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Energy Release and Plasma Heating by Reconnective Magnetic Relaxation

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Abstract

A brief review of recent progress in the theory of magnetic energy release and dissipation via magnetic reconnection is presented. The relaxation aspect of this process is emphasized, which is of interest for astrophysical plasmas such as the solar corona.

Keywords:

current sheets, forced magnetic reconnection, magnetic relaxation

1. Introduction

It is well known that magnetic reconnection, a phenomenon associated with breaking and mending of the magnetic field lines, plays an important role in laboratory and space plasmas. In fusion-oriented devices reconnection results in changes in the topology of magnetic flux surfaces which can destroy plasma confinement. For astrophysical plasma objects the energy aspect of this process is essential, since reconnection allows fast release of the excess magnetic energy which maintains the hot solar corona [1] and energizes extragalactic jets [2]. Usually, interaction of the plasma with magnetic fields has a quite complicated turbulent character. Therefore significant progress in its understanding has been achieved not starting from the first principles but by applying more general ideas of relaxation in complex systems [3]. This theory, based on magnetic reconnection as the relaxation mechanism, was very successful in predicting magnetic configurations sustained in various laboratory plasma systems. It was also applied to the problem of plasma heating in the corona and jets, though the related studies [4,5] were only semi-phenomenological. This report presents a self-consistent derivation of the energy release and plasma heating by forced magnetic reconnection [6,7]. The latter means that reconnection is triggered by the current sheet formed inside initially smooth magnetic field due to its external perturbation. Such forced reconnection seems to be a viable mechanism for solar coronal heating, because current sheets are easily formed in the corona by photospheric shuffling of magnetic footpoints [8,9].

2. Magnetic Reconnection and Energy Release

We are interested in a low-β plasma, hence we consider a sheared force-free magnetic field

$$\boldsymbol{B}_i = \{0, B_0 \sin \alpha_0 x, B_0 \cos \alpha_0 x\}, \qquad (1)$$

embedded in a planar slab of a highly conducting plasma with walls at $x = x_b^{(\pm)} = \pm a$. The initial equilibrium is perturbed by deforming the right-hand boundary $x_b^{(+)}$ as

$$x_b^{(+)} = a - \delta_0 \cos ky \, e^{-i\omega t}, \qquad (2)$$

thus making the magnetic configuration two dimensional: periodic in the y direction but invariant

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along the z-axis. For quasistatic evolution ($\omega\tau_a \ll 1$; τ_a is Alfven time-scale), a new equilibrium can be represented with the deformed magnetic field (function of x and y) as

$$\boldsymbol{B}(x, y) = [\nabla\psi(x, y)\times\hat{z}] + B_z(x, y)\hat{z}, \qquad (3)$$

and this is a force-free field if the poloidal flux function ψ and toroidal field component B_z satisfy the Grad-Shafranov equation

$$B_z(x, y) \equiv \mathcal{G}(\psi); \quad \nabla^2\psi + \mathcal{G}\frac{d\mathcal{G}}{d\psi} = 0. \qquad (4)$$

In the linear approximation ($\delta_0 \ll a$) it yields

$$\psi(x, y) = \psi_0(x) + \psi_1(x)\cos ky\, e^{-i\omega t},$$
$$\mathcal{G}(\psi) = \mathcal{G}_0(\psi), \qquad (5)$$

where $\psi_0(x)$ and $\mathcal{G}_0(\psi_0)$ relate to the initial magnetic field (1). Thus, the perturbed flux function $\psi_1(x)$ satisfies the following equation

$$\psi_1'' + [\alpha_0^2 - k^2]\psi_1 = 0. \qquad (6)$$

The solution of equation (6) subject to boundary condition (2) is (for $k < \alpha_0$)

$$\psi_1^{(r)}(x) = -B_0\delta_0 \sin\alpha_0 a \frac{\sin\kappa(x+a)}{\sin 2\kappa a},$$
$$\kappa = \sqrt{(\alpha_0^2 - k^2)}. \qquad (7)$$

