Particle Confinement Improvement in $L = 2$ Heliotrons through Plasma Boundary Modulations

YOKOYAMA Masayuki, NAKAJIMA Noriyoshi and OKAMOTO Masao
National Institute for Fusion Science, Toki 509-5292, Japan

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Abstract

The possibility of further improvement of collisionless particle confinement in $L = 2$ heliotrons has been considered with exploring the magnetic configurations by the plasma boundary modulations. An example $L = 2$ configuration with partial quasi-helical symmetry with the improved collisionless particle confinement has been obtained.

Keywords:
$L = 2$ heliotron, plasma boundary modulations, quasi-helical symmetry, collisionless particle confinement

1. Introduction

Among several helical system researches for plasma confinement, the concept of $L = 2$ helical heliotrons based on $L = 2$ continuous helical coils had been successfully developed with Heliotron E [1]. These developments for $L = 2$ heliotrons have led to the Large Helical Device (LHD) [2], which has successfully begun experiments in 1998. However, the parameter space of $L = 2$ heliotron configurations seems to have been relatively limited due to the utilization of continuous $L = 2$ helical coils.

The helical systems inherently have the three dimensionality, which can give the large possibility for attractive concepts for magnetic configurations. Recently, many theoretical optimization studies have been done for helical systems such as quasi-helical symmetry [3], quasi-axisymmetry [4] and quasi-isodynamicity [5]. These studies have been done based on the nonlinear programming method by taking the plasma boundary shape as the parameter to reach the desirable physical properties, not based on the initial definition of the coil configurations. Therefore, wide range of helical configuration space can be explored with relatively easy tailoring of the magnetic field structure.

In order to explore rather wide range of $L = 2$ magnetic configuration space, this approach has been applied for $L = 2$ heliotrons with plasma boundary modulations as the parameter for changing the magnetic configurations. In this case, $L$ does not denote the polarity of continuous helical coils, but does the poloidal mode number of the predominant helicity in the magnetic field strength. In this paper, an example $L = 2$ magnetic configuration with the partial quasi-helical symmetry is shown.

2. Partial Quasi-Helically Symmetric $L = 2$ Heliotron Configuration

The plasma boundary shape is the parameter for this study. The VMEC [6] has been utilized to calculate magnetohydrodynamic (MHD) equilibria. The plasma boundary shape can be expressed by the Fourier representation in the cylindrical coordinates $(R, \phi, Z)$ as:

\[
\begin{align*}
R (s, \theta_v, \zeta_v) &= \sum_{m \neq 0} R_{m, 0} (s) \cos (m \theta_v + nM \zeta_v), \\
Z (s, \theta_v, \zeta_v) &= \sum_{m \neq 0} Z_{m, 0} (s) \sin (m \theta_v + nM \zeta_v).
\end{align*}
\]

Corresponding author's e-mail: yokoyama@nifs.ac.jp
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Here, \( \theta(t, \zeta_t) \) is the poloidal (toroidal) angle in the VMEC, \( m(n) \) the poloidal (toroidal) mode number and \( M \) the field period number. Based on the MHD equilibrium obtained by the VMEC, transformation to Boozer coordinates \( (\psi, \theta_b, \zeta_b) \) [7] is carried to obtain the magnetic field spectra as

\[
B(\psi, \theta_b, \zeta_b) = \sum_{m} B_{m} \cos(m \theta_b + n M \zeta_b),
\]

for collisionless particle orbit calculations by the guiding center equations in the Boozer coordinates [8]. After evaluating the collisionless particle confinement properties by introducing some guidelines (e.g., loss rate, time of starting loss [5] etc.), if its property is not satisfactory for the desirable criteria, the plasma boundary shape is modulated again by varying \( R_m \) and \( Z_m \) and these calculations are iterated. Of course, several other physical aspects such as MHD stability and neoclassical transport, magnetic island properties etc. should be also taken into account for total optimization. However, as the first step for the total optimization of \( L = 2 \) heliotrons based on this approach, only the collisionless particle confinement has been considered by small number of iterations by using small number of \( R_m \) and \( Z_m \) to demonstrate the usefulness of this approach.

