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Ion-Beam Focusing by Plasma BGK Low-Frequency Oscillations in the Nonhomogeneous Magnetic Field

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Abstract

The formation of the radial electric field which is sufficient for ion beam focusing in the current-carrying plasma in the vicinity of a magnetic barrier is investigated. It is shown that the instability development in the current-carrying plasma, plasma electron trapping by wave field and their propagation along the magnetic coil axis lead to the formation of a radial electric field.

Keywords:
ion beam focusing, electric field of polarization, low-frequency instability

1. Introduction

It is well-known that in the plasma being in the external electric field in the vicinity of the magnetic barrier a formation of polarization electric fields can take place. These fields are sufficiently strong to enable the ion beam focusing. Also, in the current-carrying plasma the excitation of a wave field (instability) was observed (see, for example, [1]).

Let us show that this instability can lead to the formation of the focusing electric field in the vicinity of the magnetic barrier (for instance a coil).

2. Development of the Ion-Acoustic Instability in the Current-Carrying Plasma

The ion-acoustic instability is developed when the electron drift velocity V_d is less than the electron thermal velocity V_{th}. At the linear stage the wave amplitude increases with the growth rate $\gamma \approx (\pi m_e/8m_i)^{1/2}kV_d$ [1]. At the nonlinear stage the plasma electrons are trapped by a wave field. The nonlinear stage of the wave evolution begins when the frequency of resonant electron oscillation in the wave field $\Omega^e_{tr} = k(e\phi_k/m_e)^{1/2} = kV_{tr}/2$ exceeds the growth rate γ. Taking into account that the trapped particles are phase mixed we can obtain for the electron distribution function the following expression

$$f = \begin{cases} \left[f_e\left(\bar{u} + V_{ph} - V_d\right) + f_e\left(\bar{u} - V_{ph} + V_d\right)\right]/2, \\ \qquad |\bar{u}| \le V_{tr} \\ f_e\left(\bar{u} + V_{ph} - V_d\right), \; |\bar{u}| \ge V_{tr} \end{cases} \qquad (1)$$

$$\bar{u} = \lambda^{-1}\int_{\xi}^{\xi+\lambda} (v - V_{ph})\,d\xi.$$

The nonlinear stage begins with the small amplitude $\phi^o_k \approx (\gamma/\omega)^2(m_e/m_i)(T_e/e) << (T_e/e)$. The amplitude saturation proceeds through the mechanism of plasma ion trapping which takes place when the amplitude value reaches $\phi_k = (V_{ph} - V_{Ti})^2 m_i/e$, here $V^2_{Ti} = T_i/m_i$. This value corresponds to the trapping of a major part of plasma electrons by the nonlinear BGK wave field. The

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trapped electrons are moving with a mean velocity equal to the wave phase velocity V_{ph}.

As the magnetic coil is a barrier for electrons the latters are decelerated in the longitudinal direction depending on the invariant values $V_{//}^2 + V_\perp^2 = const$, $V_\perp^2/H = const$. Here $V_{//}$, $V_\perp$ are the transversal and longitudinal electron velocities, respectively, H is the magnetic field strength of the coil. Consequently, to maintain the mean energy of trapped electrons, the wave loses its energy by decreasing its amplitude. However, there is the mechanism of amplitude sustaining at an initial level, namely, the amplitude is pumped by the external electric field. The trapped electrons transfer the energy from E_0 to the wave by oscillating in the wave field and E_0.

Let us determine the growth rate of the instability development and the energy transfer to the wave. The expressions for the electron and ion density perturbations can be obtained from the Vlasov equation

$$\begin{aligned} &\delta n_k^e / n_0 = e^2\, i\phi_k\, E_0 / kT_e^2 - \\ &-(\pi/2k)(e/m_e)^2 \phi_k\, E_0\, \partial^3_{,v} f_0^e \mid_{V=V_{ph}} - \\ &-i(e/m_e)\pi\phi_k\, \partial_{,v} f_0^e \mid_{V=V_{ph}} - \\ &-e^3 E_0^2\, \dot{\phi}_k\, k^2\, T_e^3 + e\phi_k / T_e, \\ &\delta n_k^i / n_0 = e\phi_k / m_i\, V_{ph}^2 \end{aligned} \qquad (2)$$

The substitution of (2) into the Poisson equation gives the expression for the growth rate of instability γ_k

$$\begin{aligned} \gamma_k = \gamma + \gamma_E = (\pi/8)^{1/2} (m_e/m_i)^{1/2} \\ (V_d / V_{T_e}) k V_{gr} - eE_0 V_{gr} / 2T_e \end{aligned} \qquad (3)$$

Here γ is the linear growth rate and γ_E is the growth rate of wave amplitude pumping due to the external electric field E_0, V_{gr} is the group velocity of the wave.

The fields in the plasma outside the coil, E_p, and inside the coil, E_k, are connected by means of the plasma conductivities $E_p = E_k\sigma_k/\sigma_p$. Now, we use the expression $\sigma_k/\sigma_p \approx (v_{ef}/\omega_{ce})^2(n_k/n_p)$. For $v_{ef} << \omega_{ce}$ the plasma conductivity in the coil region σ_k is much less than the plasma conductivity outside the coil $\sigma_k << \sigma_p$. Hence, all external potential is concentrated in the coil region. Inasmuch as, the electrons are magnetized, the trapped electrons moving with the wave into the coil interior are directed to the coil axis in conformity with the magnetic field structure. In this way the radial polarization of plasma flow appears [2]. The electrons, in the crossed radial electric field E_r and the longitudinal magnetic field, move in the azimuth direction and oscillate along the radius. To describe the electron dynamics we use the equations

$$\begin{aligned} &\dot{V}_r + [V_\theta\, \omega_{ce} - (\omega_p^2 r/2 + V_\theta^2/r)] = 0, \\ &\dot{V}_\theta + V_r\,(V_\theta / r - \omega_{ce}) = 0 \end{aligned} \qquad (4)$$

(where " . " is the time derivative) similar to [3] for the electron azimuth motion and electron radial oscillations with velocities $V_\theta \equiv r\dot{\theta}$ and $V_r \equiv \dot{r}$, respectively. Here $\omega_p^2 = 4\pi e^2\delta n/m_e$, $\delta n = n_e - n_i$. From these equations it follows that the electron oscillatory velocity is equal to $V_r = (e/m\Omega_e)E_{ro}\cos(\Omega_e t)$, $\Omega_e \equiv (\omega_{ce}^2 - 2\omega_p^2)^{1/2}$ and the electron oscillation radius is $R_{ce} = eE_r/m_e(\omega_{ce}^2 - 2\omega_p^2)$. Therefore and because of the fact that the radius of a cylinder filled with electrons is less than the radius of a cylinder filled with ions, $R_{ce} < r_i$, and taking into account the balance of forces, we derive the inequality for radial plasma polarization $\delta n \le H_0^2/8\pi m_e c^2$. When $\delta n \le 1{,}25\cdot 10^{10}$ cm^{-3} this inequality provides the radial electric field $E_r = -er_p H^2/4m_e c^2$ as great as 10 kV/cm.

We have performed the computer simulation of ion beam focusing in the field of a radial polarization of electrons relatively to ions in the plasma. In this case the electrons trapped by the low-frequency wave field are moving to the coil region. The simulation shows that the size of a focused spot is not strongly influenced by the phase of the ion beam injection if the length of the coil is much longer than the wavelength.

3. Conclusion

The results of this investigation show that if the wave with trapped electrons runs into the magnetic barrier, the polarization electric field can be formed. This field is sufficient for the focusing of the high-energy ion beams.

References

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