Analysis of Current Diffusive Ballooning Mode in Tokamaks

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Abstract

The effect of finite gyroradius on the current diffusive ballooning mode is examined. Starting from the reduced MHD equations including turbulent transports, coupling with drift motion and finite gyroradius effect of ions, we derive a ballooning mode equation with complex transport coefficients. The eigenfrequency, saturation level and thermal diffusivity are evaluated numerically from the marginal stability condition. Preliminary results of their parameter dependence is presented.

Keywords: ballooning mode, current diffusivity, turbulent transport, finite gyroradius effect

1. Introduction

The self-sustained turbulence model [1] of the current diffusive ballooning mode (CDBM) is one of the most promising candidates to explain the particle and energy transport in tokamaks. This transport model has successfully described the L mode transport and improved confinement associated with the formation of the internal transport barriers [2]. Since the previous analysis [3] based on the reduced MHD equations [4] neglected the finite ion gyroradius effect, we could not separate the transport coefficients of ions and electrons. In this paper, we describe the formulation and the results of numerical analysis of the ballooning mode equation which includes the coupling with the drift motion and the finite gyro radius effect.

2. Ballooning Mode Equation

We start from a set of reduced MHD equations, i.e. the equation of perpendicular motion of ions, the extended Ohm’s law, the energy equations of electrons and ions [5],

\[
\frac{\partial}{\partial t} + \mathbf{i} \omega_n - \frac{\mu_i}{\nu_i} \nabla_i \phi = \nabla_i \tilde{J}_i + (\mathbf{b} \times \mathbf{k}) \cdot \nabla (\tilde{\rho}_i + \tilde{\rho}_e),
\]

\[
\frac{\partial}{\partial t} \tilde{A} - \nabla_i \tilde{\phi} = -\frac{1}{\omega_i} \tau_i \nabla_i \tilde{\rho}_e - \lambda \nabla_i^2 \tilde{J}_i,
\]

\[
\frac{\partial}{\partial t} \tilde{\rho}_e - \chi_e \nabla_i^2 \tilde{\rho}_e + \mathbf{b} \cdot \left( \nabla_i \tilde{\phi} \times \nabla_i \rho_e \right) = 0,
\]

\[
\frac{\partial}{\partial t} \tilde{\rho}_i - \chi_i \nabla_i^2 \tilde{\rho}_i + \mathbf{b} \cdot \left( \nabla_i \tilde{\phi} \times \nabla_i \rho_i \right) = 0,
\]

\[
\nabla_i^2 \tilde{A} = \tilde{J}_i,
\]

where \( \tau_i = qR/v_i, \mu_i, \) \( \chi_i, \) and \( \chi_e \) the ion viscosity, \( \chi_i, \) and \( \chi_e \) the ion and electron thermal diffusivities, \( \lambda \) the current diffusivity due to the electron viscosity \( \mu_e \) and other notations are standard. The streaming function for ions \( \tilde{\phi} \) is reduced from the electrostatic potential \( \tilde{\phi} \) by a factor
of \( \Lambda_0(b) = I_0(b)e^b \) where \( I_0 \) is the modified Bessel function of the first kind and \( b = k^2_1q^2 \). Length and time are normalized by the minor radius \( r \) and the poloidal Alfvén time \( \tau_A \), respectively. Using the ballooning transformation [6], we obtain

\[
\frac{d}{d\eta} \left( \frac{f}{-i\omega + \lambda n^2 q^4f^2} \right) \frac{d}{d\eta} \left( \frac{1}{-i\omega + \lambda n^2 q^4f^2} \right) \phi \\
+ \left( \frac{\alpha_s}{-i\omega + \lambda n^2 q^4f^2} \right) \frac{\alpha - i\omega + \lambda n^2 q^4f^2}{-i\omega + \lambda n^2 q^4f^2} H(\eta) \phi \\
+ \left( i\omega + A_0 \right) \lambda n^2 q^4f^2 \right) f \Lambda_0(\tilde{\phi}) = 0,
\]

