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δf Simulation of Ion Neoclassical Transport

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Abstract

Ion neoclassical transport with finite orbit width dynamics is calculated over whole poloidal cross section by using accurate δf method which employs an improved like-particle collision operator and an accurate weighting scheme to solve drift kinetic equation. Ion thermal transport near magnetic axis shows a great reduction from its conventional neoclassical level due to non-standard orbit topology, like that of previous δf simulation. On other hand, the direct particle loss from confinement region may strongly increase ion energy transport near the edge. It is found that ion parallel flow near the axis is also largely reduced due to non-standard orbit topology. In the presence of steep density gradient, ion thermal conductivity is significantly reduced, and an ion particle flux is driven by self-collision alone.

Keywords:
neoclassical transport, finite orbit width dynamics, δf method

The study of neoclassical transport phenomena is newly motivated by recent development in fusion experiments where plasmas are operated in parameter regimes beyond the valid scope of the conventional neoclassical theory [1]. The modifications to the neoclassical theory are required for taking into account finite orbit width dynamics, strong radial electric field with large radial gradient, and non-standard orbit topology near magnetic axis. The low noise δf particle simulation, solving drift kinetic equation, can be a powerful tool for neoclassical transport calculation [2,3].

We solve linearized drift kinetic equation (distribution function $f = f_0 + f_1$ with f_0 and f_1 being the zeroth and first order parts, respectively)

$$\frac{\mathrm{D}}{\mathrm{D}t} f_1 = -\vec{v}_\mathrm{d} \cdot \nabla f_0 + C(f_0, f_1), \qquad (1)$$

from the dynamics of a finite number of particles (markers), where the notation D/Dt denotes, for an arbitral function f_A,

$$\frac{\mathrm{D}}{\mathrm{D}t} f_\mathrm{A} = \frac{\partial f_\mathrm{A}}{\partial t} + (\vec{v}_\parallel + \vec{v}_\mathrm{d}) \cdot \nabla f_\mathrm{A} - C(f_\mathrm{A}, f_0), \qquad (2)$$

and C is the Coulomb collision operator. Here the drift term is retained to the leading order in order to take into account finite orbit effects. Each marker is pushed in the extended phase space $(\vec{r}, \vec{v}, w)$ along characteristic

$$\frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = \dot{\vec{r}}, \quad \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = \dot{\vec{v}}, \quad \text{and} \quad \frac{\mathrm{d}w}{\mathrm{d}t} = \dot{w}. \qquad (3)$$

The numerical representation of f_1 is

$$f_1(\vec{r}, \vec{v}, t) = \sum_j w_j \delta(\vec{r} - \vec{r}_j(t)) \delta(\vec{v} - \vec{v}_j(t)). \qquad (4)$$

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The particle weight w is calculated from

$$\dot{w} = \frac{1}{g}\left[-\int w S_{\mathrm{M}} \mathrm{d}w - \vec{v}_{\mathrm{d}} \cdot \nabla f_0 + C(f_0, f_1)\right], \quad (5)$$

where S_{M} is the marker source used to control marker population and g is marker density [4].

Accurate implementation of Coulomb collisions is an important ingredient for neoclassical transport calculation. Collisions between ions and electrons can be simply modeled by employing the large mass ratio approximation. Here we present a linear like-particle collision model, accurately conserving all particle number, momentum, and energy for δf simulation. Our collision model reads

$$C^{\ell}(f_1) = C_0^{\ell} + [P_1 + Q_1 + P_2 + Q_2] f_{\mathrm{M}}, \quad (6)$$

where

$$C_0^{\ell}(f_1) = C_{TP}(f_1) + P f_{\mathrm{M}}, \quad (7)$$

$$P(\vec{r}, \vec{v}) = \vec{A} \cdot \frac{\mathrm{d}}{\mathrm{d}t}\left\langle \vec{v} \right\rangle_{TP} + B\frac{\mathrm{d}}{\mathrm{d}t}\left\langle v^2 \right\rangle_{TP}, \quad (8)$$

