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Reactively Driven Drift Modes in Sheared Flow

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Abstract

Equations describing drift waves in a strongly inhomogeneous plasma with a sheared flow and a sheared magnetic field are derived. New stabilization criteria are obtained in the linear regime. In the nonlinear regime, it is found that the governing equations admit stationary solutions in the form of tripolar vortex and vortex chains that are driven by the equilibrium plasma flow and magnetic shear. The tripolar vortex obtained in this paper is surprisingly similar to corresponding structures in two-dimensional flows in ordinary fluids.

Keywords:

drift waves, sheared flow, tripolar vortices, vortex chains.

1. Introduction

Magnetic field shear has a strong stabilizing effect on drift waves, especially for exponential density profiles when they are completely stabilized. This stabilization follows as a result of the convection of the wave energy from the mode rational surface towards the ion Landau damping layer. In realistic geometries, inhomogeneities along the magnetic field lead to toroidal coupling which may destabilize drift waves by a complete elimination of the shear effects. Standard approach to the problem of shear stabilization of drift waves is to treat the diamagnetic drift velocity v_* as a constant, which is hardly satisfied in the edge of a tokamak plasma. In configurations of this type, with a steep density gradient and radially sheared transverse velocity field, the magnetic shear stabilization criteria are severely restricted, and the velocity profile curvature $v_0''(x)$ is found to play a crucial role. In this paper, we first derive linear equations for drift waves in astrongly inhomogeneous plasma with a sheared plasma flow and a sheared magnetic field, keeping the terms which describe the modification of the ion polarization drift associated with the equilibrium flow velocity. New stabilization criteria are obtained in the linear regime. In the nonlinear regime, it is found that the corresponding governing equations admit stationary solutions in the form of a tripolar vortex and vortex chain.

2. Basic Equations and Derivations

We study low-frequency electrostatic perturbations, $\partial/\partial t \ll \Omega_i$, of a collisionless, spatially nonuniform plasma, $n_0 = n_0(x)$, with a sheared flow, $\vec{v}_0 = v_0(x)\vec{e}_y$, placed in a sheared magnetic field of the form $\vec{B}_0 = \vec{e}_z B_0 + \vec{e}_y B_0 (\mathrm{d}/\mathrm{d}x) f(x)$, where $|(\mathrm{d}/\mathrm{d}x) f(x)| \ll 1$. We take ions to be cold, and the electron temperature gradient effects are neglected.

The nonlinear ion continuity and the parallel motion equations describing the system [2] can be written as

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$$\left[\frac{\partial}{\partial t}+\frac{1}{B_0}\vec{e}_z\times\nabla_\perp(\Phi+B_0\varphi)\cdot\nabla_\perp\right]$$
$$[(1-\rho^2\nabla_\perp^2)(\Phi+B_0\varphi)-B_0(\varphi+\Psi)]+$$
$$\frac{T_e}{e}\nabla_z\cdot\delta\vec{v}_z=0, \tag{1}$$

$$\left[\frac{\partial}{\partial t}+\frac{1}{B_0}\vec{e}_z\times\nabla_\perp(\Phi+B_0\varphi)\cdot\nabla_\perp\right]$$
$$[\delta v_z-\Omega_i f(x)]=0. \tag{2}$$

Here, we have introduced the stream function φ of the zero-order shear flow $v_0(x)=\mathrm{d}\varphi/\mathrm{d}x$, and also $v_{*e}=\mathrm{d}\Psi/\mathrm{d}x=-n_0' c_s^2/n_0\Omega_i$.

