β Limit and Stabilization of Resistive Interchange Instability in Reversed Shear Tokamak

FURUKAWA Masaru, NAKAMURA Yuji and WAKATANI Masahiro
Graduate School of Energy Science, Kyoto University, Uji 611-0011, Japan

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Abstract
Resistive interchange modes are studied by using local stability criterion, and it is shown that these modes become unstable in reversed shear tokamaks for finite beta plasmas. However the stability of these modes can be improved by optimizing the flux surface shape and choosing an appropriate q profile, where q is a safety factor. Here the beta limit under a fixed pressure profile is studied for various cross-sections and q profiles, which shows that the normalized beta (\(\beta_n\)) above 2 is possible. For the reversed shear JT-60U model, it is difficult to obtain \(\beta_n \sim 2\) due to the steep pressure gradient.

Keywords:
resistive interchange mode, reversed shear tokamak, flux surface shape, q profile, β limit

1. Introduction
Reversed shear tokamaks [1-3] are recent topics for experiments and theory, since high performance discharges have been realized. However, discharges of this type were often terminated by the beta collapse. Thus the stability analysis is one of the important issues. Most dangerous instabilities are the ideal MHD modes, because they grow in a fast time scale, and often lead to disruptive termination of a discharge. However, these modes have been studied in detail, thus the various ways to avoid them have been discussed. For example, it is reported that the external kink modes can be stabilized by placing conducting walls near the plasma [4]. In terms of the ballooning modes, it is reported that D-shaped cross-section is favorable for stability [4]. Also, ballooning modes have been found to be stable in negative magnetic shear configurations [5]. Furthermore, ideal interchange modes are stable in configurations with sufficiently high safety factor [6].

Thus it is considered that the study of resistive MHD modes is important as a next step; when the ideal modes are stabilized, resistive modes could have significant roles.

The purpose of this paper is to clarify the equilibrium parameter dependences of the beta limit due to the resistive interchange (RI) modes. Here the beta limit \(\beta_\text{c}\) is obtained for a fixed pressure profile by changing a central pressure, therefore the RI modes disappear completely for \(\beta \leq \beta_\text{c}\). The pressure profile is not optimized here with respect to the RI modes. Since we use the local stability criterion, it is possible to obtain a pressure profile marginal to the RI modes everywhere. This profile may give the higher beta limit than the simply assumed profile.

In Sec. 2, the procedure for studying local stability is explained first, and the results of the beta limit are shown. Concluding remarks are given in Sec. 3.

2. Beta Limit Estimation
The stability is analyzed by the following way. First 1) an MHD equilibrium is calculated with VMEC [7] for a pressure profile, a safety factor profile, a boundary shape, and a central beta, then 2) the local...
stability criterion derived by Glasser, Greene and Johnson [8] is checked. If the equilibrium is stable, the central beta is raised while other profiles are fixed, and the procedures 1) and 2) are continued until the local stability criterion is violated on a magnetic surface. Then the beta limit can be obtained. Also it should be noted that the MHD equilibria studied in this section are stable for ideal interchange modes.

For calculating an MHD equilibrium with VMEC, a pressure profile, a $q$ profile, a boundary shape of outermost flux surface, and a central beta are assumed. For studying the beta limit, we assume the pressure profile as,

$$P = P_0(1 - s)^2$$  \hspace{1cm} (1)

where $P_0$ is the pressure at the magnetic axis and $s = \delta r/\delta r_{edge}$ is the normalized toroidal magnetic flux. The profile given by Eq. (1) is often used for MHD equilibrium and stability analyses. Also the boundary shape is given as,

$$R_b = R_{b0} + R_{b1} \cos \theta + R_{b2} \cos 2\theta,$$  \hspace{1cm} (2)

$$Z_b = Z_{b1} \sin \theta,$$  \hspace{1cm} (3)

where $R$ is a major radius, $Z$ is a height from the midplane, and $\theta$ is a poloidal angle. $R_{b1}$ corresponds to the aspect ratio $A$. $Z_{b1}$ to the ellipticity $\kappa$, and $R_{b2}$ to the triangularity $\delta$. The definitions of $A$, $\kappa$, and $\delta$ are the same as those in ref. [9].

First we show the $q_0$ dependence of the beta limit, where $q_0$ is the safety factor on the magnetic axis. Here the marginal stability is determined by $D_R = 0$, where $D_R$ is defined in ref. [8]. In Fig. 1, the $q$ profiles used in the MHD calculations are plotted versus the normalized minor radius $q/\langle q \rangle_{edge}$. Also, in Fig. 1, $q_{min} = 3.8$ and $q_{edge} = 6.0$ are fixed and $q_0$ is changed, where $q_{edge}$ is the $q$ value at the plasma edge. The beta limit is plotted versus $q_0$ in Fig. 2, where $\beta_b = \langle \beta \rangle a B_0/\rho_1$ [mT/MA], $B_{10} = 2\mu_0 P_0/B_{10}$ [\%]. ($\beta_b$) is a volume averaged beta, $a$ is a horizontal minor radius, $B_{10}$ is a vacuum toroidal field at the plasma center, $\rho_1$ is a toroidal plasma current, and $\mu_0$ is the vacuum permeability. In the VMEC code, $a = 1$ [m] and $B_{10} = 1$ [T] were used. It is seen that the beta limit increases as $q_0$ is decreased. It should be noted that, for the equilibria with the reversed shear $q$ profiles in Fig. 1, the violation of the stability criterion occurred in the reversed shear region. Thus the shear reversal seems to have an important role. However, for the equilibrium with positive shear $q$ profile in Fig. 1, the violation of the criterion also occurred for high beta regime with $\beta_n \geq 4.5$. In this case, the unstable magnetic surface locates around $\langle q \rangle/\langle q \rangle_{edge} = 0.3$. Therefore, it can be considered that the most important factor for this instability is not the shear reversal (but the other change of the equilibrium, which, of course, is enhanced when the shear reversal exists). Even for the positive shear equilibrium, the enhancement of the Pfirsch-Schlüter current destabilized the RI modes. Large amount of Pfirsch-Schlüter current flows on a magnetic surface with high $q$ and high pressure gradient.