This solution is regular and contains nothing special unless at some location $x = x_0$ within the slab $B_{iy}(x_0) = 0$, i.e. $x = x_0$ is the resonant surface where $\boldsymbol{k}\cdot\boldsymbol{B}_i = 0$. Since $\psi_1^{(r)}(x_0)$ is, generally, non-zero, the new equilibrium (5) has topology different from that of (1), with magnetic islands of width $\Delta = 4(|\psi_1^{(r)}(x_0)|/\alpha_0 B_0)^{1/2}$ formed at the resonant surface. Obviously, such a transformation cannot be realized in a perfectly conducting medium for which magnetic reconnection is prohibited. Therefore the solution for this case, denoted as $\psi_1^{(i)}$, has the additional requirement $\psi_1^{(i)}(x_0) = 0$, *i.e.*

$$\psi_1^{(i)}(x) = \begin{cases} 0, & -a \le x \le x_0 \\ -B_0\delta_0 \sin\alpha_0 a \dfrac{\sin\kappa(x-x_0)}{\sin\kappa(x-x_0)}, & x_0 \le x \le a. \end{cases} \qquad (8)$$

This solution, though preserving the field topology, has a singularity which is allowed in ideal MHD: the current sheet at $x = x_0$, where $B_y = \partial\psi/\partial x$, is discontinuous. If a finite plasma resistivity is present, it tries to destroy the current sheet by reconnective transition from $\psi_1^{(i)}$ to $\psi_1^{(r)}$. This transition is also associated with the release of magnetic energy [10], which, being related to a unit volume, is equal to

$$\Delta\mathcal{E} = \frac{B_0^2}{2\mu_0}\left(\frac{\delta_0}{a}\right)^2 f(\alpha_0 a, ka, x_0), \qquad (9)$$

where

$$f = \frac{\sin^2\alpha_0 a}{8}\kappa a[\cot\kappa(a - x_0) - \cot 2\kappa a].$$

The dependence of f on the scale length ka of perturbation is shown in Figure 1 for various values of $\alpha_0 a$.

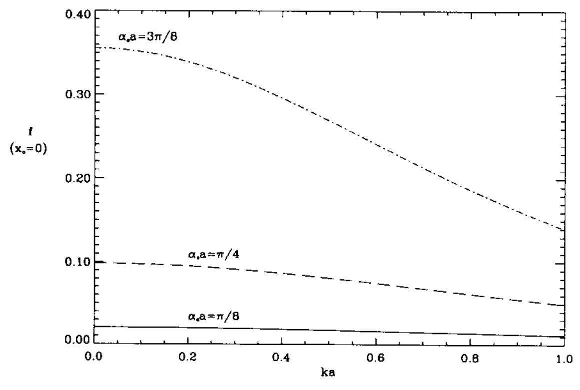

Fig. 1 The factor f (see Equation (9) plotted as a function of ka for three different values of $\alpha_0 a$ i.e. $\alpha_0 a = \pi/8$, $\pi/4$ and $3\pi/8$ ($x_0 = 0$). Note that the magnitude of f increases with increasing $\alpha_0 a$ but decreases with increasing ka.

3. Plasma Heating by Magnetic Relaxation

In this section we discuss the possibility of plasma heating that can be provided if this forced reconnection is continuously generated by the external perturbation (2). In a plasma with finite resistivity a general equilibrium configuration can be represented as a linear superposition of the two equilibria derived in the previous section

$$\psi_1 = A\psi_1^{(i)} + (1 - A)\psi_1^{(r)}. \qquad (10)$$

Since the solution $\psi_1^{(i)}$ is singular, its presence in (10) means that the resulting equilibrium acquires a current sheet whose strength can be characterized by the parameter Δ'- the jump in the logarithmic derivative of ψ_1 at the resonance surface $x = x_0$. It follows from (7) and (8) that