In the linear limit, combining Eqs. (1)-(2), we obtain:

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}x^2}+F(x)\right]\Phi\equiv\left\{\frac{\mathrm{d}^2}{\mathrm{d}x^2}-k_y^2+\frac{1}{a+bx+cx^2}\left[a_1+b_1x+c_1x^2+\frac{a_0 f'^2(x)}{a+bx+cx^2}\right]\right\}\Phi=0, \tag{3}$$

$$a_0=k_y^2\frac{c_s^2}{\omega^2\rho^2},\quad a=1-V_0,\quad b=-\frac{V_0}{L_1},\quad c=-\frac{V_0}{L_2^2},$$
$$a_1=-1+2\frac{V_0}{L_2^2}+V_{*0}+V_0,$$
$$b_1=-b,\quad c_1=\frac{V_0}{L_2^2}-\frac{V_{*0}}{L_*^2}.$$

Here we used the following normalization

$$\rho\frac{\mathrm{d}}{\mathrm{d}x}\to\frac{\mathrm{d}}{\mathrm{d}x},\quad k_y\frac{V_0}{\omega\rho}\to V_0,\, k_y\frac{V_{*0}}{\omega\rho}\to V_{*0},\quad \frac{L_{1,2,*}}{\rho}.$$

and, in order to simulate conditions typical for a H-mode plasma [3,4], we modeled the unperturbed profiles of the radial and diamagnetic velocities in the form

$$v_0(x)=V_0\left(1+\frac{1}{L_1}+\frac{x^2}{L_2^2}\right),\quad\text{and}$$
$$v_{*0}(x)=V_{*0}\left(1-\frac{x^2}{L_*^2}\right). \tag{4}$$

For the linear profile of the magnetic shear, $f'(x)=x/L_S$, we obtain

$$l\equiv\lim_{x\to\infty}F(x)=\frac{c_1-ck_y^2}{c}=\frac{V_{*0}}{V_0}\left(\frac{L_2}{L_*}\right)^2-k_y^2-1.$$

For $l>0$, which is equivalent to $L_2^2/L_*^2>(k_y^2+1)V_0/V_{*0}$ any nontrivial solution of Eq. (3) is an oscillatory function along the x axis for $x\to\infty$, with the distance $\delta(x)$ between its zeros going to $\delta_0\approx\pi/\sqrt{l}$ at large x. Thus, in this case we have an oscillatory, and possibly stabilized solution along the x-axis, without any explicit dependence on the magnetic shear. On the other hand, when the above inequality is reversed, the solution is unstable. Assuming that $V_0\approx V_{*0}$, we obtain the following condition for the oscillatory solution $k_y^{-1}>\sqrt{L_*^2/(L_2^2-L_*^2)}$, where obviously $L_2^2>L_*^2$. According to the experimental data $L_*\approx L_n\approx 1$ cm; consequently, for a stabilized drift wave, we must have $|L_2|>1$ cm.

On the other hand, for a quadratic magnetic shear determined by $f'(x)=x^2/L_S^2$, we have the following limit for $F(x)$:

$$l=\frac{L_2^4}{V_0^2}\left[\frac{a_0}{L_S^4}-\frac{V_0}{L_2^2}\left(\frac{V_0}{L_2^2}-\frac{V_{*0}}{L_*^2}\right)-k_y^2\frac{V_0^2}{L_2^4}\right]. \tag{5}$$

An oscillatory, and thus stabilized solution for $l>0$, is obtained if

$$\frac{L_2^4}{L_S^4}\frac{a_0}{V_0^2}+\frac{V_{*0}}{V_0}\frac{L_2^2}{L_*^2}>k_y^2+1, \tag{6}$$

which for $V_0\approx V_{*0}=\rho\, c_S/L_n$ yields

$$k_y^{-1}>\left[\left(\frac{L_2}{L_S}\right)^4\frac{a_0}{V_0^2}+\frac{L_2^2}{L_*^2}-1\right]^{-1/2},\quad\text{and}$$
$$\left(\frac{L_2}{L_S}\right)^4\frac{c_S^2}{V_0^2}+\frac{L_2^2}{L_*^2}>1. \tag{7}$$

However, for typical experimental values of the above quantities [3,4], *i.e.*, $\rho\approx 0.1$ cm, $L_S\approx 70$ cm, and $L_n\approx L_*\approx 1$ cm, the shear term in Eq. (7) is much smaller than the others, and the stability condition is practically the same as for the linear magnetic shear.

