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Does the Ultimate State of Thermal Turbulence Exist?

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Abstract

It is predicted that the ultimate state of thermal turbulence should exsist in natural convection for large enough Rayleigh (*Ra*) number. In 80's Chicago group [1] found hard turbulent state in thermal turbulence which is characterized by the scaling relation between the Nusselt number (*Nu*) and Ra number as $Nu \sim Ra^{2/7}$. The ultimate state was predicted to have a new scaling relation; $Nu \sim Ra^{1/2}$, but it is never be observed in any experiment. In this paper, the prediction was tested by using mercury (Hg) for *Ra* numbers up to 3 order magnitude larger than the predicted value. However, our experiments in Hg found no eveidence of a transition and suggest the assumptions of thories are invalid. Thus it appears that the hard turbulence is the ultimate state.

Keywords:
thermal turbulence, Rayleigh-Benard convection, hard turbulence

1. Introduction

What is the character of turbulence in the limit of infinitely large fluid velocities? Is this turbulence universal? The simplest experimental system to test thermal turbulence is known as Rayleigh-Bénard (RB) convection. The control parameter in RB convection is Rayleigh (*Ra*) number,

$$Ra = \frac{\alpha g \Delta T L^3}{\kappa \nu}, \qquad (1)$$

where α is the coefficient of thermal expansion, κ the thermal diffusivity, ν the kinematic viscosity, g the gravitational acceleration, L the typical length and ΔT the temperature difference across the cell.

Theoretical predictions [2,3] that a new, final and universal type of turbulence, ultra-hard turbulence, should exist in *RB* for very large *Ra* numbers stimulated current experiments. The new state was predicted to have a new scaling relation between the Nusselt number (the ratio between advective thermal transport and diffusive thermal transport in the absence of flow, *Nu*) and *Ra* with

$$Nu \sim Ra^{1/2}, \qquad (2)$$

rather than the relation for hard turbulence [1] of

$$Nu \sim Ra^{2/7}. \qquad (3)$$

Such a transition would have important consequences for all calculations of thermal transport at very high *Ra* number. Kraichnan [4] originally suggested this limiting state for turbulence in 1962, with later theories by Howard [5] and Busse [6]. Cioni *et al.* [7] in Hg and Chavanne *et al.* [8] in gaseous He have claimed to detect this transition. However, our experiments in Hg,

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which reached higher effective Ra, found no evidence of a transition and suggest that the assumptions of the theories are invalid. Thus it appears that ultimate turbulence is hard turbulence.

2. Search for the New Regime

Convective turbulence is inevitably anisotropic and non-homogeneous at large length scales. Two types of boundary layers are important. Near the top and bottom container walls, flow velocity vanishes due to the non-slip condition which creates an highly sheared viscous boundary layer. The temperature profile becomes linear near the walls, because only diffusion transports heat since advection is suppressed. The time averaged temperature in the bulk is constant and equal to the average of the top and bottom plate temperatures due to strong mixing by the turbulent flow. The time averaged profile is steep and linear in the thermal boundary layer. The thickness of the thermal boundary layer, which is the inverse of the temperature gradient, limits the gross heat transport across the turbulent cell. Thus Nu is proportional to the ratio between the cell height and the thickness of the thermal boundary layer.

The theories predicting $Nu \sim Ra^{1/2}$ make one of two assumptions. The first is that, since the thickness of the viscous boundary layer varies as $\sim Ra^{-1/2}$, it becomes negligible at very high Ra. Thus heat is advected by buoyant structures (*e.g.* plumes or thermals) which move at the free fall velocity, *i.e.*, since viscous forces are negligible compared with inertial forces in this regime, the thermals accelerate as if they were free, undamped particles, subject to the buoyancy force produced by gravity. Since the free fall velocity, V scales as $\sim (\alpha g \Delta TL) \sim Ra^{1/2}$, the heat flux scales as $Nu \sim Ra^{1/2}$. The other argument is that when the viscous boundary layer becomes thinner than the thermal boundary layer (about $Ra = 10^{14}$ for He [9] and $Ra = 10^5$ for Hg [7,10]), the thickness of the viscous boundary layer limits the heat flux, which also gives $Nu \sim Ra^{1/2}$. The critical Ra for the transition, Ra_{crit} is predicted to depend on the Prandtl number of the fluid (the ratio of thermal and viscous dissipation, $Pr = \kappa/\nu$) as $Ra_{crit} \sim Pr^4$ [2].