$$\Delta' = \frac{2\kappa A}{(1 - A)}\cot\kappa a \qquad (11)$$

(here $x_0 = 0$ is assumed for simplicity). The particular value of Δ' and hence of A is determined by the internal structure of the current sheet where both inertial and resistive effects are essential. Therefore the situation is similar to usual tearing stability analysis [11], although in the tearing mode Δ' is prescribed by the global magnetic configuration (the external solution), and then the current sheet (the internal solution) determines the instability growth rate whereas here the perturbation frequency ω is fixed by (2), and so is the internal solution; thus the external equilibrium (10) which is specified by yet unknown amplitude A should adjust to this current sheet. The plasma motion is incompressible inside the current sheet *i.e.* $v = [\nabla\phi(x,y,t) \times \hat{z}]$, where $\phi = \phi(x)\sin ky\, e^{-i\omega t}$ is the stream function. Then the magnetic induction equation and equation of motion can be written there in the following dimensionless form:

$$-i\omega\psi_1 = -ikx\phi + \frac{\psi_1''}{\omega\tau_\eta};$$
$$-\omega^2\tau_a^2\phi'' + kx\psi_1'' = 0, \qquad (12)$$

where $\tau_a = \sqrt{\rho_0}/B_0\alpha_0$ is the Alfven time, $\tau_\eta = (\eta\alpha_0^2)^{-1}$ is the resistive time ($\tau_a \ll \tau_\eta$) with x scaled by α_0^{-1}, ψ_1 by B_0/α_0 and ϕ by $-i\omega/\alpha_0^2$. Adopting the constant-ψ approximation [11] and using new variables:

$$\zeta = x\left(\frac{\tau_\eta}{\omega\tau_A^2 k^2}\right)^{1/4}, \quad \chi = \frac{\phi}{\psi_1(0)}\left(\frac{\omega\tau_A^2}{\tau_\eta k^2}\right)^{1/4}, \qquad (13)$$

equations (12) can be reduced to

$$\chi'' - i\zeta^2\chi + i\zeta = 0. \qquad (14)$$

The appropriate solution of (14) which is an odd function of ζ and behaves as $1/\zeta$ at $\zeta \to \infty$ to match the external solution is

$$\chi(\zeta) = \chi_1 + i\chi_2$$
$$= \frac{(1+i)\zeta}{2\sqrt{2}}\int_0^1 \frac{\exp\left\{-\frac{(1+i)\zeta}{2\sqrt{2}}\mu\zeta^2\right\}}{(1-\mu^2)^{1/4}}\,d\mu. \qquad (15)$$

Knowing the stream function ϕ, one can use the second of equations (12) to derive the current density inside the current sheet, which yields $\psi_1'' = \frac{\omega^2\tau_a^2}{kx}\phi''$. Therefore

$$\Delta' = \frac{\int \psi_1''\,dx}{\psi_1(0)} = \frac{\omega^2\tau_a^2}{k\psi_1(0)}\int\frac{\phi''}{x}\,dx$$
$$= \frac{\alpha_0(\omega\tau_a)^2}{(k/\alpha_0)^{1/2}}\left(\frac{\tau_\eta}{\omega\tau_a^2}\right)^{3/4}\int_{-\infty}^{+\infty}\frac{\chi''}{\zeta}\,d\zeta. \qquad (16)$$

Equating (16) and (11), the following equation which determines the amplitude A is found:

$$(\omega\tau_r)^{5/4}(a+ib) = \frac{A}{1-A};$$
$$(a+ib) = \int_{-\infty}^{+\infty}\frac{\chi''}{\zeta}\,d\zeta = -\frac{2\pi\Gamma(3/4)}{\Gamma(1/4)}e^{i\frac{3\pi}{8}}. \qquad (17)$$

Here $\tau_r = \tau_a^{2/5}\tau_\eta^{3/5}\left[\frac{\alpha_0\tan\kappa a}{2\kappa(k/\alpha_0)^{1/2}}\right]^{4/5}$ is the effective internal reconnection time. It has usual tearing mode scaling [11] with respect to τ_a and τ_η, but it also depends on the shear parameter α_0, so that $\tau_r \to \infty$ when the initial configuration (1) approaches the tearing instability threshold $\kappa a = \pi/2$. The solution of (16) is

$$A = A_1 + iA_2 =$$
$$\frac{(\omega\tau_r)^{5/4}(a^2+b^2)\left\{\left[a + (\omega\tau_r)^{5/4}(a^2+b^2)\right] + ib\right\}}{\left[a + (\omega\tau_r)^{5/4}(a^2+b^2)\right]^2 + b^2}. \qquad (18)$$

