J. Plasma Fusion Res. SERIES, Vol. 2 (1999) 371–375

Nonlinear Dispersion Relation for Beam-Exited Plasma

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(Received: 9 December 1998 / Accepted: 16 June 1999)

Abstract

Results of experimental study for generation of envelope soliton and the corresponding nonlinear theory and also the analysis of dispersion relation in the beam-plasma system are discussed. We succeed to obtain the nonlinear dispersion relation with a Gaussian type Green's function. We also tried to solve the nonlinear dispersion relation numerically. As an example, a solution of the dispersion relation showing two curves in ω-$\boldsymbol{k}$ plane is represented which correspond to the beam electron dispersion diagram. Both curves are quite similar to the solution derived from the linear dispersion relation.

Keywords:

nonlinear, soliton, beam, coherent, dispersion

1. Introduction

Hitherto, analysis of a dispersion relation has been performed extensively because the dispersion relation shows an important character of the propagation of waves in a plasma. In the analysis, we usually use the linearlization technique so that we can not discuss, for example, an amplitude dependent phenomenon (*Kerr effect*). Accordingly, we need something new method to treat a nonlinear effect. We intend to solve this problem by starting from a renormalization technique and by using a Gaussian type Green's function.

In this paper we consider a high frequency electric field (elementary excitation) emitted by an electron beam by so called 'large signal theory', when the beam injected into plasma. The trajectory of a beam electron is changed from linear straight line to curved line in the presence of the electric field, i.e., when an electron emits a quantum (elementary excitation) its direction is changed to opposite side against the direction of quantum by momentum conservation, while for the absorption of a quantum, the quantum strikes the electron to bend its direction. We introduce a quantity concerned with this emission and subsequent absorption as Σ_α which means collision frequency or self-energy term.

2. The Nonlinear Theory

To introduce the curvature effect by the electric field into electron's trajectory, we use the renormalization technique which is initially developed by the authors: Karpman [1], Dupree [2], Weinstock [3], Kono and Ichikawa [4]. However the method of these authors can not be applied directly to our case, since we treat a coherent interactions, i.e., $\boldsymbol{k} = \boldsymbol{k}_1$, between beam's modulated wave number $\boldsymbol{k}$ and the wave number $\boldsymbol{k}_1$ of surrounded plasma wave (plasmon's wave number) and further we require the finite frequency width at resonance, $\mathrm{Im}\Sigma_\alpha$, where the life time of the elementary excitation, τ, is given as $\tau = 1/(2\mathrm{Re}\Sigma_\alpha)$, therefore we use a Gaussian-type Green's function, which is utilized initially by Horton, in ion-acoustic turbulence [5] and define a diffusion tensor $\boldsymbol{D}_{ij}$ instead of scalar function, then the Green's function is described as:

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$$G_\alpha(\boldsymbol{k},\boldsymbol{v},t;\boldsymbol{k}',\boldsymbol{v}',t') = \mathrm{i}\left\{\frac{4\pi}{\boldsymbol{k}^2}\boldsymbol{k}_i\boldsymbol{k}_j\boldsymbol{D}_{ij}(\boldsymbol{k},\boldsymbol{v})t\right\}^{-3/2} \cdot \exp\left[-\frac{\boldsymbol{k}_i(\boldsymbol{v}-\boldsymbol{v}')_i\boldsymbol{k}_j(\boldsymbol{v}-\boldsymbol{v}')_j}{4\boldsymbol{k}_i\boldsymbol{k}_j\boldsymbol{D}_{ij}(\boldsymbol{k},\boldsymbol{v})t} - \frac{\boldsymbol{k}_i\boldsymbol{k}_j\boldsymbol{D}_{ij}(\boldsymbol{k},\boldsymbol{v})}{12}t^3 - \frac{\mathrm{i}}{2}\boldsymbol{k}\cdot(\boldsymbol{v}+\boldsymbol{v}')t\right]\delta_{\boldsymbol{k},\boldsymbol{k}'}.$$