In the nonlinear regime, we look for travelling solutions of Eqs. (1), (2), which are stationary in the reference frame moving with a constant velocity u almost perpendicularly to the magnetic field lines and the unperturbed gradients, *i.e.*, $\partial/\partial t=-u\partial/\partial y$. Integrating Eq. (2) we obtain

$$\delta v_z=\Omega_i f(x)+\mathcal{F}(\Phi+B_0\varphi-uB_0x). \tag{8}$$

Here, $\mathcal{F}$ is an arbitrary function of the given argument, which we adopt it in the linear form $\mathcal{F}=F\cdot(\Phi+\varphi-ux)$, where $\mathcal{F}$ is an arbitrary constant. From the condition of vanishing perturbations for $x\to\infty$, we have

$$\delta v_z=F\cdot\Phi,\quad\text{and}\quad \Omega_i f(x)=-F\varphi(x)+Fux. \tag{9}$$

By using Eq. (9), we may integrate Eq. (1), yielding

$$(\nabla_\perp^2 - 1)\,\Phi + \varphi''(x) + \Psi(x) + f(x) = \mathcal{G}\,[\Phi + \varphi(x) - ux], \tag{10}$$

where we have set $\rho^2\nabla^2 \to \nabla^2$, $B_0\varphi \to \varphi$, $B_0 u \to u$, $B_0\Psi \to \Psi$, and $T_e F B_0 f/e \to f$. Here, $\mathcal{G}(\xi)$ is also an arbitrary function of the given argument, and we take it in the form $\mathcal{G}(\xi) = G_0 + G_1 \cdot \xi$. Such simple expressions for the functions $\mathcal{F}$ and $\mathcal{G}$ are possible if the unperturbed quantities $\varphi(x)$, $\Psi(x)$ and $f(x)$ satisfy

$$\varphi(x) - ux = \kappa_1 x^2, \quad \varphi''(x) + \Psi(x) + f(x) = \kappa_2 x^2, \tag{11}$$

and, consequently, Eq. (10) can be rewritten in the form:

$$(\nabla_\perp^2 - 1)\,\Phi + \kappa_2 x^2 = G_0 + G_1\,(\Phi + \kappa_1 x^2). \tag{12}$$

Equation (12) will be solved separately inside and outside of a circle with an arbitrary radius r_0, allowing for different values of $G_{0,1}^{\rm out}$ and $G_{0,1}^{\rm in}$ outside and inside of the vortex core, on condition of localized solutions for $r \to \infty$. In the cylindrical co-ordinates r, θ, Eq. (12) separates variables, and its solution can be written in the form

$$\Phi^{\rm out}(r,\theta) = B_0\,\mathrm{K}_0(\lambda_1 r) + \beta_2 \mathrm{K}_2(\lambda_1 r)\cos 2\theta, \quad r > r_0, \tag{13}$$

$$\Phi^{\rm in}(r,\theta) = a_0 \mathrm{J}_0(\lambda_2 r) - A\frac{r^2}{2} - B + \left[a_0 \mathrm{J}_2(\lambda_2 r) - A\frac{r^2}{2}\right]\cos 2\theta. \tag{14}$$

Here, $G_0^{\rm out} = 0$, $G_1^{\rm out} = \kappa_2/\kappa_1$, and $K_{0,2}$, $J_{0,2}$ are modified Bessel, and Bessel functions of the given order, respectively, and

$$\lambda_1^2 = 1 + G_1^{\rm out} = 1 + \frac{\kappa_2}{\kappa_1}, \qquad \lambda_2^2 = -1 - G_1^{\rm in},$$

$$A = \kappa_1 + \frac{\kappa_1 + \kappa_2}{\lambda_2^2},$$

$$B = -\frac{2}{\lambda_2^2}\left(\frac{G_0^{\rm in}}{2} + \kappa_1 + \frac{\kappa_1 + \kappa_2}{\lambda_2^2}\right).$$

