J. Plasma Fusion Res. SERIES, Vol. 2 (1999) 12–14

Parallel Plasma Transport at Long Mean-Free Path

HAZELTINE Richard D.
Institute for Fusion Studies, The University of Texas at Austin Austin, Texas 78712, USA

(Received: 8 December / Accepted: 13 May 1999)

Abstract

Transport parallel to the magnetic field of a toroidal plasma confinement system is investigated through kinetic theory, with emphasis on the long mean-free path limit. It is noted that a full transport matrix does not exist in the collisionless limit. A collisionless transport law, involving a non-local operator that accounts for toroidal topology, is derived for parallel heat conduction on ergodic magnetic surfaces. In the rational surface case, perpendicular diffusion must be included in the kinetic equation to avoid singularity; this allows a calculation of the width and amplitude of resonant temperature perturbations that will be excited by heat sources with sufficiently broad Fourier spectra.

Keywords:
plasma trasnport, long mean free path

1. Introduction

Collisionless transport parallel to the magnetic field of a toroidal confinement system deserves study for several reasons. First, the transit frequency in many such systems far exceeds the collision frequency, so that the long mean-free path regime has practical importance, especially outside the separatrix, where the assumption of full parallel equilibration is not adequate to describe relevant phenomena. Second, the topic bears on general issues of fluid closure: under what circumstances can a nearly collisionless plasma be described by fluid (or fluid-like) equations? Finally the subject has intrinsic interest, especially in comparison to more conventional (collisional) relaxation mechanisms.

We begin by reviewing the well-known [1] slab-model description of collisionless parallel heat transport. In this regard we note that the full transport matrix is singular in the collisionless limit. Indeed, the Chapman-Enskog model for fluid closure, even after it is generalized to allow for non-local effects, is not appropriate outside the short mean-free path regime for which it was constructed. The point is that transport is generally driven by particle and energy sources; the density and temperature profiles mediate this driving mechanism only when collisions dominate.

When the same problem is considered in toroidal topology, with magnetic shear, the distinction between rational and ergodic flux surfaces becomes crucial. On an ergodic flux surface we derive an expression for the heat flow closely analogous to that of the slab, with a modified kernel to account for toroidal topology. But rational surfaces require separate treatment, because of the limited ability of closed field lines to distribute heat. Including radial diffusion to resolve the rational surface resonance, we compute the temperature perturbation corresponding to a localized heat pulse and compare its amplitude to that of the ergodic case.

2. Slab Geometry

Perpendicular transport coefficients often change their functional form as the mean-free path (λ) increases, but parallel transport changes more fundamentally, losing its spatially local character when

Corresponding author's e-mail: rdh@physics.utexas.edu

λ becomes comparable to the gradient scale length. Thus the long mean-free path heat flux h is no longer proportional to the local temperature gradient, but rather responds to the global temperature profile:

$$h(x) = \frac{n_0 v_t}{\pi^{3/2}} \int_0^\infty \mathrm{d}x' \frac{T(x-x') - T(x+x')}{x'}, \qquad (1)$$

where n_0 is the lowest order density, T the temperature and v_t the thermal speed. This result, due to Hammett and Perkins [1], suggests that transport coefficients pertinent at short mean-free path become non-local transport operators at long mean-free path. Unfortunately, while reproducing (1) we find that non-locality is only part of the story. Indeed the full transport matrix becomes singular as $\lambda \to \infty$.

We solve a one-dimensional kinetic equation, representing collisions by a particle and energy conserving Krook model. The transit and collisions frequencies are assumed comparable—there is no collisionality ordering—but the sources, necessary for the presumed steady state, are taken to be weak. The resulting solution is used to compute the 2×2 transport matrix at arbitrary mean-free path. In the collisional limit, this matrix is diagonal but otherwise agrees with conventional short mean-free path results. When the collision frequency is allowed to vanish, the known result of (1) is reproduced, but the corresponding form of the particle flux, Γ, is surprising:

$$\Gamma = -\frac{1}{2} \nu \lambda^2 n_0 \left(\nabla \log p - \frac{e \nabla \Phi}{T} \right). \qquad (2)$$

This is Fick's law, with classical diffusion coefficient $D_c = 0.5\ \nu\lambda^2 = (1/2)\ v_t^2/\nu$ (ν is the collisional frequency) and conventional driving force, $A_1 \equiv \nabla \log p - e\nabla\Phi/T$. What is surprising is that Fick's law represents the exact solution to the kinetic equation for any $\nu > 0$. In the collisionless limit, $D_c \to \infty$, whence $A_1 \to 0$, as indeed the kinetic analysis predicts: $\nabla \log p \to e\nabla\Phi/T$ as $\nu \to 0$, corresponding to the Maxwell-Botzmann response. Note that neither the flow, Γ, nor the force A_1, shows singularity at $\nu = 0$.