Fourier transformation of the above Green's function becomes:

$$G(\boldsymbol{k},\boldsymbol{v},\omega;\boldsymbol{k}',\boldsymbol{v}',\omega') = -2\mathrm{i}\,\{4\pi(\frac{\boldsymbol{k}_i\boldsymbol{k}_j}{\boldsymbol{k}^2})\cdot D_{ij}(\boldsymbol{k},\boldsymbol{v})\}^{-3/2} \sum_{m=0}^{\infty}\frac{(-b)^m}{m!}\{\frac{a}{\mathrm{i}(-\omega+c)}\}^{3m/2-1/4} \cdot K_{3m-1/2}\{2\sqrt{ia(-\omega+c)}\}\cdot\delta_{\boldsymbol{k},\boldsymbol{k}'}\delta_{\omega,\omega'}, \quad (1)$$

$$a = \frac{k_i(\boldsymbol{v}-\boldsymbol{v}')_i k_j(\boldsymbol{v}-\boldsymbol{v}')_j}{4\boldsymbol{k}_i\boldsymbol{k}_j\cdot D_{ij}(\boldsymbol{k},\boldsymbol{v})}, \quad b = \frac{\boldsymbol{k}_i\boldsymbol{k}_j\cdot D_{ij}(\boldsymbol{k},\boldsymbol{v})}{12},$$

$$c = k\cdot\boldsymbol{v} - \mathrm{i}\,\Sigma_\alpha,$$

where the function $K_{3m-1/2}(x)$ is the modified Bessel function and m = 0 term is very important and $K_{-1/2}(x) = K_{1/2}(x) = \sqrt{\pi/(2x)}\cdot\exp(-x)$. We denote that $f_\alpha^{(0)}(\boldsymbol{k}',\boldsymbol{v}',\omega')$, $\Sigma_\alpha(\boldsymbol{k},\boldsymbol{v},\omega)$, $\boldsymbol{E}(\boldsymbol{k},\omega)$ are the initial distribution function and the collision frequency, the electric field respectively. Then we can add the following fundamental equations (2), (3), (4) and (5).

$$D_{ij}(\boldsymbol{k},\boldsymbol{v}) = (\frac{e_\alpha}{m_\alpha})^2\sum_{k_1}\int\frac{d\omega_1}{2\pi}\frac{\boldsymbol{k}_{1i}\boldsymbol{k}_{1j}}{\boldsymbol{k}_1^2}\left|\boldsymbol{E}(\boldsymbol{k}_1,\omega_1)\right|^2 G_\alpha(\boldsymbol{k}-\boldsymbol{k}_1,\boldsymbol{v},\boldsymbol{k}\cdot\boldsymbol{v}-\omega_1;\boldsymbol{k}',\boldsymbol{v}',\omega'), \quad (2)$$

$$\boldsymbol{E}(\boldsymbol{k},\omega) = -\sum_\alpha\frac{e_\alpha}{\varepsilon_0}\cdot\frac{i\boldsymbol{k}}{\boldsymbol{k}^2}\int d\boldsymbol{v}\int d\boldsymbol{v}'\int\frac{d\boldsymbol{k}'}{(2\pi)^3} G_\alpha(\boldsymbol{k},\boldsymbol{v},\omega;\boldsymbol{k}',\boldsymbol{v}',\omega')\cdot f_\alpha^{(0)}(\boldsymbol{k}',\boldsymbol{v}',\omega'), \quad (3)$$

$$f_\alpha(\boldsymbol{k},\boldsymbol{v},\omega) = \int d\boldsymbol{v}'\,G_\alpha(\boldsymbol{k},\boldsymbol{v},\omega;\boldsymbol{k}',\boldsymbol{v}',\omega')\cdot f_\alpha^{(0)}(\boldsymbol{k}',\boldsymbol{v}',\omega'), \quad (4)$$