Shraiman and Siggia predict $Ra_{crit} \sim 10^6$–10^8 for Hg [2,3]. Cioni *et al.* proposed modifications to the theory which predicted $Ra_{crit} = 5 \times 10^5$ in Hg [7].

The Chicago experiments [8,11,12] on turbulence in RB convection in low temperature gaseous He, saw no indication of a shift in Nu scaling up to $Ra \sim 10^{14}$ but did see a suggestive change in the temperature power spectrum.

Because $Pr = 0.025$ at 20°C in Hg compared to $Pr = 0.7$ for He gas below 1 atm at 5°K, searching new range in Hg required only a Ra number of order 10^8. Our initial Hg experiments used aspect ratio one half, one, and two cylindrical cells to reach $Ra \sim 2 \times 10^9$. Our results were compatible with the classical Nu vs Ra scaling exponent of 2/7 = 0.285 [13]. The break in the aspect ratio two data at $Ra \sim 2 \times 10^5$ results from a pattern competition instability in the flow [10]. For small $Ra \sim 10^6$–10^8, strong steady bulk mean flow in the aspect ratio 1 cell, reduces the measured scaling exponent to 0.25 [14]. The exponent increases towards 2/7 with increasing Ra.

These measurements appear to invalidate both of the mechanisms proposed for the onset of ultra-hard turbulence. The thermal and viscous boundary layers cross below $Ra \sim 10^5$, so we should definitely have seen the transition if the boundary crossing theory were correct. The boundary layer thickness argument also seems to fail: in Hg, after the viscous boundary layer crosses the thermal they shrink together with a nearly constant ratio and a scaling exponent of -0.20 ± 0.02 different from the theoretical value of $-1/2$ [14].

Cioni *et al.* in a larger Hg cell of aspect ratio one, measured up to $Ra \sim 5 \times 10^9$ [7]. They claimed that the Nu vs Ra scaling exponent increased above 2/7 at a single point at the very top of their Ra range, but could not measure the new value. Chavanne *et al.* studied aspect ratio one-half RB convection in gaseous and liquid He. In gas near the gas-liquid critical point they saw an increasing Nu vs Ra scaling exponent above $Ra \sim 10^{11}$ [8] and measured an approximate power law of $Nu \sim Pr^{0.072} Ra^{0.389 \pm 0.005}$. Chavanne *et al.* claimed that the new exponent was compatible with 1/2 scaling due to a logarithmic correction.

3. Experimental Setup

Our current experiment uses an aspect ratio one half cylinder (30cm × 60cm). The top and bottom plates are solid copper 5cm thick, coupled on top to cooling water via 126 vertical, thermally anchored cooling pipes. Heating is supplied by four resistance coils soldered to bottom plate and driven by four regulated DC power supplies (Takasago, GP110-30 × 4) with an accuracy of 0.01% and a maximum combined heat supply of approximately 12,000Watts. Cooling is supplied by two building air-conditioning units (capacity ~15000Watts) with PID temperature regulation controlled by a flow rate throttle. Temperature control of the top plate is accurate to 2% of the total temperature difference (ΔT) at the maximum power. Top plate temperature ranges from 15°C to 42°C and bottom plate temperature from 18°C to 130°C. The maximum variation of Prandtl number occurs at maximum power when it is 0.024 at the top plate and 0.018 at the bottom plate. The horizontal temperature non-uniformity across the plate diameter is much less than 1%. The size and power capacity were dictated by our need to search three decades higher in *Ra* number than the highest value predicted for the transition so that we could definitively determine whether the transition exists.

A thermal shield surrounds the cell. It can be temperature matched to the cell to eliminate radiative and convective heat losses. Without the thermal shield, heat loss from the bottom plate is less than 1% at the highest *Ra* and much lower for smaller *Ra*. With the thermal shield the worst case heat loss is less than 0.1%. Thermal conduction through the cell walls is negligible. The power supplied by the heater thus gives the heat flux to an accuracy of better than 1%.