$$P_1(\vec{r}, \vec{v}) = \vec{A} \cdot \frac{\mathrm{d}}{\mathrm{d}t}\left\langle \vec{v} \right\rangle_{TP} + B\frac{\mathrm{d}}{\mathrm{d}t}\left\langle v^2 \right\rangle_{TP+P}, \quad (9)$$

$$Q_1(\vec{r}, \vec{v}) = D\frac{\mathrm{d}}{\mathrm{d}t}\left\langle v^0 \right\rangle_{P}, \quad (10)$$

$$P_2 = \vec{A} \cdot \frac{\mathrm{d}}{\mathrm{d}t}\left\langle \vec{v} \right\rangle_{TP+P+P_1+Q_2} + B\frac{\mathrm{d}}{\mathrm{d}t}\left\langle v^2 \right\rangle_{TP+P+P_1+Q_1}, \quad (11)$$

$$Q_2(\vec{r}, \vec{v}) = D\frac{\mathrm{d}}{\mathrm{d}t}\left\langle v^0 \right\rangle_{P+P_1+Q_1}, \quad (12)$$

$$\vec{A} = -6\sqrt{\frac{\pi}{2}}\,\phi\,\frac{v_{\mathrm{th}}\vec{v}}{v^3}, \quad (13)$$

$$B = -2\sqrt{\frac{\pi}{2}}\left[\phi - \frac{\mathrm{d}\phi}{\mathrm{d}y}\right]\frac{1}{v v_{\mathrm{th}}}, \quad (14)$$

$$D(\vec{r}, \vec{v}) = 3\sqrt{\frac{\pi}{2}}\left[\phi - \frac{\mathrm{d}\phi}{\mathrm{d}y}\right]\frac{v_{\mathrm{th}}}{v} - 1, \quad (15)$$

and $\phi(y)$ is Maxwellian integral with $y = v^2/v_{\mathrm{th}}^2$. The $C_{TP}(f_1)$ is drag and diffusion part that describes the test particle (f_1) collisions with Maxwellian field particles (f_{M}). The Pf_{M} originating from $C(f_{\mathrm{M}}, f_1)$ works for compensating $\frac{\mathrm{d}}{\mathrm{d}t}\langle\vec{v}\rangle_{TP}$ and $\frac{\mathrm{d}}{\mathrm{d}t}\langle v^2\rangle_{TP}$, the averaged momentum and energy losses due to C_{TP}, respectively. C_0^{ℓ} represents the previous collision operator [2,5]. It is shown that the accuracy in particle, momentum, and energy conservation is insufficient with using C_0^{ℓ}. There are still considerable non-vanishing $\frac{\mathrm{d}}{\mathrm{d}t}\langle\vec{v}\rangle_{TP+P}$, $\frac{\mathrm{d}}{\mathrm{d}t}\langle v^2\rangle_{TP+P}$ and $\frac{\mathrm{d}}{\mathrm{d}t}\langle v^0\rangle_{P}$ coming from the use of a finite number of particles, and the numerical errors may get so enlarged as to distort simulation results. The compensation terms are then developed to restore the conservations. For example, $P_1 f_{\mathrm{M}}$ is introduced to compensate the momentum and energy loss, $\frac{\mathrm{d}}{\mathrm{d}t}\langle\vec{v}\rangle_{TP+P}$ and $\frac{\mathrm{d}}{\mathrm{d}t}\langle v^2\rangle_{TP+P}$, and $Q_1 f_{\mathrm{M}}$ is used to compensate $\frac{\mathrm{d}}{\mathrm{d}t}\langle v^0\rangle_{P}$ (the particle loss coming from the implementation of Pf_{M}). With the suggested compensation terms, all the three quantities are conserved nearly perfectly. The calculations of neoclassical transport using the present collision scheme demonstrate the increased accuracy in results. A shifted Maxwellian solution of ion drift kinetic equation under $\nabla T = 0$, which gives zero particle and energy flux, is recovered (Fig.1) owing to the use of the improved operator.