The simple \( L = 2 \) heliotron configuration (Conf. 1) is the initial configuration of this study. It has elliptic magnetic surfaces which rotate in the toroidal direction. The field period number is 10 as an example. This configuration can be expressed by \( R_m \) and \( Z_m \) as listed in Tab. I, whose magnetic surfaces at \( \phi = 0 \), 1/4 period and 1/2 period are shown in Fig. 1 with solid, chain and dotted curves, respectively. This configuration has the toroidicity in the magnetic field spectra \( (B_{1,0}) \) as almost equal to the geometrical inverse aspect ratio. The toroidicity should be eliminated to realize a quasi-helically symmetric configuration. The reduction of the toroidicity can be performed with the helical magnetic axis as shown in Ref. [9]. Therefore, \( R_{0,1} \) and \( Z_{0,1} \) components are added to construct the plasma boundary shape. After reducing \( |B_{1,0}| \) and tailoring several magnetic field spectra, an example \( L = 2 \) configuration (Conf. 2) with the partial helical symmetry (although \( B_{1,1} \) term still remains) is obtained, whose magnetic surfaces are shown in Fig. 2. The magnetic field spectra are also shown in Fig. 3, where \( B_{0,0} \) curve corresponds to the difference, \( B_{0,0}(r) - B_{0,0}(0) \) with \( r \) the averaged minor radius. The Fourier coefficients for the plasma boundary shape are summarized in Tab. II.

Collisionless particle confinement is studied by following the protons launched from \( r/a = 0.25, 0.5, 0.75 \) magnetic surfaces with assumed temperature profile: \( T(r) = 1.0(1 - r/a)^2 \) keV. The number of launched particles from uniformly distributed points in the Boozer poloidal and toroidal angles is defined by considering the area element, \( dS = J|\nabla \psi|d\theta_0 d\zeta_b \), to take into account its variation on a magnetic surface. Here, \( J \) denotes the Jacobian of the Boozer coordinates. The particles are followed for 2 ms or until they cross the plasma boundary. The loss rate is decreased from

<table>
<thead>
<tr>
<th>( (m,n) )</th>
<th>( R_{mn}[m] )</th>
<th>( Z_{mn}[m] )</th>
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</thead>
<tbody>
<tr>
<td>(0,0)</td>
<td>5.0</td>
<td>0.0</td>
</tr>
<tr>
<td>(1,0)</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>(1,1)</td>
<td>-0.39</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Fig. 1 Magnetic surface cross sections at \( \phi = 0, (1/4)(2\pi/M) \) and \( (1/2)(2\pi/M) \) for a simple \( L = 2 \) heliotron configuration (Conf. 1).

Fig. 2 Magnetic surface cross sections at \( \phi = 0, (1/4)(2\pi/M) \) and \( (1/2)(2\pi/M) \) for an \( L = 2 \) heliotron configuration (Conf. 2) with the partial quasi-helical symmetry.
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### Table II

<table>
<thead>
<tr>
<th>((m,n))</th>
<th>(R_{\text{out}})</th>
<th>(Z_{\text{out}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>((0,0))</td>
<td>5.0</td>
<td>0.0</td>
</tr>
<tr>
<td>((0,1))</td>
<td>0.2</td>
<td>-0.2</td>
</tr>
<tr>
<td>((1,0))</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>((1,1))</td>
<td>-0.39</td>
<td>0.4</td>
</tr>
<tr>
<td>((2,0))</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>((2,1))</td>
<td>-0.03</td>
<td>-0.035</td>
</tr>
</tbody>
</table>

6.2\% (among 11549 particles) to 3\% (among 9255) for Conf. 1 to Conf. 2. It is noted that the loss rates almost saturate until 2 ms orbit following time. This improved particle confinement can be explained by considering the magnetic field topography. The distribution of (a) magnetic field strength \(B\), (b) \((1/B)(\partial B/\partial \rho_h)\) and (c) \((\psi/B)(\partial B/\partial \psi)\) on \(r/a = 0.75\) magnetic surface (one period) are shown in Fig. 4.1 (4.2) for Conf. 1 (2), respectively.