Next we show the dependence of beta limit on the flux surface shape. In Fig. 3, the beta limit for the...
stable (B⁺)
unstable (B⁻)

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Fig. 3 Dependence of beta limit on triangularity \( \delta \) for the equilibria with \( A = 3, \kappa = 1.6 \), and \( q \) profile with \( q_o = 5.35 \) in Fig. 1.

Fig. 4 \( q \) profiles with various \( q_{\text{min}} \) as functions of normalized minor radius. Here \( q/q_{\text{min}} = 2.0 \) and \( q_{\text{edge}}/q_{\text{min}} = 1.8 \) are fixed and \( q_{\text{min}} \) is changed.

Fig. 5 Dependence of beta limit on \( q_{\text{min}} \) for the equilibria with \( A = 3, \kappa = 1.8, \delta = 0.4 \) and \( q \) profiles in Fig. 4.

Fig. 6 Pressure and \( q \) profiles for a model of JT-60U reversed shear configuration.

Equilibria with \( A = 3, \kappa = 1.6 \), and \( q \) profile with \( q_o = 5.35 \) in Fig. 1 is plotted versus \( \delta \). It is noted that the elliptic deformation only (\( \kappa > 1 \) and \( \delta = 0 \)) has also a stabilizing effect on the RI modes. The normalized beta limit \( \beta_n = 2.3 \) can be obtained with \( \kappa = 1.6 \) and \( \delta = 0.4 \). The increase of the marginal beta relative to the circular cross-section (see Fig. 2) can be explained as follows; the Pfirsch-Schlüter current decreases by the elliptic deformation, also the triangular deformation leads to the deeper magnetic well.

Next, we study the \( q_{\text{min}} \) dependence of the beta limit. In Fig. 4, the \( q \) profiles for various \( q_{\text{min}} \) are plotted, where \( q/q_{\text{min}} = 2.0 \) and \( q_{\text{edge}}/q_{\text{min}} = 1.8 \) are fixed. This variation corresponds to the change of the total plasma current under the fixed current density profile. The dependence of the beta limit on \( q_{\text{min}} \) is plotted in Fig. 5 for the equilibria with \( A = 3, \kappa = 1.8 \) and \( \delta = 0.4 \). It is seen that the beta limit increases as \( q_{\text{min}} \) is decreased. This can be also explained by the reduction of the Pfirsch-Schlüter current due to the decrease of \( q \).

In Fig. 5, the marginal beta \( \beta_m = 1.55 \) is obtained for \( q_{\text{min}} = 1.8 \); however, \( \beta_n = 2 \) is also possible by decreasing \( q_o \) at fixed \( q_{\text{min}} \) (see Fig. 2).

Finally we examine the beta limit for the MHD equilibrium with the reversed shear \( q \) profile similar to the experimental results in JT-60U. The pressure and the \( q \) profiles are assumed as shown in Fig. 6. In this case, unlike the previous cases, the pressure profile has a
steep gradient in the reversed shear region. The stability criterion is violated when $\beta_N = 0.387$ or $\beta_{\text{ry}} = 0.8$ [%], although $\beta_N = 2$ has been obtained in JT-60U experiments. Since it is known that the sheared poloidal flow has a stabilizing tendency for the RI modes, the discrepancy may be mitigated by including the plasma flow in the stability study. For the RI modes in a sheared slab plasma, it is shown that the poloidal shear flow suppresses growth rates. For the toroidal model, this is a future subject.

3. Conclusions

We estimated the beta limit due to the RI modes. The beta limit can be increased 1) by reducing $q_0$ at fixed $q_{\text{min}}$ and $q_{\text{edge}}$, 2) by reducing $q_{\text{min}}$ at fixed $q_d/q_{\text{min}}$ and $q_{\text{edge}}/q_{\text{min}}$, and 3) by making the plasma boundary D-shaped. The decrease of $q$ and the elliptic deformation correspond to the reduction of the Pfirsch-Schlüter current, and the triangular deformation corresponds to the deeper magnetic well. When $q_0 = 5.35$ for $q_{\text{min}} = 3.8$ and $q_{\text{edge}} = 6.0$, the marginal beta $\beta_N \simeq 2.3$ is obtained for the equilibrium with $\Delta = 3$, $\kappa = 1.6$, and $\delta = 0.4$. Also when $q_{\text{min}} = 1.8$ for $q_0 = 3.6$ and $q_{\text{edge}} = 3.24$, the marginal beta $\beta_N \simeq 1.55$ is obtained for the equilibrium with $\Delta = 3$, $\kappa = 1.8$, and $\delta = 0.4$. From Fig. 2, the higher beta limit $\beta_N \gtrsim 2$ are expected for the configuration with low magnetic shear near the magnetic axis.

For the JT-60U model, it is found that the violation of the local stability criterion occurs at much lower beta than the experimentally obtained ones. The inclusion of the effects, such as the mass flow, is our future issue.

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References