The calculation of particle weight is crucial in a δf simulation. It is found that valid results essentially rely on the correct evaluation of marker density g in weight calculation. The function g satisfies

$$\frac{\mathrm{D}}{\mathrm{D}t} g = \int S_{\mathrm{M}} \mathrm{d}w. \quad (16)$$

Previous weighting schemes including the nonlinear weighting scheme [2,6] employ an assumed g in weight equation for advancing particle weights. Such a schems is found to be ineffective or inaccurate for solving the drift kinetic equation because of a severe constraint that the real marker distribution must be consistent with the initially assumed g during the simulation. Otherwise, correct results can not be guaranteed. Instead using an approximation of g for advancing particle weights, we solve g directly from its kinetic equation using the idea of δf method. To this end, we employ a new weight function ω and a marker source Ω_M. The accurate weight equation for each marker is derived as

$$\dot{w} = \frac{1-\omega}{f_0}\left[-\int w S_{\mathrm{M}} \mathrm{d}w - \vec{v}_{\mathrm{d}} \cdot \nabla f_0 + C(f_0, f_1)\right], \quad (17)$$

$$\dot{\omega} = \frac{1-\omega}{f_0}\left[-\int \omega \Omega_{\mathrm{M}} \mathrm{d}\omega - \vec{v}_{\mathrm{d}} \cdot \nabla f_0 + \int S_{\mathrm{M}} \mathrm{d}w\right], \quad (18)$$

Now each simulation particle is assigned two weights rather than one. A convienient choice for the sources is $S_{\mathrm{M}} = \nu(t) s(\vec{r}) f_0 \delta(w)$, and $\Omega_M = \nu(t) s(\vec{r}) f_0 \delta(\omega)$. This choice means that new Maxwellian markers with $w = \omega = 0$ are added in terms of the rate $\nu(t)$ and spatial