$$\Sigma_\alpha(\boldsymbol{k},\boldsymbol{v},\omega) = (\frac{e_\alpha}{m_\alpha})^2\sum_{k_1}\frac{d\omega_1}{2\pi}\boldsymbol{E}(\boldsymbol{k}_1,\omega_1)\frac{\partial}{\partial\boldsymbol{v}} \{G_\alpha(\boldsymbol{k}-\boldsymbol{k}_1,\boldsymbol{v},\boldsymbol{k}\cdot\boldsymbol{v}-\omega_1)\cdot\boldsymbol{E}(-\boldsymbol{k}_1,-\omega_1)\cdot\frac{\partial}{\partial\boldsymbol{v}}\}. \quad (5)$$

The eqs. (1)–(5) construct a closed set of fundamental equation to analyse the nonlinear dispersion relation or the soliton emission. To discuss the amplitude dependence, a coherent 2nd-order quantity $\boldsymbol{E}(\boldsymbol{k},\omega)\boldsymbol{E}(-\boldsymbol{k}_1,-\omega)\delta_{k,k_1}\delta_{\omega,\omega_1} = |\boldsymbol{E}(\boldsymbol{k},\omega)|^2$ is important. If we substitute a condition of coherence, $\boldsymbol{k} = \boldsymbol{k}_1$, $\omega = \omega_1$, into the 2nd-order quantity, eqs. (2) and (5), we find that an importance of the Green's function $G_\alpha(\boldsymbol{k}-\boldsymbol{k}_1\to 0,\boldsymbol{v},\boldsymbol{k}\cdot\boldsymbol{v}-\omega_1-\mathrm{i}\Sigma_\alpha)$ is crucial, while in the Green's function, eq. (1), the terms m $\neq$ 0 are proportional to $\boldsymbol{k}^{2m}$ then they vanish when we consider the limit $\boldsymbol{k}\to 0$, therefore the most significant term is the m = 0-th one.

$$G_\alpha^{(m=0)}(\boldsymbol{k},\boldsymbol{v},\omega;\boldsymbol{k}',\boldsymbol{v}',\omega') = -\frac{\mathrm{i}}{4\pi D_{ij}}\cdot\frac{1}{|\boldsymbol{v}-\boldsymbol{v}'|} \cdot\exp\left(-\frac{|\boldsymbol{v}-\boldsymbol{v}'|}{\sqrt{D_{ij}}}\cdot\sqrt{i(-\omega+c)}\right)\cdot\delta_{\boldsymbol{k},\boldsymbol{k}'}\delta_{\omega,\omega'}. \quad (6)$$

We define that 'y' equals the square root of the ratio: Σ_α and D_{ij}, which appears in the exponential function in eq. (6) at resonance $\omega - \boldsymbol{k}\cdot\boldsymbol{v} - \mathrm{i}\Sigma_\alpha = 0$, then the following relation is obtained:

$$y \equiv \frac{|\boldsymbol{v}-\boldsymbol{v}'|\sqrt{\Sigma_\alpha}}{\sqrt{D_{ij}}}. \quad (7)$$

As the eq. (5) is a operator, eigenvalue must be derived for Σ_α, i.e., we multiply Green's function from the right side to the eq. (5) and after integration, we divide the Green's function again. It can be obtained also D_{ij} by substitution of eq. (6) into eq. (2) then the resultant relation becomes:

$$\boldsymbol{D}_{ij}(\boldsymbol{k},\boldsymbol{v}) = \frac{1}{2}\frac{C_1\left|\boldsymbol{E}(\boldsymbol{k},\boldsymbol{k}\cdot\boldsymbol{v})\right|^2}{\left|\boldsymbol{v}-\boldsymbol{v}'\right|^3}\cdot(1-y)\cdot e^y. \quad (8)$$

where the y (in eqs. (7), (8)) has a value of 0~1 and the C_1 is denoted as $C_1 = \{1/(2\pi)\}(e_\alpha/m_\alpha)^2\{\boldsymbol{k}_i\boldsymbol{k}_j/\boldsymbol{k}^2\}$. If we define a new quantity: $C_1^* = \{1/(2\pi)\}(e_\alpha/m_\alpha)^2\{\boldsymbol{k}_i\boldsymbol{k}_j/\boldsymbol{k}^2\}(1-y)e^y$, then the D_{ij} in eq. (6) can be replaced with $|\boldsymbol{E}(\boldsymbol{k},\boldsymbol{k}\cdot\boldsymbol{v})|^2$.

