J. Plasma Fusion Res. SERIES, Vol.1 (1998) 522–525

Natural Shaping of Spherical Tokamak Plasma

LI Fangzhu, ZHANG Jinhua, GAO Qingdi, WANG Zhongtian and XU Wenbin
Southwestern Institute of Physic, Chengdu 610041, China

(Received: 30 September 1997/Accepted: 2 March 1998)

Abstract

The dependence of the natural elongation, triangularity and shaping factor for spherical tokamak plasma on the aspect ratio for different internal inductance and poloidal beta is investigated numerically.

Keywords:
spherical tokamak, natural shaping

1. Introduction

Peng and Strickler were the first to systematically investigate the characteristics of spherical tokamaks and pointed out favourable features of such configurations[1]. Stable tokamak discharges with the aspect ratio, $A \approx 1.25$ have been produced in START[2,3].

An interesting feature of equilibrium configurations with low aspect ratio is that the cross-section of plasma column has high natural elongation, triangularity and shaping factor ($S = I_p / aB_T \cdot q_\psi$) even when the vertical equilibrium field is uniform.

In this paper, the natural shaping is investigated numerically in detail. It is shown that the natural elongation strongly depends not only on the internal inductance of plasma column, but also on the poloidal beta as well.

2. MHD Equilibrium Configuration

In this paper the toroidal current density is taken to have the form

$$j_t = j_0 \left[\beta_{P0} \frac{r}{R_0} + (1 - \beta_{P0}) \frac{R_0}{r} \right] (\psi - \psi_b)^{a_1} f(\psi)$$

where

$$f(\psi) = \begin{cases} \psi_m - \psi + a_2 (\psi_m - \psi_b) & if\, a_2 \neq 0 \\ 1 & if\, a_2 = 0 \end{cases} \qquad (1)$$

ψ_b is the flux on the boundary of plasma column and is determined by the boundary condition on limiter,

$$\psi_b = \max\, \psi(r, z), \qquad (r, z) \subset \mathrm{L}\,, \qquad (2)$$

ψ_m is the flux on magnetic axis. R_0 is the major radius of the plasma center, j_0 is obtained by the condition $\int j_t \mathrm{d}s = I_p$. β_{p0}, α_1 and α_2 are parameters which can be adjusted to give poloidal beta β_p and internal inductance l_i:

$$\beta_P = \frac{4 \int P \mathrm{d}v}{\mu_0 R_0 I_P^2}\,, \qquad l_i = \frac{2 \int B_P{}^2 \mathrm{d}v}{\mu_0^2 R_0 I_P^2} \qquad (3)$$

In Eq.(1), if $\alpha_1 = \alpha_2 = 0$, the current profile is quasi-uniform; when $\alpha_2 = 0$, $\alpha_1 \neq 0$, it is peaked, for $\alpha_1 \neq 0$ and $\alpha_2 \neq 0$, then the current profile may be hollow, which corresponds to a large fraction of bootstrap current.

The poloidal flux ψ can be divided into two parts

$$\psi = \psi_P + \psi_e \qquad (4)$$

where ψ_p is produced by the plasma current and can be expressed as

$$\psi_P = \iint G(r, z, r', z') \mathrm{j}_t (r', \psi(r', z')) \mathrm{d}r' \mathrm{d}z' \qquad (5)$$

where G is the Green function.

Corresponding author's e-mail: yegaoy@public2.bta.net.cn

The exact solution for ψ_e, produced by the currents in the external coils, was given by Pfirsch and Rebhan[4].

$$\psi_e = \sum_n [A_n P_n(r, z) + B_n Q_n(r, z)]$$

where the lowest P_n and Q_n for even n's are

$$P_2 = \frac{1}{2} r^2, \qquad Q_2 = \frac{z^2}{2} + \frac{1}{4} r^2 - P_2 \ln r,$$

$$P_4 = \frac{1}{4} r^2 z^2 - \frac{1}{16} r^4, \qquad (6)$$

$$Q_4 = \frac{z^4}{24} + \frac{r^2 z^2}{8} - \frac{5}{64} r^4 - P_4 \ln r$$

corresponding to serial multipole fields. An uniform external vertical field is obtained only if A_2 is not zero in Eq.(6). For given uniform vertical field and current parameters I_P, R_0, β_{P0}, α_1 and α_2, by solving the free boundary toroidal plasma MHD equilibrium Grad-Shafranov equation[5,6] with the boundary condition Eq.(2), we can determine the configurations with different aspect ration A, poloidal beta β_p and internal inductance l_i.