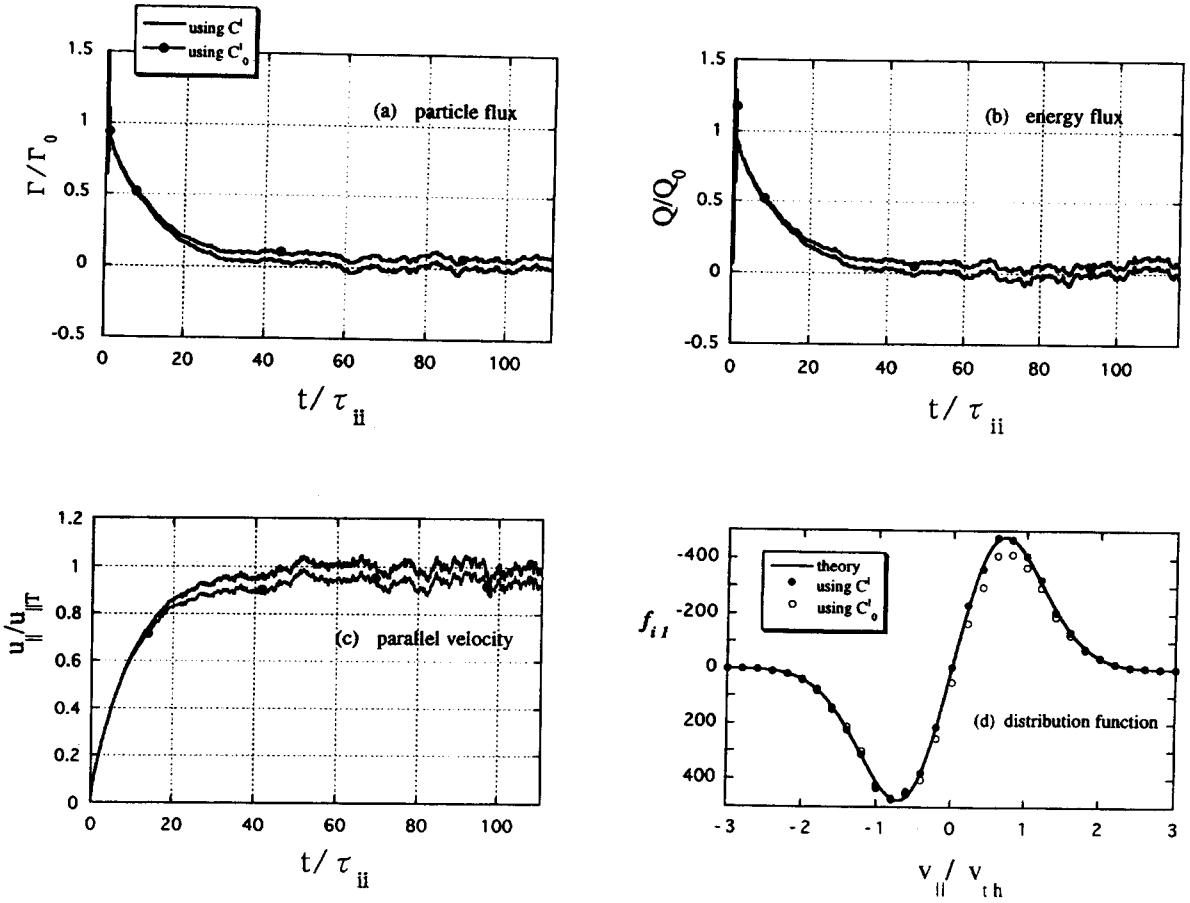

Fig. 1 Comparison of simulation results from using present and previous collision operators, with zero orbit width and $\nabla T = 0$. Γ_0 and Q_0: fluxes computed from using non-conservation operator C_{TP}; $u_{\parallel T}$: analytical parallel velocity.

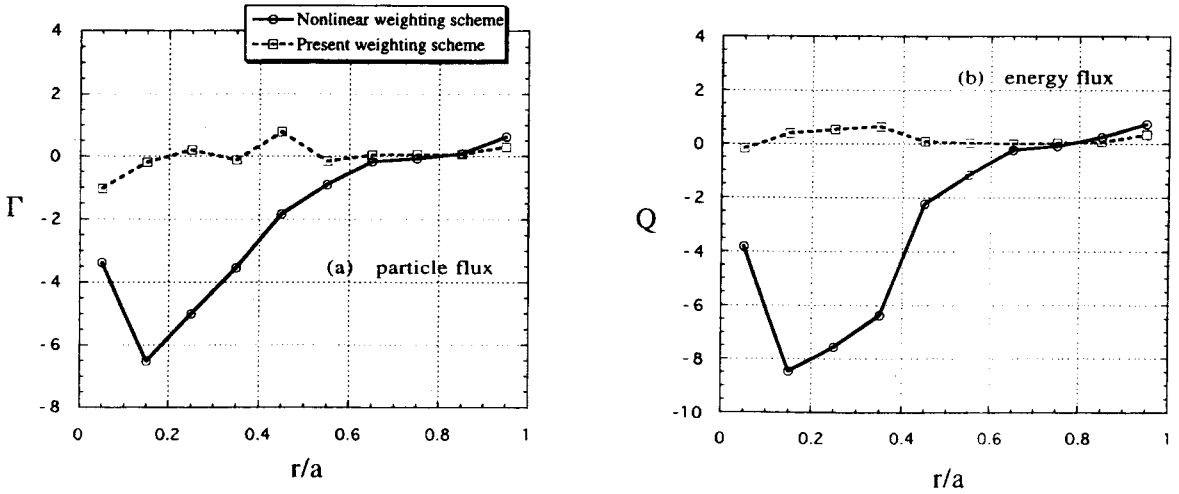

Fig. 2 Comparison of simulation results from using present weighting scheme and the nonlinear weighting scheme, with finite orbit width and $\nabla T = 0$.

distribution $s(\vec{r})$.

The results of simulations (solving Eq.(1) with $\nabla T = 0$) using the previous nonlinear scheme and the present scheme are compared in Fig.2. Because $g = f$, which is assumed for advencing particle weights, is not recognized in the simulation, the nonlinear weighting scheme fails to give the correct particle flux and energy flux. While the accurate weighting scheme works for the problem effectively; it leads to the correct results with vanishing particle and energy fluxes over the whole cross section.