$$G(\boldsymbol{k},\boldsymbol{v},p;\boldsymbol{k}',\boldsymbol{v}',p') = -\frac{\mathrm{i}}{4\pi}\cdot\frac{|\boldsymbol{v}-\boldsymbol{v}'|^2}{C_1^*|\boldsymbol{E}|^2}$$

$$\cdot\exp\left\{-\frac{|\boldsymbol{v}-\boldsymbol{v}'|^{5/2}}{\sqrt{C_1^*|\boldsymbol{E}|}}\cdot\sqrt{p+ic}\right\}\cdot\delta_{k,k'}\,\delta_{p,p'}\,. \quad (9)$$

where $p = -\mathrm{i}\omega$ and $|\boldsymbol{E}|^2 = |\boldsymbol{E}(\boldsymbol{k},p)|^2$.

The electric field can be calculated from eq. (3), the initial distribution function, $f_\alpha^{(0)}(\boldsymbol{k}',\boldsymbol{v}',p')$, $(p' = -\mathrm{i}\omega')$, is given as:

$$f_\alpha^{(0)}(\boldsymbol{k}',\boldsymbol{v}',p') = f_1^{(0)}(\boldsymbol{k}')\,n_b\delta(\boldsymbol{v}'-\boldsymbol{v}_0)$$

$$+n_p\left(\frac{m_\alpha}{2\pi kT_\alpha}\right)^{3/2}\exp\left\{-\frac{m_\alpha(\boldsymbol{v}')^2}{2\pi kT_\alpha}\right\}. \quad (10)$$

where the $f_1^{(0)}(\boldsymbol{k}')$, n_b, n_p are the no-dimensional beam forming factor, the beam electron density and the plasma density respectively. In actual calculation, we put $f_1^{(0)}(\boldsymbol{k}') \simeq O(1) = 1$. Thus the electric field, $\boldsymbol{E}(\boldsymbol{k},p)$, can calculate as:

$$\boldsymbol{E}(\boldsymbol{k},p) = \sum_\alpha \frac{e_\alpha}{\varepsilon_0}\cdot\frac{\mathrm{i}\boldsymbol{k}}{\boldsymbol{k}^2} f_1^{(0)}(\boldsymbol{k})\int \mathrm{d}\boldsymbol{v}$$

$$\frac{\mathrm{i}\,n_b|\boldsymbol{v}-\boldsymbol{v}_0|^2}{4\pi C_1^*|\boldsymbol{E}|^2}\cdot\exp\left\{-\frac{|\boldsymbol{v}-\boldsymbol{v}_0|^{5/2}}{\sqrt{C_1^*|\boldsymbol{E}|}}\cdot\sqrt{p+\mathrm{i}c}\right\}$$

$$-\sum_\alpha\frac{e_\alpha}{\varepsilon_0}\cdot\frac{\mathrm{i}\boldsymbol{k}}{\boldsymbol{k}^2}\int\mathrm{d}\boldsymbol{v}\int\mathrm{d}\boldsymbol{v}'$$

$$\left[-\frac{\mathrm{i}\,n_b}{4\pi}\cdot\frac{|\boldsymbol{v}-\boldsymbol{v}'|^2}{C_1^*|\boldsymbol{E}|^2}\cdot\exp\left\{-\frac{|\boldsymbol{v}-\boldsymbol{v}'|^{5/2}}{\sqrt{C_1^*|\boldsymbol{E}|}}\cdot\sqrt{p+\mathrm{i}c}\right\}\right]$$

$$\cdot\left(\frac{m_\alpha}{2\pi kT_\alpha}\right)^{3/2}\exp\left\{\frac{m_\alpha(\boldsymbol{v}')^2}{2\pi kT_\alpha}\right\}. \quad (11)$$