The latter two quantities strongly affect the particle orbit behavior through the radial and poloidal drift in the Boozer coordinates \([7]\)

\[
\psi = \frac{\delta}{\gamma} \left( \frac{\partial B}{\partial \rho_h} I - \frac{\partial B}{\partial \theta_h} \right),
\]

\[
\theta_h = \frac{e}{\gamma} \left( \frac{\partial B}{\partial \psi} + e \frac{\partial \Phi}{\partial \psi} \right) - \frac{e^2 B^2}{m \rho_c} \left( \rho_c \gamma - \frac{\theta_h}{\gamma} \right),
\]

where the same notations as in Ref. [7] are used. When the currentless \((I = 0)\), zero beta \((\gamma' = 0)\) and zero radial electric field \((\Phi = 0)\) cases are considered, \(\partial B/\partial \rho_h\) \((\partial B/\partial \psi)\) contributes to the radial (poloidal) drift. The region denoted by \(Min\) in Figs. (a) corresponds to the local minimum of \(B\), where the locally trapped particles (or blocked particles) appear. It is noted that the unclosed contours exist surrounding this region in Fig. 4.2(a), which is considered to have quasi-helically symmetric

![Fig. 3 Magnetic field spectra in the Boozer coordinates for Conf. 2.](image)

![Fig. 4.1](image)

![Fig. 4.2](image)

Fig. 4 The distribution of (a) magnetic field strength \(B\), (b) \((1/B)(\partial B/\partial \rho_h)\) and (c) \((\psi/B)(\partial B/\partial \psi)\) on \(r/a = 0.75\) magnetic surface (one period) for Conf. 1 (Fig. 4.1) and Conf. 2 (Fig. 4.2).
property. The structure of the contour of $(1/B)(\partial B/\partial \theta_0) = 0$ (zero radial drift) does not change compared to Figs. (b). However, the distribution of $(\psi/B)(\partial B/\partial \psi)$ is significantly varied and rather well alignment of fast poloidal drift region to the local minimum of $B$ is seen in Fig. 4.2(c). This is considered to be the main reason for the improved collisionless particle confinement in Conf. 2 compared to Conf. 1.

The region of local minimum of $B$ exists around $\theta_0/(2\pi) = 0$ (outerside of a torus) due to the toroidicity in Conf. 1 (cf., Fig. 4.1(a)). As shown in Fig. 4.2(a), this region is shifted to around $\theta_0/(2\pi) = 0.5$ (innerside of a torus) due to the elimination of the toroidicity based on the plasma boundary modulation approach. Thus, this approach would give the large possibility to explore wide $L = 2$ heliotron configuration space, which has not yet explored based on the definition of the coil system. Consideration of the magnetic field topography is the one important guidance to approach the good collisionless particle confinement as demonstrated here.

3. Summary
The possibility of further improvement of collisionless particle confinement in $L = 2$ heliotrons has been considered by plasma boundary modulations. This approach has been successful to decrease (even eliminate) the toroidicity in the magnetic field, which has not been realized in the $L = 2$ heliotrons obtained from the definition of coil system. An example $L = 2$ heliotron configuration with good collisionless particle confinement is shown, where the region of fast poloidal drift aligns significantly well to the local minimum of the magnetic field strength on a magnetic surface. This configuration even has a partial quasi-helically symmetric property. These results demonstrate the usefulness of the plasma boundary modulation approach in $L = 2$ heliotron concepts. Thus, further detailed optimization study based on large computations should be explored including several other physical aspects.

For experimental realization, it is necessary to find a distribution of the external coil currents. The NESCOIL code [10] can be utilized to examine the possibility of reasonable coil configurations, which also should be taken into account for optimization study.

References