It shows that magnetic equilibrium is governed by the parameter $\omega\tau_r$ which is the ratio of the internal reconnection time τ_r to the time scale of the external driver (2). In the limit of fast internal reconnection, when $\omega\tau_r \ll 1$, $A \to 0$, i.e. the system is very close to the reconnected equilibrium $\psi_1^{(r)}$. In the opposite limit of slow reconnection, $\omega\tau_r \gg 1$, $A \to 1$ and the ideal equilibrium $\psi_1^{(i)}$ with strong current sheet is present. In general case, when both $\psi_1^{(i)}$ and $\psi_1^{(r)}$ contribute to the equilibrium state, the amplitude A is a complex quantity. This means a phase shift (temporal delay) between the external driver and internal equilibrium, and it is this delay that determines a continuous dissipation of magnetic energy. A straightforward calculation of energy dissipation rate per unit volume results in

$$Q = \frac{\Delta\mathcal{E}}{\tau_r}\mathcal{F}(\omega\tau_r); \quad \mathcal{F}(\omega\tau_r) = -\omega\tau_r A_2(\omega\tau_r), \qquad (19)$$

where $\Delta\mathcal{E}$ is the excess magnetic energy given by (9). The function $\mathcal{F}(\omega\tau_r)$, plotted in Figure 2, has its maximum at $\omega\tau_r \sim 1$ and tends to zero as $(\omega\tau_r)^{9/4}$ for $\omega\tau_r \ll 1$, and as $(\omega\tau_r)^{-1/4}$ for $\omega\tau_r \gg 1$. In general, $\mathcal{F}(\omega\tau_r)$ shows features typical for a relaxation process when dissipation is most effective for the external driver's time-scale comparable to the internal relaxation time. It also demonstrates explicitly that the faster reconnection, *i.e.* shorter τ_r, does not necessarily result in an increased heating. Though this is true for $\omega\tau_r > 1$, the dissipation rate becomes decreasing with shorter τ_r when $\omega\tau_r < 1$, simply because too fast reconnection does not allow build up of the excess magnetic energy (the amplitude A of the current sheet is small).

4. Conclusions

In this work, energy release and plasma heating by forced magnetic reconnection is studied self-consistently. The most interesting regime of reconnective relaxation occurs when the system is initially close to marginal tearing stability *i.e.* when α_0 approaches $(\alpha_0)_{cr}$ (= $\pi/2a$ for the above model). In this case both $\Delta\mathcal{E}$ and Q become very large (and it follows from (9), (16) and (18) that they scale as $\Delta\mathcal{E} \sim [(\alpha_0)_{cr} - \alpha_0]^{-1}, Q \sim [(\alpha_0)_{cr} - \alpha_0]^{-1/5}$).

From a general physics point of view this plasma heating by forced magnetic reconnection is analogous to the well-known "second viscosity" phenomenon in fluids [12]. In both cases relaxation processes result in enhanced dissipation with a characteristic frequency dispersion as in Figure 2.

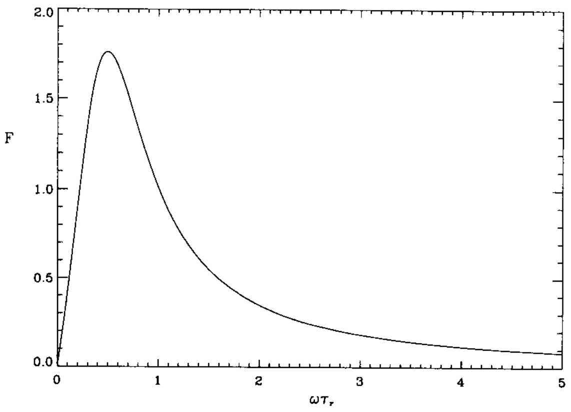

Fig. 2 The relaxation function $\mathcal{F}(\omega\tau_r)$.

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