Thus the transport matrix, even as a matrix of non-local operators, becomes singular in the collisionless limit. While it is always possible to relate the flows of particles and heat to corresponding sources, a non-singular relation between flows and profiles is possible only at short mean-free path.

3. Entropy Production

The heat flow described above is associated with entropy increase, even in the absence of collisions. Thus entropy increases—information is lost—as heat spreads out from a local energy source. In the absence of physical boundaries, this spreading is irreversible.

We denote the particle source in phase space by $S_0(x, v)$ and the heat source by $S_2(x, v)$. Then the conventional rate of entropy production, denoted by

$$\Theta = -\int \mathrm{d}v \frac{f}{f_M} C(f), \qquad (3)$$

where f is the perturbed distribution function, f_M a Maxwellian distribution and C the collision operator, can be computed exactly for arbitrary collisionality. It includes an entropy flow term, expressed as a divergence (or x-derivative in the one-dimensional case). We annhilate the flow term by performing a spatial average,

$$\langle \Theta \rangle \equiv -\int_{-\infty}^{\infty} \mathrm{d}x \Theta,$$

and find that

$$\langle \Theta \rangle = \left\langle S_0 \frac{\Delta n}{n_0} + S_2 \frac{\Delta T}{2T_0} \right\rangle. \qquad (4)$$

The kinetic equation shows that this quantity is equivalent to the conventional entropy production rate

$$\langle \Theta \rangle = -\int \mathrm{d}x \left[\Gamma \left(\frac{\mathrm{d}\log p}{\mathrm{d}x} - \frac{e\mathrm{d}\Phi}{T\mathrm{d}x} \right) + h \frac{\mathrm{d}\log T}{\mathrm{d}x} \right]. \qquad (5)$$

This result is valid for arbitrary collisionality. In the collsionless limit, the term involving Γ vanishes, as we have remarked. But the heat-flow term remains finite and can be seen, from (1), to make a positive-definite contribution.

4. Toroidal Geometry

To treat toroidal geometry we introduce the poloidal and toroidal angles, θ and ζ, use r for the radial coordinate and denote that field line pitch or safety factor by q. We also suppose a quasi-static state in which the net input of particles and energy to any flux surface vanishes. Then the steady state response to localized sources and sink on an ergodic surface can be computed analogously to the slab case, with the resulting heat flux

$$h(\theta, \zeta) = -\frac{N_0\ v_t}{2\pi^{3/2} T_0} \oint \mathrm{d}\theta' \Delta T(\theta', \zeta + [q](\theta' - \theta)) \cot\left(\frac{\theta' - \theta}{2} \right). \qquad (6)$$

where $[q]$ is the largest integer contained in q. This is the version of (1) appropriate to toroidal topology; it has some interesting features, but does not address the most important aspect of parallel heat transport in a torus: resonance.

On rational flux surfaces $r = r_0$, where $q = m_0/n_0$ for integers m_0 and n_0, the kinetic equation for steady state parallel dynamics is singular for a general source configuration on the surface. The problem is that a closed field line that makes contact with a localized energy source, say, may not connect to the balancing sink. (The requirement that sources and sinks balance on each closed line, rather than merely on the surface as a whole, is the well-known Newcomb condition.) To avoid the resulting singularity we must include radial diffusion, with diffusion coefficient D, as a perturbation to parallel flow. The resulting 2-dimensional kinetic equation can be reduced to an Airy equation and solved exactly; it is then used to compute the temperature perturbation on a surface with a specified source, with the result

$$\frac{\Delta T_{m_0 n_0}}{T} = \frac{S_{m_0 n_0}}{\omega_t} \frac{r_0}{n \omega q'(r_s)}$$

where $S_{m_0 n_0}$ is the Fourier component of the source, ω_t is the transit frequency and $\omega = (D r_0 / n \omega_t q'(r_0))^{1/3}$ is the width of the resonant layer. For comparable source-strengths, this perturbation exceeds that on an ergodic surface by about 2 orders of magnitude. Thus, if a tokamak is penetrated by a narrow heating or cooling spike (as from an ECH pulse or pellet), (low-order) rational surfaces are preferentially affected.

References

[1] G.W. Hammett and F.W. Perkins, Phys. Rev. Lett. **64**, 3019 (1990).