Equation (11) is a nonlinear integral equation for electric field, $\boldsymbol{E}(\boldsymbol{k},p)$, the first term of which shows a contribution of beam with initial speed $\boldsymbol{v}_0$ and it behave actively by generating mainly the electric field, while the second term represent the contribution of maxwellian plasma. The both term interact each other through the electric field. After some calculation with respect to the first term, a relative permittivity, $\varepsilon_s(\boldsymbol{k},p,|\boldsymbol{E}|^2,\Sigma_\alpha)$, is obtained. Therefore dispersion relation becomes as following:

$$\varepsilon_s(\boldsymbol{k},p,|\boldsymbol{E}|^2,\Sigma_\alpha) = 1+\sum_\alpha\frac{e_\alpha}{\varepsilon_0}\cdot\frac{\mathrm{i}}{|\boldsymbol{k}|}f_1^{(0)}(\boldsymbol{k})$$

$$\cdot\frac{2}{5}\cdot\frac{n_b}{\{|\boldsymbol{E}|^2\}^{1/2}}\cdot\frac{(a_1)^{2/5}}{(p+ic)}\cdot\left(\frac{\mathrm{Im}\,\Sigma_\alpha}{2k}\right)$$

$$\cdot\Gamma\left(q,a_1\left|\frac{\mathrm{Im}\,\Sigma_\alpha}{2k}\right|^\nu\right)-\sum_\alpha\frac{e_\alpha}{\varepsilon_0}\cdot\frac{\mathrm{i}\boldsymbol{k}}{|\boldsymbol{k}|}\int\mathrm{d}\boldsymbol{v}\int\mathrm{d}\boldsymbol{v}'$$

$$\left[-\frac{\mathrm{i}\,n_b}{4\pi}\cdot\frac{|\boldsymbol{v}-\boldsymbol{v}'|^2}{C_1^*\{|\boldsymbol{E}|^2\}^{1/2}}\cdot\exp\left\{-\frac{|\boldsymbol{v}-\boldsymbol{v}'|^{5/2}}{\sqrt{C_1^*|\boldsymbol{E}|}}\cdot\sqrt{p+\mathrm{i}c}\right\}\right]$$

$$\cdot\left(\frac{m_\alpha}{2\pi kT_\alpha}\right)^{3/2}\exp\left\{\frac{m_\alpha(\boldsymbol{v}')^2}{2\,kT_\alpha}\right\}=0, \quad (12)$$

$$q=\frac{8}{5},\ \nu=\frac{5}{2},\ p=-\mathrm{i}\omega,\ c=\boldsymbol{k}\cdot\boldsymbol{v}+\mathrm{i}\,\Sigma_\alpha,$$

$$a_1=\frac{\sqrt{2}\,\sqrt{p+\mathrm{i}c}}{\sqrt{C_1^*}\,|\boldsymbol{E}|}.$$

$$C_1^*=\frac{1}{2\pi}\left(\frac{e_\alpha}{m_\alpha}\right)^2\left(\frac{\boldsymbol{k}_i\boldsymbol{k}_j}{\boldsymbol{k}^2}\right)(1-y)e^y. \quad (13)$$

The equations (11), (12) include the Σ_α and it is important especially when a resonance condition, $\omega = \boldsymbol{k}\cdot\boldsymbol{v}$, is satisfied, however an eigenvalue of the operator eq. (5) includes also $|\boldsymbol{E}|^2,\omega,\boldsymbol{k},\boldsymbol{v}$ so that the situation is complicated, so, we will discuss the most probable value of them from our experimental data.