where \( \alpha = -q^2R(d\beta/dr) \) is the normalized pressure gradient, \( s = (r/R)(d\beta/dr) \) the magnetic shear and \( \eta \) the ballooning coordinates. Taking account of the Shafranov shift of the magnetic surface, we take \( f = 1 + (s\eta - \alpha \sin \eta)^2 \) and \( H(\eta) = \kappa + \cos \eta + (s\eta - \alpha \sin \eta) \sin \eta \) where \( \kappa = (r/R)(1 - 1/q^2) \) is the average magnetic curvature. The first term in eq.(6) has a stabilizing effect due to the bending of the magnetic field and is reduced by the current diffusivity \( \lambda \). The second term coming from the pressure gradient destabilizes the mode in the bad curvature region and is reduced by the thermal diffusivity. The last term represents the stabilizing effect of \( \omega_n \) and the ion viscosity.

The transport coefficients are determined by the relation for the nonlinear transfer rate \( \gamma \), e.g. \( \chi_s = \langle \dot{\phi}^2 \rangle / \gamma \) and \( \gamma = -i\omega + k^2_1\chi_s \), where the bracket \( \langle \rangle \) denotes the average over the wave number and the \( \eta \) dependence of \( \langle \dot{\phi}^2 \rangle \) is neglected in the present analysis [1]. Since mode frequency has a real part, the transport coefficients are written as

\[
\mu_s = \chi_s = \sqrt{\left( \frac{\dot{\phi}^2}{n^2 q^4f^2} \right) - \frac{1}{4} \left( \omega + \frac{\omega_n}{n^2 q^4f^2} \right)^2 + \frac{i}{2} \frac{\omega}{n^2 q^4f^2} + \frac{i}{2} \frac{\omega_n}{n^2 q^4f^2}},
\]

\[
\chi_s = \sqrt{\left( \frac{\dot{\phi}^2}{n^2 q^4f^2} \right) - \frac{1}{4} \left( \omega - \frac{\omega_n}{n^2 q^4f^2} \right)^2 + \frac{i}{2} \frac{\omega}{n^2 q^4f^2} - \frac{i}{2} \frac{\omega_n}{n^2 q^4f^2}},
\]

\[
\chi_i = \sqrt{\left( \frac{\dot{\phi}^2}{n^2 q^4f^2} \right) - \frac{1}{4} \left( \omega - \frac{\omega_n}{n^2 q^4f^2} \right)^2 + \frac{i}{2} \frac{\omega}{n^2 q^4f^2} + \frac{i}{2} \frac{\omega_n}{n^2 q^4f^2}},
\]

where \( n \) is the toroidal mode number and \( \xi = \omega_n^2 r^2 / c^2 \). When \( \langle \dot{\phi}^2 \rangle \) is smaller than critical values, we take that the square roots have negative imaginary values so that the turbulent transport coefficients vanish for \( \langle \dot{\phi}^2 \rangle = 0 \). A pure imaginary value for small \( \langle \dot{\phi}^2 \rangle \) implies nonlinear frequency shift rather than diffusion. We should note that the electron thermal diffusivity \( \chi_e \) is less than \( \chi_i \) by a factor of \( \Lambda_0 \) for large \( \langle \dot{\phi}^2 \rangle \). With the increase of the perpendicular wave number \( k_\perp \sim nq \), the finite gyroradius effect reduces \( \chi_e \).

With the marginal stability condition (\( \text{Im} \omega = 0 \)), we numerically solve the ballooning mode equation (6) to find the eigen modes and evaluate the real part of the

![Spatial structure of the eigen mode for the parameters](image1)

![\( nq \) dependence of the thermal diffusivity \( \chi_e \) derived from the marginal stability condition for the parameters](image2)
mode frequency \( \omega_s \) and the average amplitude of the fluctuation \( \langle \phi^2 \rangle \). Fig. 1 shows an example of the CDBM eigenmode for \( s = 0.4 \), \( \alpha = 0.4 \), \( q = 20 \), \( \rho/r = 0.01 \) and \( nq = 134 \). It has a small negative real frequency \( \omega_s = -0.05 \) and propagates in the direction of ion drift motion. The saturation level of the fluctuation is \( \langle \phi^2 \rangle = 6 \times 10^4 \) and this mode is unstable for smaller \( \langle \phi^2 \rangle \). The eigen mode structure indicates that the dominant real part localizes in the outer region \( (\eta = 0) \), while the out-of-phase imaginary part in the inner region \( (\eta = \pm \alpha) \).

