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Behavior of Interchange Mode in Heliotron and Tokamak

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Abstract

When the pressure profile becomes flat locally at the resonant surface, the beta limit due to the ideal interchange mode increases, since the highly localized radial mode structure is suppressed. It is also found that non-resonant ideal modes become unstable near the magnetic axis, when the pressure profile is highly peaked in Heliotron E. However, the beta limit of non-resonant mode is higher than that of resonant mode. On the other hand for negative shear tokamaks, resistive interchange modes are destabilized, when q_0 (q value at magnetic axis) is larger than $q_{\min}$ (minimum q value). Here the non-resonant ideal modes similar to those in Heliotron E also become unstable.

Keywords:

interchange mode, pressure profile, heliotron, negative shear tokamak

1. Properties of Interchange Mode

General properties of interchange modes were known for tokamaks [1] and stellarators [2]. Particularly for heliotrons with large rotational transform and high shear, the pressure-driven interchange modes are dangerous [3]. In Heliotron E the origin of the observed sawtooth oscillations is considered as the resistive or ideal interchange mode [4]. For explaining the crash phase of soft-X ray fluctuation, the ideal interchange mode seems relevant [5].

For the theoretical study of interchange modes smooth pressure profiles decreasing toward to edge are usually assumed. In Heliotron E with a wide magnetic hill region, resistive interchange modes are unstable even for extremely low beta plasmas and these modes may affect the pressure profile to flatten at the resonant surface.Thus we consider a stair-like pressure profile or a profile with a locally flat region for the stability study in Section 2.

One well-known property of the localized interchange mode is that the growth rate becomes extremely small near the marginal state, which is shown as $\gamma \propto \exp\,[-\mathrm{const}/(\beta(0)/\beta_c(0) - 1)^{1/2}]$, where $\beta_c(0)$ is the beta limit of relevant ideal interchange mode [6]. The associated radial mode structure becomes highly localized at the resonant surface even for low mode number interchange instabilities. This fact means that the stability condition called Mercier-criterion for toroidal plasmas or Suydam criterion for cylindrical plasmas is applicable to the lowest mode number case with $m = 1/n = 1$, where $m(n)$ is a poloidal (toroidal) mode number.

For studying the stability of interchange mode in a sheared magnetic field, usually resonant modes are considered dangerous. However, this is not valid when a

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pressure profile is highly peaked or a low shear region exists. Several examples are shown in Section 3.

2. Interchange Modes for Locally Flat Pressure Profile

When the pressure gradient vanishes at the mode resonant surface, the properties of interchange mode change and the beta limit increases. For the locally flat pressure profile given by $P/P_0 = 1 - r^2 + \lambda\,(r - r_s)\exp[-(r - r_s)^2/2W^2]$ (see Fig. 1), the growth rate of $m = 2/n = 1$ mode is plotted as a function of $\beta(0) = P_0/(B^2/2\mu_0)$ in Fig. 2, where $\lambda = -\mathrm{d}P/\mathrm{d}r|_{r=r_s}$ and r_s is the resonant surface. Here both pressure and rotational transform profiles are fixed, when $\beta(0)$ is increased. The current effect is negligible, since $\beta(0)$ is low. The eigenfunctions for $W = 0$ and $W = 0.01$ are also shown here. Even for the case of $W = 0.01$ the beta limit increases and the radially localized mode structure disappears for the nearly marginal state. Also it is noted that this tendency has been seen for the toroidal model [7].

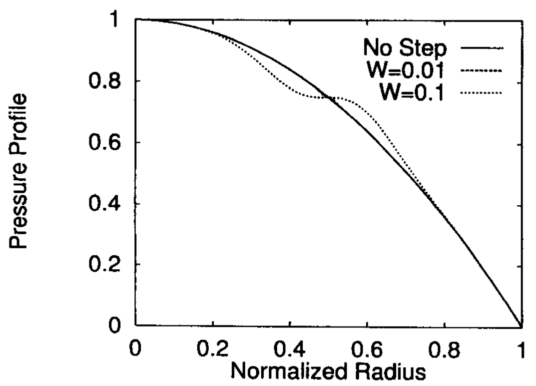

Fig. 1 Locally flat pressure profiles for $W = 0.01$ and $W = 0.1$. As a reference $W = 0$ case is also plotted.