3. The Experimental Data

In experiment we use mirror magnetic field of 80 (Gauss) in center and mirror ratio of 1.4. A stainless vessel for plasma region is 16 cm in diameter and 42 (cm) in length. The region is fed with argon gas from 10^{-4} to 7×10^{-4} Torr. A beam of 8 (mm) in diameter, 1.92 keV-18 (mA), is injected into plasma. Any plasma density of 10^{14}~5×10^{15} (m^{-3}) in center is controllable. A solitary wave appears as burst with time width 150~500 (nsec). The carrier frequency is ≃400 (MHz). The solitary waves are constantly generated as an intermittent burst in suspended time of every 4≃8 (μsec). The carrier frequency is $\omega = 2.505\times10^9$ (rad/s) while the accelerating voltage is given by 1.92 (kV), so that the beam velocity $v_0 \simeq 2.59 \times 10^7$ (m/s) then from the resonance condition, $\omega = \boldsymbol{k}_{\parallel} \cdot \boldsymbol{v}_0$, we can get the value of $\boldsymbol{k}_{\parallel}$ as $\boldsymbol{k}_{\parallel}$ = 103.3 (rad/m) $\simeq$ 100 (rad/m), while by our report [6], the $|\boldsymbol{k}|$ is given as: $|\boldsymbol{k}| \simeq 120\pi$ (rad/m).

Figure 1 shows a envelope soliton obtained by the experiment. The probes are used as antennae. The signal pass through coaxial cable with 50-Ω impedance and the cable is directly connected to the HP-54542A 2GHz-oscilloscope. The total length of probe (antenna) is 5 (cm), however only $\simeq$1 (cm) of the tip will be submerged into plasma. The terminal voltage at the oscilloscope is $\simeq$0.1 (V), so that the electric field is expected as $\simeq$0.1 (V/cm).

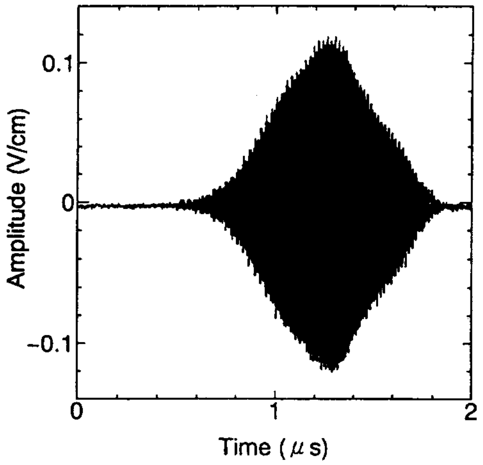

Fig.1 A solitary wave detected by 2-GHz oscilloscope is shown. Since the carrier frequency is 400 MHz, then the phase is crowded, however, the phase is confirmed to be continuous.

4. Soliton from the Theory

To get a soliton we limit the interacting velocity region,

$$\boldsymbol{v}_0 - \left(\frac{\mathrm{Im}\Sigma_\alpha}{2\boldsymbol{k}}\right) < \boldsymbol{v} < \boldsymbol{v}_0 + \left(\frac{\mathrm{Im}\Sigma_\alpha}{2\boldsymbol{k}}\right),$$

then the main contribution comes from the first term of eq. (11), and we get $\boldsymbol{E}(\boldsymbol{k},t)$ by inverse transformation.