The saturation level of the average fluctuation amplitude depends on the mode number \( nq \). Figure 2 illustrates the \( nq \) dependence of the electron thermal diffusivity \( \chi_e \) derived from the saturation fluctuation amplitude \( \langle \phi^2 \rangle \). The maximum value of \( \chi_e \) near \( nq = 134 \) gives the thermal diffusivity necessary for stabilizing the ballooning modes for all \( nq \).

### 3. Parameter Dependence

In order to study the parameter dependence of the mode frequency and the thermal diffusivity, we look for the maximum value of \( \chi_e \) by numerically solving the ballooning mode equation (6) for wide range of \( nq \) and for various values of \( s \) and \( \alpha \). We have been much benefitted by parallel processing on a PC cluster of 8 CPUs with a numerical code using the Message Passing Interface (MPI) library.

The \( (s, \alpha) \) dependence of the electron thermal diffusivity is shown in Figs. 3 and 4 for \( \xi = 10^4 \) and \( q = 10^3 \). Fig. 3 depicts the case of small gyroradius, \( \rho/r = 10^{-3} \), while Fig. 4 the case of larger gyroradius, \( \rho/r = 10^{-2} \). The \( (s, \alpha) \) dependence in Fig. 3 is almost same as that of \( \rho/r = 0 \) [2]. For larger \( \rho/r \), \( \chi_e \) is enhanced, typically, by a factor of 3 and the shear dependence is slightly reduced.

For small \( \alpha (\alpha < 0.4) \), \( \chi_e \) is a strongly increasing function of \( \alpha (\chi_e \propto \alpha^{1/2}) \) and decreases for small and negative values of \( s \). For larger value of \( \alpha (\alpha > 0.4) \), \( \chi_e \) is enhanced in the stronger shear region \( (s > 0.5) \) where the ideal ballooning mode is unstable. The reduction of \( \chi_e \) in the weak and negative shear region is more prominent for larger \( \alpha \) (larger pressure gradient). These behavior is similar to the result of the previous analyses without coupling to the drift motion, \( \omega_n = \omega_\perp = 0 \). One of the reasons is the large \( q \) value in the present calculation \( (q = 10^3) \). For realistic values of \( q \), the effect of finite \( \omega_n \) and \( \omega_\perp \) affects the mode structure, but conclusive results have not been obtained yet.

The mode frequency is usually negative except in the region, \( \chi_e > 10^4 \), where the ideal mode is unstable.
The mode number $n_{q}$ decreases with the increase of $\chi_{e}$. Typical value of $n_{q}$ is 300 for $\chi_{e} = 10^{-3}$ and 100 for $\chi_{e} = 10^{-4}$. As the mode number increases, the finite gyroradius effect enhances $\chi_{e}$ and reduces $\chi_{i}$. Fig. 5 indicates the contour of the ratio $\chi_{i}/\chi_{e}$ on the $(s, \alpha)$ plane. In the region where $\alpha > 0.4$ and $\chi_{i}$ approaches to $\chi_{e}$, otherwise $\chi_{i}/\chi_{e}$ is less than about 0.4.

4. Conclusion

Taking account of the gyroradius effects, we found that $\chi_{e}$ of the CDBM transport model is enhanced by a factor of 3 for $\rho_{i}/r = 10^{-2}$ and that the ratio $\chi_{i}/\chi_{e}$ is considerably reduced for low $\alpha$ (L-mode) region. In the present calculation, the safety factor $q$ is limited to a rather large value. The calculation with realistic values of $q$ as well as the effect of the $E \times B$ rotation shear will be reported in future.

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References