A δf simulation code "FORTEC" (Finite ORbit Transport study by the Extensive Code) has been developed based on the above new method. The modifications to ion neoclassical transport from the finite orbit width dynamics are investigated by "FORTEC". The simulations which are performed over global poloidal cross section enable us to calculate the neoclassical transport in both the region near magnetic axis and that far from it (including the edge region of confined plasma). Moreover, the global simulations enable us to calculate nonlocal transport fluxes which depend not only on the local gradients but on the entire density and temperature profiles as well. Our calculations focus on the banana-plateau regime in which the present fusion plasmas are most operated.

A typical feature of ion thermal conductivity over the whole cross section is shown in Fig.3. The ion thermal conductivity in the region within inner half radius is greatly reduced from its conventional neoclassical level due to non-standard orbit topology near the axis. This modification due to finite banana width dynamics was previously proposed [3] to interprete the very low ion thermal transport observed in the enhanced reversed shear (ERS) core plasmas. The reduction of ion thermal conductivity is attributed to both the decrease of orbit size and the increase of the trapped particle fraction. This finite orbit effects commonly exist in the tokamak plasmas near the axis. However, the reduced ion neoclassical thermal transport is revealed only as turbulence transport is suppressed, as achieved in ERS plasmas. In contrast to the region near the axis, the ion energy transport near the edge is strongly enhanced by the direct particle loss from the confinement region.

Another important transport quantity that is modified by the finite orbit effect is ion parallel flow. A typical simulation result of ion parallel flow is shown in Fig.4. It is found that the ion parallel flow near the axis is also largely reduced as the finite orbit width dynamics

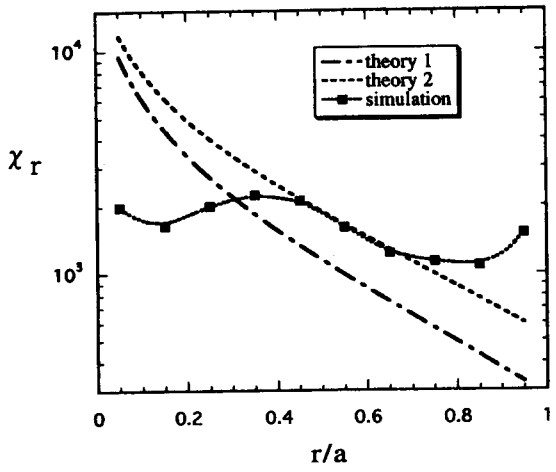

Fig. 3 Ion thermal conductivity χ_r vs. minor radius r (theory 1-Ref. [1], theory 2-Ref. [8]). An uniform q-profile (q = 1.1) is used here as well as in Fig.4 below.

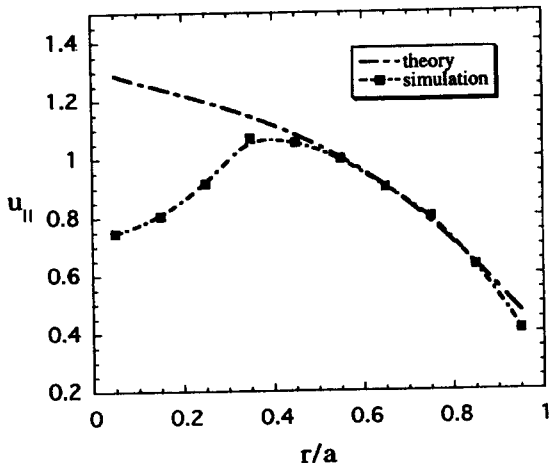

Fig. 4 Ion parallel flow $u_\parallel$ vs. r.