In a realistic situation, the flattering of pressure profile may occur at many resonant surfaces, when the resistive interchange modes are unstable in the wide region. For a cylindrical model of Heliotron E, a stair-like pressure profile has been obtained to improve the beta limit several times by considering resonant surfaces for $n \leq 4$ [8].

3. Non-resonant Modes in Heliotron and Tokamak

First we found that non-resonant resistive modes become unstable in the central region for a highly peaked pressure profile case in Heliotron E [9]. Recently we also found that the non-resonant ideal modes become unstable in the similar situation [5]. For toroidal currentless plasmas of Heliotron E, the rotational transform at the magnetic axis increases from 0.47 at $\beta(0) = 0$ to the values larger than 0.5, when $\beta(0) \gtrsim 1$ % for a highly peaked pressure profile [5]. The property of these non-resonant modes is similar to that of the infernal mode in low shear tokamaks [10].

It is shown that the resistive interchange modes become unstable in negative shear tokamaks [11,12]. It is noted that the localized ideal interchange modes are stable in tokamaks when $q > 1$ in the whole plasma region [1]. In order to destabilize the localized resistive interchange modes the condition, $q_0 > q_{\min}$, is necessary;

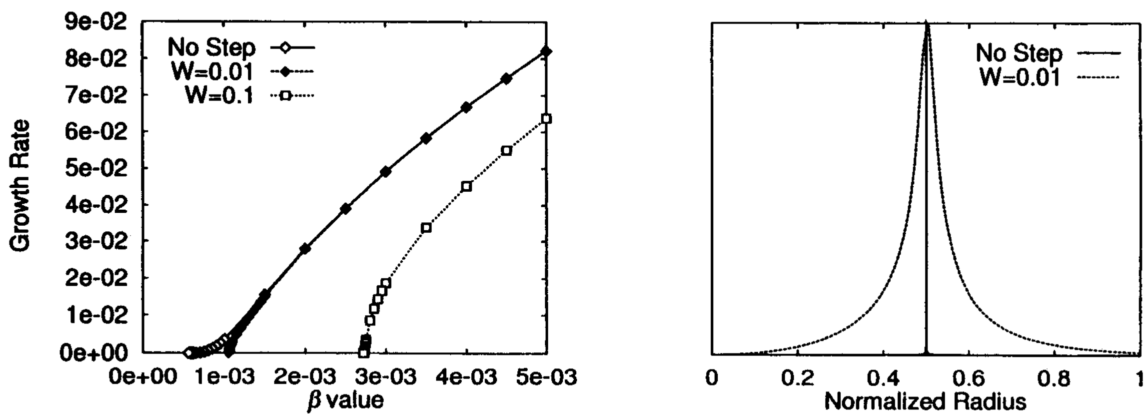

Fig. 2 Growth rate of $m = 2/n = 1$ mode is shown as a function of $\beta(0)$ for a cylindrical plasma with rotational profile of $\iota(r) = 0.4 + 0.2\,r^2$ in the left hand side figure. Eigenfunctions for $W = 0$ and $W = 0.01$ at nearly marginal state are plotted in the right hand side figure.

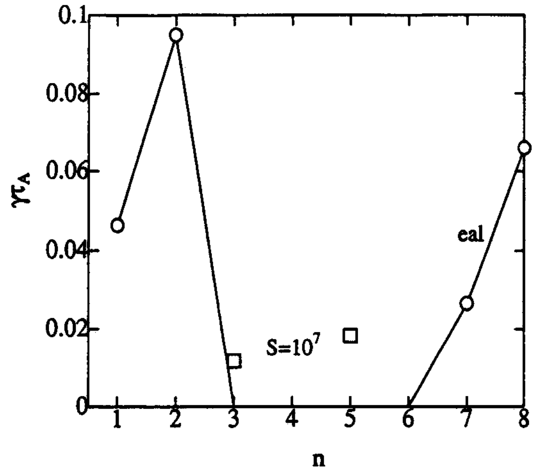

Fig. 3 Growth rates for interchange modes with $1 \le n \le 8$ in a negative shear tokamak. For n = 3 and n = 5, only the resistive interchange mode becomes unstable and growth rate for $S = 10^7$ is shown. For n = 1, 2, 7 and 8, the non-resonant ideal mode becomes unstable.