$$\begin{aligned}\boldsymbol{E}(\boldsymbol{k},t) &= \sum_\alpha \left(\frac{e_\alpha}{\varepsilon_0}\right)\left(\frac{\mathrm{i}\boldsymbol{k}}{\boldsymbol{k}^2}\right) f_1^{(0)}(\boldsymbol{k}) \\ &\cdot \left(\frac{4^q \cdot n_b \cdot \boldsymbol{v}_0}{\pi^{1/2}\{C_1^* |\boldsymbol{E}|^2 t\}^{(3/2-q)}}\right) \\ &\cdot \left(\frac{1}{\boldsymbol{k}\cdot\boldsymbol{v}_0 t}\right) \cdot \exp(-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{v}_0 t) \cdot \sinh\left(\frac{\mathrm{Im}\Sigma_\alpha}{2} t\right) \\ &\cdot \Gamma\left(q, a_1 \left|\frac{\mathrm{Im}\,\Sigma_\alpha}{2k}\right|^\nu\right), \end{aligned} \tag{14}$$

$$a_1 = \frac{1}{4C_1^* |\boldsymbol{E}|^2 t}, \quad q = 1.3, \quad \nu = 5,$$

where the $\Gamma(q,x)$ is an incomplete Gamma function. At the limit $t \to 0$, $\boldsymbol{E}(\boldsymbol{k},t) \to 0$ owing to sinh $\{(\mathrm{Im}\Sigma_\alpha/2)t\}$ and $\Gamma(q, x \to \infty) \to 0$, while from the character $\mathrm{Im}\Sigma_\alpha \propto \exp\{-\mathrm{Re}\Sigma_\alpha t\}$ we get $\boldsymbol{E}(\boldsymbol{k}, t \to \infty) \to 0$ then the elementary excitation has finite life time.

Figure 2 shows the calculated $\boldsymbol{E}(\boldsymbol{k},t)$ when the beam electrons collide to ion wave. The following constants are used: correlative length in velocity space $|\boldsymbol{v} - \boldsymbol{v}'| \simeq 10^4$ (m/s), the slowing down ratio $\sqrt{m_e/m_i}$ = (1/500), no dimensional constant y = 0.508.

We find that the collision frequency, $\Sigma_\alpha(\boldsymbol{k},\boldsymbol{v},t)$, becomes usually a function of carrier frequency, $\boldsymbol{k}\cdot\boldsymbol{v}_0$, in the electron-electron collision, then the soliton does not appear because the $\boldsymbol{k}\cdot\boldsymbol{v}_0$, oscillates so fast, however if the response is retarded by ions corresponding to the frequency $\sqrt{m_e/m_i} \cdot (\boldsymbol{k}\cdot\boldsymbol{v}_0)$, then the soliton, having a time width $(1/\omega_i)$, appears.

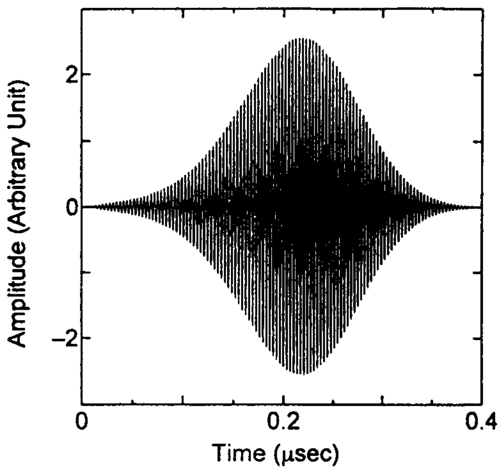

Fig.2 An example of the calculated wave packet (soliton) is shown. The following constants are used. $|\boldsymbol{v} - \boldsymbol{v}'| \simeq 10^4$ (m/s), $y = 0.508$, $\gamma = \{m_e/m_i\}^{1/2} \simeq$ (1/500).