is taken into account. The modification of the parallel flow again results from the non-standard orbit topology. For trapped ions, a net parallel flow results from the noncancellation of the parallel momentum carried by co-moving ions coming from the high temperature and density inner region and antiparallel momentum carried by counter-moving ions coming from the low temparature and density outer region. Therefore, the net parallel flow is driven by the plasma gradients, and is proportional to the radial excursion of trapped particles, the banana width Δ_b. It is found that the real banana width near the axis is smaller than the conventional neoclassical estimation. Then the finite banana effect

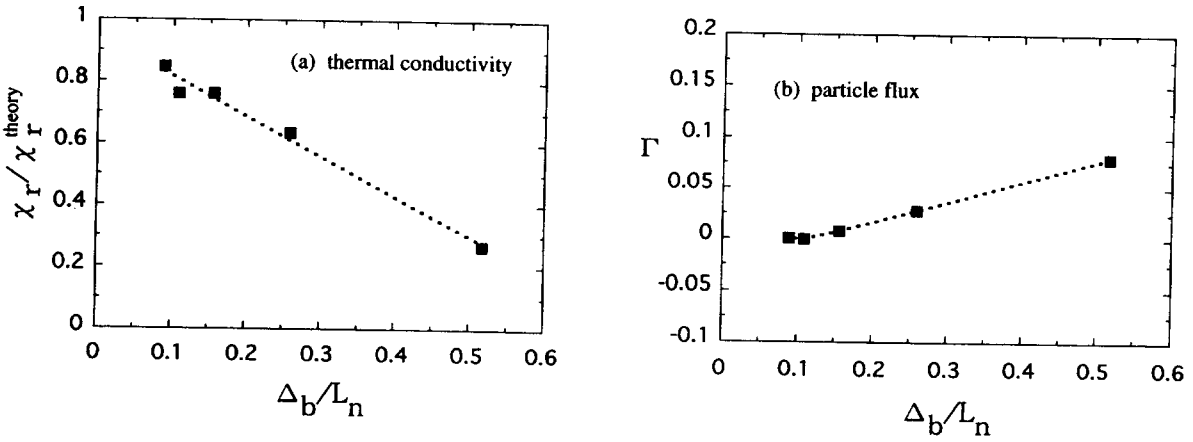

Fig. 5 Ion thermal conductivity χ_r and particle flux Γ vs. Δ_b (banana width)/L_n (density gradient scale length).

could help reduce the parallel flow near the axis. But this reduction is small because the ion parallel flow is mostly contributed by passing ions rather than trapped ions. The point is as follows. The collisions between trapped and passing ions force the passing ion distribution function to be continuous with the trapped distribution. A displacement δ of particles from their home flux surface Ψ is definrd by [7] $f = f_M(\varepsilon, \Psi) - \delta\partial f_M/\partial r$ (ε: energy). For trapped ions, δ is about the half-width of the banana orbits. The continuity of the distribution function across the trapped-passing boundary implies that $\delta_p \approx \delta_b$. That is, the relative displacement of co-moving passing ions from counter-moving passing ions is determined by the orbit width of barely trapped ions, which is, in the region near the axis, proportional to the local minor radius r and is significantly smaller than the conventional neoclassical estimate. The parallel flow arising from the relative displacement between co- and counter-moving passing ions is therefore reduced. The ion parallel flow represents a friction on electrons. As an interesting consequence, the modification of ion parallel flow may modify the standard neoclassical bootstrap current near the axis.

Figure 5(a) shows the dependence of ion thermal conductivity on density gradient length. When the banana width and the gradient length become comparable, the ion thermal conductivity is significantly reduced from its conventional neoclassical result. It is found from Fig.5(b) that in the presence of steep density gradient ion particle flux can be driven by ion-ion collisions alone. The simulation results suggest that the reduction of the conductivity and the particle flux depend linearly on (1/n)dn/dr. Since the finite orbit effect on electrons is very weak, the electron transport is unchanged in the presence of steep density gradient. Then the neoclassical particle transport could be non-ambipolar, or, a radial electric field can be established within the neoclassical framework in order to maintain an ambipolar neoclassical transport. The neoclassical transport with steep gradients is under investigation.

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