We use the following physically justified continuity conditions at $r = r_0$, which allows for determining the unknown constants in Eqs. (13) and (14): continuity of the potential Φ, yielding finiteness of the electric field at $r = r_0$, continuity of $(\partial/\partial r)\Phi$, yielding the absence of surface charges at $r = r_0$, continuity of the function $\mathcal{G}(\xi)$ to avoid singularities of the starting equation. The argument of the function $\mathcal{G}(\xi)$ is constant on the separatrix $r = r_0$. The above continuity conditions yield:

$$a_0\left(\mathrm{J}_0\,\mathrm{K}_0' + \frac{\lambda_1^2}{\lambda_2^2}\mathrm{J}_0'\,\mathrm{K}_0\right) = A\left[\left(\frac{r_0^2}{2} - \frac{1}{\lambda_2^2}\right)\mathrm{K}_0' + r_0\frac{\lambda_1^2}{\lambda_2^2}\mathrm{K}_0\right] - \frac{\lambda_1^2 + \lambda_2^2}{\lambda_2^2}\frac{\kappa_1 r_0^2}{2}\mathrm{K}_0', \quad \alpha_2 = -\frac{\kappa_1 - A}{\mathrm{J}_2}\frac{r_0^2}{2}, \tag{15}$$

$$\beta_0\left(\mathrm{J}_0\,\mathrm{K}_0' + \frac{\lambda_1^2}{\lambda_2^2}\mathrm{J}_0'\,\mathrm{K}_0\right) = A\left[\left(\frac{r_0^2}{2} - \frac{1}{\lambda_2^2}\right)\mathrm{J}_0' - r_0\,\mathrm{J}_0\right] - \frac{\lambda_1^2 + \lambda_2^2}{\lambda_2^2}\frac{\kappa_1 r_0^2}{2}\mathrm{J}_0',$$

$$B = \alpha_0\,\mathrm{J}_0 - \beta_0\,\mathrm{K}_0 - A\frac{r_0^2}{2}, \tag{16}$$

$$\kappa_1 r_0\frac{\mathrm{K}_2'}{\mathrm{K}_2} = (\kappa_1 - A)\,r_0\frac{\mathrm{J}_2'}{\mathrm{J}_2} + 2A,$$

$$\beta_2 = -\frac{1}{\mathrm{K}_2}\frac{\kappa_1 r_0^2}{2}. \tag{17}$$

As an illustration, we take $\kappa_1 = 1$, $\kappa_2 = 1$ and $r_0 = 2$, and solve the nonlinear dispersion equation (17) numerically in the vicinity of the first zero of the Bessel function J_2, giving $\lambda_2 = 2.68$. This yields the following values for the above constants: $\beta_0 = -53.85$, $\beta_2 = -25.87$, $\alpha_0 = 5.75$, $\alpha_2 = -7.46$, $A = 1.28$, and $B = -0.6$. A typical profile of the tripolar vortex, given by Eqs. (13), (14), corresponding to these numerical values, is presented in Figs 1. and 2.

Equation (10) admits also several types of vortex chain solutions. Here we shall present only one. For the shear flow in the form of an asymmetric function around the velocity u, given by $v_0(x) = u + A_0\kappa\tanh(\kappa x)$, and in the case $\Psi(x) + f(x) = 0$, taking the function $\mathcal{G}$ in the form:

$$\mathcal{G}\,(\Phi + \varphi - ux) = A_0\,\kappa^2 \exp\left[-\frac{2}{A_0}(\Phi + \varphi - ux)\right],$$

Eq. (10) can be rewritten in the following way:

$$(\nabla_\perp^2 - 1)\,\Phi - \mathcal{J}(x)\left(e^{-\Phi} - 1\right) = 0, \tag{18}$$

where $\mathcal{J}(x) = 8\kappa^2/(e^{2\kappa x} + e^{-2\kappa x} + 2)$. An equation of the type of Eq. (18) has been derived and solved earlier in our paper [5], where we studied the problem of self-generation of the magnetic field in a plasma with a shear flow, thus for a completely different physical problem with different spatial and time scales. Following the same procedure we may write $\Phi(x, y) = \Phi_1(x) + \delta\Phi(x)$