5. Dispersion Relation

As shown in the nonlinear dispersion relation eq. (12) with eq. (13), the relative permittivity $\varepsilon_s(\boldsymbol{k},\omega,|\boldsymbol{E}|^2,\Sigma_\alpha)$ is a function of a Σ_α while the Σ_α

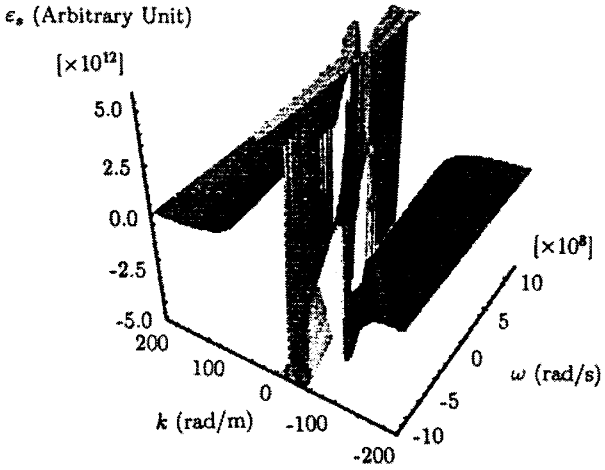

Fig.3 The relative permittivity $\varepsilon_s(\boldsymbol{k},\omega,|\boldsymbol{E}|^2,\Sigma_\alpha)$ of electron gas is shown, where the following constants are used. $|\boldsymbol{E}| = 0.01$, $|v - v'| = 10^4$ (m/s) and $y = 0.72$.

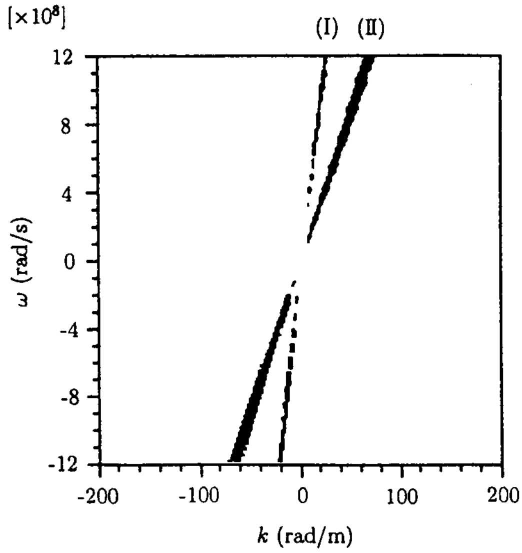

Fig.4 The cross section cutting by zero plane in the manifold of fig. 3, i.e., $\varepsilon_s(\boldsymbol{k},\omega) = 0$ is represented. There are two curves, where the curve (II) will be a slow space charge wave of beam electron and the curve (I) seems to be a fast space charge wave.

depends on $\boldsymbol{k}$, ω and $|\boldsymbol{E}|^2$, so we must solve them simultaneously. If we slice a manifold $\varepsilon_s(\boldsymbol{k},\omega,|\boldsymbol{E}|^2,\Sigma_\alpha)$ with the surface $|\boldsymbol{E}|$ = constant and y = constant, then the situation is some what simpler and we can get a cross section of zero-plane which will be expected to show the dispersion relation under the condition $|\boldsymbol{E}|$ = constant. Matters are too complicated, so, as an example we neglect maxwellian term in eq. (12) and leave only a beam term and examine to solve hereafter by numerical method.

Figure 3 represents the case $|\boldsymbol{E}| = 0.02$ and $y = 0.6$. Figure 4 shows two curves (I, II) in ω-$\boldsymbol{k}$ plane. It seems that the curve (II) is a slow space charge wave of beam electron and the other curve (I) is a fast space charge wave. Both curve are quite similar to those derived from the linear dispersion relation.

References

[1] I.M. Al'tshul and V.I. Karpman, Sov. Phys. JETP **22**, 361 (1966).
[2] T.H. Dupree, Phys. Fluids **9**, 1773 (1966).
[3] J. Weinstock, Phys. Fluids **12**, 1045 (1969).
[4] M. Kono and Y.H. Ichikawa; Prog. Theor. Phys. **49**, 754 (1973).
[5] C.W. Horton, *Longtime Prediction in Dinamics* (John Wiley, 1983) p.311.
[6] I. Mori, T. Morimoto, R. Kawakami and K. Tominaga, *to be published in* J. Plasma Fusion Res. **2** (1999).