in Fig. 2.

Here and below we use the term "longitudinal" current for z-component of the electric current (a) and "transverse" current for its poloidal component $(j_x^2 + j_y^2)^{1/2}$ (b). The magnetoacoustic wave changes the right-left symmetry of the initial magnetic field. Two currents sheets are formed by the magnetoacoustic perturbation near the two null lines. The transverse current is concentrated near magnetoacoustic shock waves located in the vicinity of the magnetic separatrices.

The second series of numerical calculations was performed for the initially structurally stable magnetic configuration. The configuration of this magnetic field is shown in Fig. 3. The results of the simulations are shown in Fig. 4.

The nonlinear interaction of the magnetoacoustic and Alfvén waves leads to a situation where only one null point dominates. The longitudinal and transverse currents are concentrated near the dominant null line.

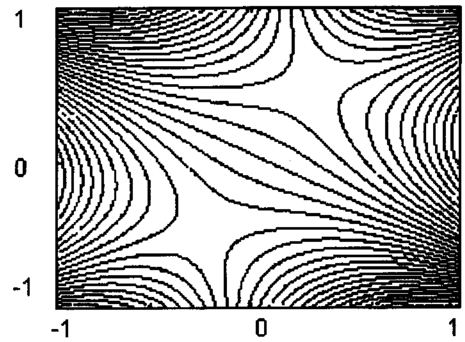

Fig. 3 Structurally stable magnetic configuration with two X-lines.

The main result shown in Figure 8 is in the symmetry breaking due to the nonlinear interaction of the magnetoacoustic and of the Alfvén perturbations on null lines. The transverse electric current associated with Alfvén type perturbations is localized mainly in the vicinity of the null lines and of the separatrices at the top of the computational region.

We have studied the transformation of structurally unstable magnetic configurations into structurally stable configurations. We have observed a global redistribution of the electric current in the $x - y$ plane which is due to the nonlinear interaction of the magnetioacoustic and of the Alfvén perturbations.

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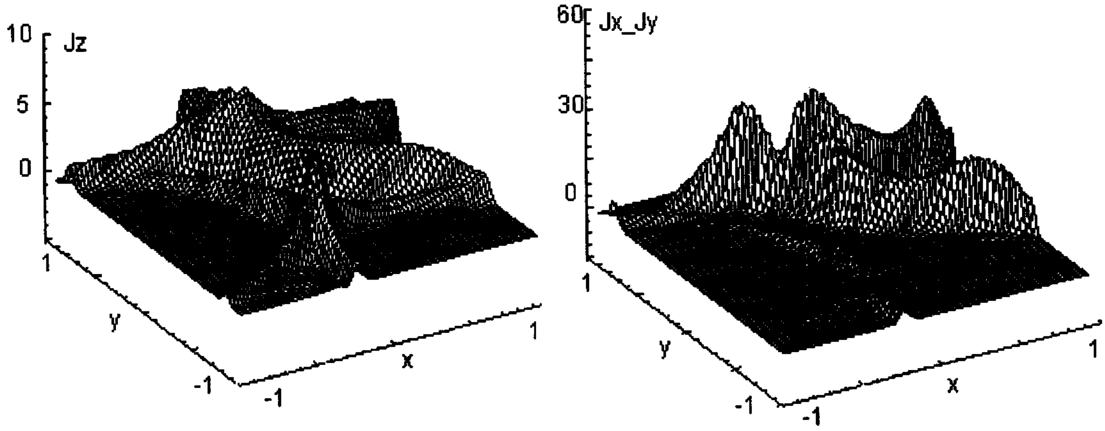

Fig. 4 Distributions of the electric current density for magnetoacoustic (a) and Alfvén (b) perturbation.

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