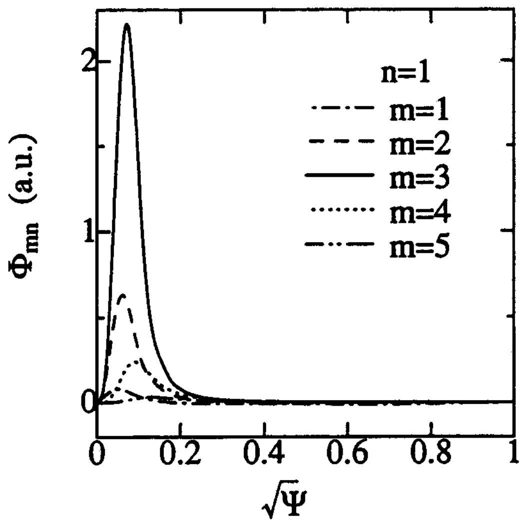

Fig. 4 Radial mode structure of n = 1 non-resonant ideal mode in a negative shear tokamak. The growth rate is shown in Fig.3. Here Ψ denotes a poloidal flux function.

however, by deforming the outermost flux surface to elliptic or dee-shape, the stability is improved and $\beta_N \sim 2$ is possible for fairly broad pressure profiles, where β_N is a normalized beta given by $\beta_N = \beta(\mathrm{a(m)}B(\mathrm{T})/I_p$ (MA)). Here a is a radius, B is a toroidal field and I_p is a plasma current. By applying the RESORM code to a negative shear tokamak with a circular cross-section, low-n resistive modes are shown unstable. Figure 3 shows the growth rate of unstable mode for different toroidal mode number with $1 \le n \le 8$ [12]. It is noted that n = 3 and n = 5 cases for $S = 10^7$ shown with squares belong to the resistive interchange modes; however, other cases with n = 1, 2 and n = 7, 8 are non-resonant ideal modes. Here S is a magnetic Reynolds number. For the cases in Fig. 3, Mercier modes are stable and the unstable non-resonant modes have similarity to the infernal modes. The radial mode structure of the n = 1 non-resonant mode is shown in Fig. 4. Here q-profile is almost flat with q_0 = 2.8 near the magnetic axis and the (m, n) = (3, 1) mode gives the minimum of $|q_0 - m/n|$. It is quite similar to that obtained for the Heliotron E [12].

4. Nonlinear Interchange Mode

When we include both ideal and resistive interchange modes in the nonlinear evolution of Heliotron E plasma, the flattening of pressure profile occurs self-consistently. For the multi-helicity case including several toroidal mode numbers, a stair-like pressure profile may be obtained at the saturated state [8]. For this profile the beta-limit due to the linear ideal interchange will be higher than the Mercier or Suydam limit for a standard smooth pressure profile [7]. On the other hands, when the source term or heating term is included in the pressure evolution equation, a relaxation oscillation similar to the sawtooth appears for the dominant mode with $m = 1/n = 1$ [4] or $m = 2/n = 1$ [5] in the case of Heliotron E. When the magnetic axis is shifted inward and the pressure profile is highly peaked, the $m = 2/n = 1$ resonant interchange mode is easily destabilized. Nonlinear evolution of this mode seems consistent with the sawtooth observed by the soft-X ray tomography.

Non-linear evolution was also studied for the non-resonant mode with $m = 2/n = 1$ [5]. When the mode amplitude becomes finite, the rotational transform changes and the mode resonant surface of $\iota = 0.5$ for the $m = 2/n = 1$ mode appears and the reconnection of magnetic field line occurs in the nonlinear stage. Thus the global nonlinear behavior of the non-resonant mode looks similar to that of the resonant ideal interchange mode. Thus the $m = 2/n = 1$ resonant or non-resonant interchange mode may explain the sawtooth observed in the Heliotron E.

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