3. Dependence of Natural Shaping on the Aspect Ratio

For a plasma with constant current density, Mukhovatov and Shafranov found that the elongation, κ in a equilibrium field with index $n = 0$ was given by

$$\kappa = 1 + \frac{3}{4A^2}\left(\ln 8A - \frac{17}{12}\right) \qquad (7)$$

in large aspect ratio approximation[7]. It is shown that the natural elongation decreases with the increase of aspect ratio. This is in agreement with our numerical calculations. For low aspect ratio or non-constant current density, however, Eq.(7) is not valid. For $l_i = 0.5$, using computed results, we have fitted the natural elongation with a polynomial of the inverse aspect ratio:

$$\kappa = 1 + 1.654A^{-2} - 0.624A^{-4} \qquad (8)$$

We find also that the natural elongation has a sensitive dependence on the internal inductance l_i which characterizes the peakedness of current profile. This dependence is based upon the change of effective current channel. It can be described in terms of an effective aspect ratio. The natural elongation as a function of the aspect ratio is shown in Fig. 1 for $l_i = 0.5$ and $l_i = 1.0$. We see that the natural elongation of flat current profile is larger than that of the peaked current profile. The coefficients on the right hand of Eq.(8) can be

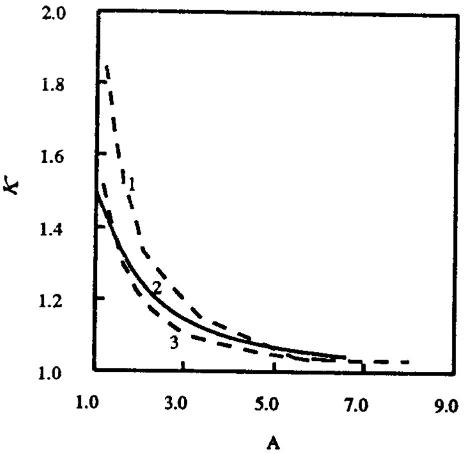

Fig. 1 Natural elongation *vs.* aspect ratio. 1, l_i = 0.5, β_p = 0.5; 2, from M-S Eq.(7); 3, l_i = 1.0, β_p = 0.5.

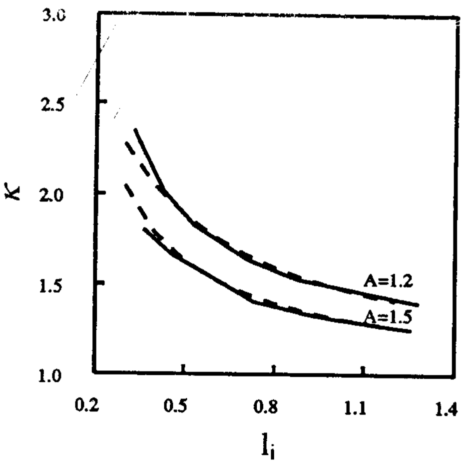

Fig. 2 Natural elongation *vs.* internal inductance for different aspect ratio at β_p = 0.5.

expressed in terms of internal inductance. According to reference[8] it can be written as $\kappa \propto c_1/l_i + c_2/l_i^2$. Using natural elongations for $l_i = 0.3$, 0.5 and 1.0, the general dependence of natural elongation on the internal inductance is given by

$$\kappa = 1 + \left[\frac{c_1}{l_i} + \frac{c_2}{l_i^2}\right]A^{-2} + \left[\frac{c_3}{l_i} + \frac{c_4}{l_i^2}\right]A^{-4} \qquad (9)$$

where c_1 = 0.6306, c_2 = 0.1064, c_3 = 0.1952, c_4 = −0.2439. This dependence is exhibited in Fig. 2. The natural elongation as a function of internal inductance for different values of aspect ratio at $\beta_p = 0.5$ is shown there and the dashed lines are the results from Eq.(9). The natural elongation decreases with the increase of internal inductance for all aspect ratio values studied here.

The effect of poloidal beta on the natural elongation is also investigated. A finite poloidal beta introduces an outward displacement of the magnetic axis, which, in the quasi-uniform current profile approximation, is written as[7]

$$\Delta = R\frac{\left(1+3\left(\beta_P+\frac{1}{4}\right)^2/A^2\right)^{1/2}-1}{3\left(\beta_P+\frac{1}{4}\right)} \tag{10}$$

It changes the effective radial position of plasma column. The computed dependence of the natural elongation on the internal inductance for different poloidal beta values at aspect ratio A = 1.3 is shown in Fig. 3. Increasing the internal inductance, the effect of poloidal beta becomes more effective. This is important for the design of low aspect ratio tokamaks.