4. Experimental Results

In Fig. 1 we show the *Nu* vs *Ra* curve for the combined data for our four experiments. From $Ra = 2 \times 10^5$ to 8×10^{10} the Nusselt numbers lie on top of each other with a constant scaling exponent of 0.29 ± 0.01. This *Ra* number is the highest yet achieved in a low *Pr* number fluid, and the estimated Reynolds number (*Re*) is 5×10^5 [14], which is higher than that of Chavanne *et al.* Even at the highest *Ra* numbers, the data show no indication of an increase in power law.

The *Nu* vs *Ra* curve is a highly averaged characterization of the fluid flow. In particular, transitions which change the temperature histograms and power spectra (about which the theory is mute) might not change the *Nu* vs *Ra* exponent. Figure 2 compares the power spectra and histograms for temperature time series taken for $Ra = 1.85 \times 10^9$ and $Ra = 5.14 \times 10^{10}$. Except for the expected increase in inertial scaling range and shrinking width of exponential decay in the temperature histogram, the two curves correspond exactly to each other. The small asymmetry in the histogram shape results from the location of the temperature probe above the center of the cell, 15 cm from the cold top plate [10]. Again we see no sign of a qualitative change in the turbulence.

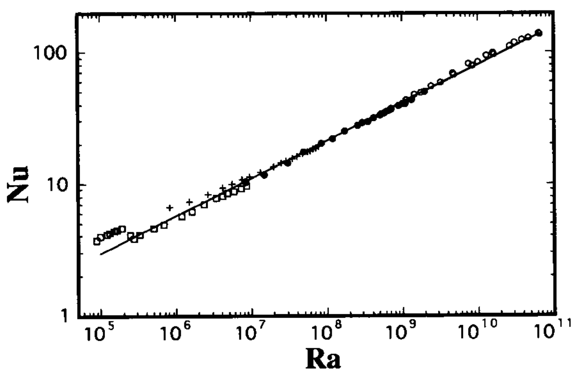

Fig. 1 Nusselt number as a function of Rayleigh number for: circles, large aspect ratio one half cell; crosses, small aspect ratio 1 cell; squares small aspect ratio two cell; bullets, small aspect ratio one half cell. The solid line line is $Nu \sim Ra^{0.285}$. The discontinuity in the small aspect ratio two data is a result of a pattern competition instability. The reduced slope for small *Ra* for the aspect ratio one data results from strong bulk circulation.

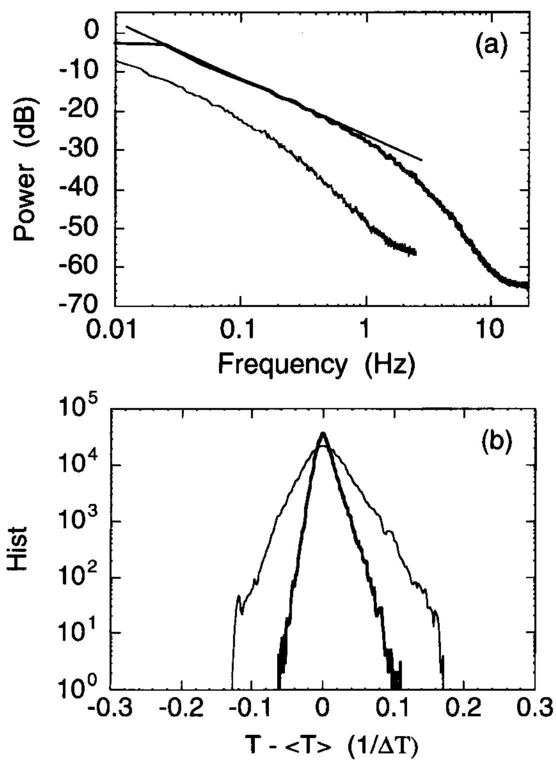

Fig. 2 (a) Power spectra of the temperature fluctuations. (b) Histograms of the temperature fluctuations. Thin line: $Ra = 1.85 \times 10^9$. Thick line: $Ra = 5.14 \times 10^{10}$.

5. Conclusion

Chavanne *et al.* clearly see an interesting effect in gaseous He – an increase of *Nu* at very high *Ra* – but the transition is gradual and never shows a clear power law, certainly not $Ra^{1/2}$. One complication in interpreting their results is that *Pr* variations near the critical point. In Hg, in which these complicating factors do not contribute (and for substantially higher *Re*), we see no evidence of a transition. Based on these results it appears that classical hard turbulence holds for arbitrarily high *Ra*, up to the limit where shocks and molecular granularity become important.

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