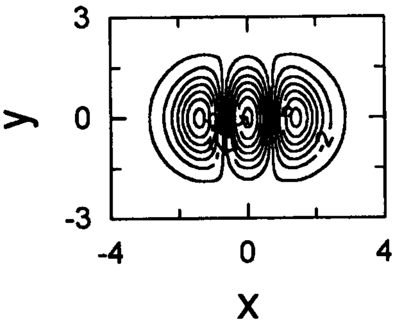

Fig. 1 Contour plots of the tripolar vortex describing the potential $\Phi(r, \theta)$ for $\kappa_1 = 1$, $\kappa_2 = 1$, $r_0 = 2$, $\beta_0 = -53.85$, $\beta_2 = -25.87$, $\alpha_0 = 5.75$, $\alpha_2 = -7.46$, $A = 1.28$, $B = -0.6$.

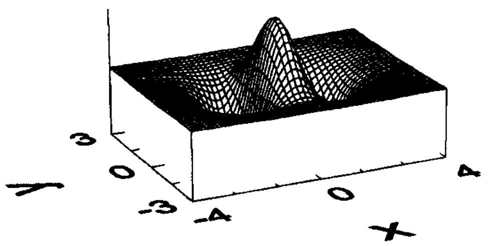

Fig. 2 Three dimensional view of the potential from Fig. 1.

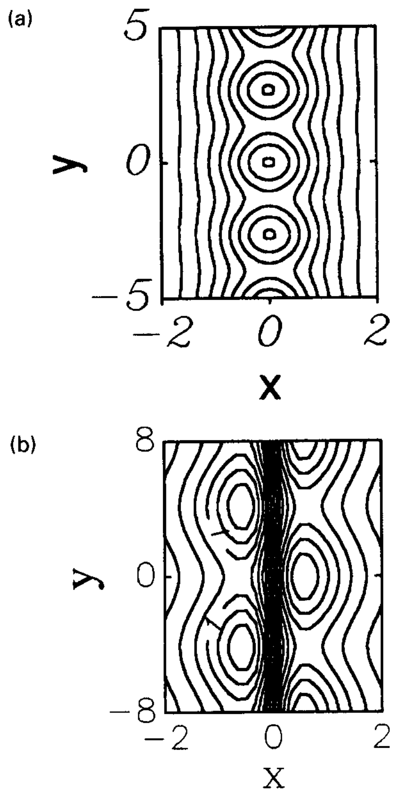

Fig. 3 The sketch of the single (a) and double (b) vortex chain solution.

$\cos(k_y y)$, where $|\delta\Phi(x)| << |\Phi_1(x)|$, and we find two types of localized solutions, in the form of single and double vortex chains presented in Fig. 3, which are structures localized in the x-direction, and periodic in y-direction.

3. Summary and Conclusions

We have studied the behavior of linear and nonlinear drift waves in strongly inhomogeneous systems with sheared plasma flows. By a careful inclusion of the shear flow effects in corresponding equations describing drift waves in such plasma systems, the stabilization criteria are substantially changed. For the given density profile and sheared plasma flows, the linear magnetic field shear has no effects on the behavior of drift waves; it is the sheared flow whose influence is essential, while the role of the magnetic shear is much smaller. It enters into the stability conditions provided that its profile can be approximated by a quadratic function. Yet, for typical parameters in the H-mode of a tokamak plasma, the influence of the magnetic shear on the stability of drift waves is insignificant. In the strongly nonlinear regime, we have solved the appropriate equations using the standard vortex scenario, and the formation of a tripolar vortex and vortex chain is presented. The tripolar vortex structure consists of the monopolar, and quadrupolar part (resulting from the x^2 terms by which the unperturbed state is modeled). For some specific profiles of the functions $\varphi(x)$, $\psi(x)$, and $f(x)$ describing the unperturbed state, the vortex chain solutions can also be found [2]. Finally, it should be emphasized that the nonlinear solutions presented here are analogous to the well-known structures in ordinary fluids, and we believe that they may arise as the final stage in the development of instability of drift waves in both laboratory, and space plasma configurations of this type.

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