The triangularity of plasma column plays an important role in plasma confinement and advanced tokamak design. It is found from our numerical results that the natural triangularity decreases with the increase of aspect ratio. Generally speaking, in conventional tokamaks with aspect ratio $A > 2.5$, the natural triangularity, δ is lower than 0.05, whereas in low aspect ratio tokamaks with aspect ratio $A < 1.5$, δ is larger than 0.20. The dependence of natural triangularity on the aspect ratio for $l_i = 0.5$, $\beta_p = 0.5$ is given in Fig. 4. It is clearly shown that δ decreases sharply with the increase of aspect ratio up to $A \sim 2.0$.

The dependencies of the natural shaping factor on A for different l_i are investigated numerically. There are

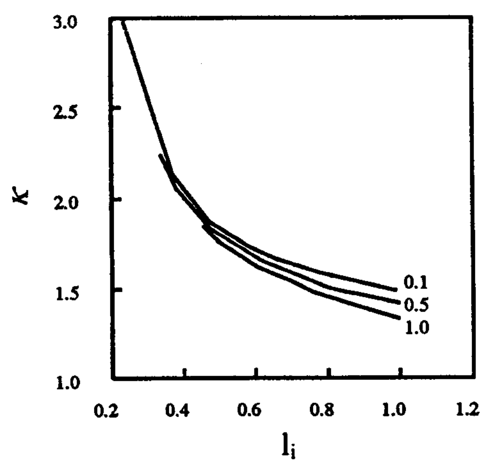

Fig. 3 Effect of poloidal beta on natural elongation.

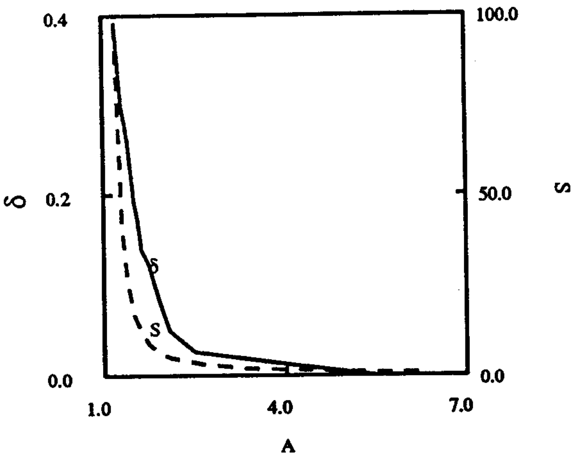

Fig. 4 Natural triangularity and shaping factor *vs* aspect ratio.

so slight differences between the results for $l_i = 0.5$ and $l_i = 1.0$, although they are hardly distinguished in the figure. The results for the case of $l_i = 0.5$, $\beta_p = 0.5$ are also shown in Fig. 4. The natural shaping factor increases rapidly as A is reduced. This dependence can be fit by the expression

$$S = 1 + 2.0(A-1)^{-1} + 2.7(A-1)^{-2} + 0.1(A-1)^{-3} \tag{11}$$

For spherical tokamak with $A = 1.2$, $l_i = 0.5$ and $\beta_p = 0.5$, The natural shaping factor can reach 97.

4. Discussion

In this paper, we have shown that the natural elongation of spherical tokamak plasmas has an explicit dependence on the plasma internal inductance and that the effect of poloidal beta (or the pressure profile) is not negligible. When the effect of the poloidal beta is not included, the natural elongation calculated from Eq.(3) of Ref.[8] is about 1.76 for A = 1.3 and l_i = 0.5. Taking the β_p effects into account, we have found numerically that the natural elongations are 1.76, 1.80 and 1.91 for β_p = 1.0, 0.5 and 0.1, respectively, when A and l_i are the same as above. Thus, the significant effects of the poloidal beta on the natural elongation are clearly demonstrated.

No explicit dependence of the natural triangularity on internal inductance and poloidal beta has been found from our numerical results.

References

[1] Y.K.M. Peng and D. J. Strickler, Nucl. Fusion **26**, 769 (1986).

[2] A. Sykes *et al.*, Nucl. Fusion **32**, 694 (1992).

[3] A. Sykes *et al.*, IAEA-CN-60, 111 (1994).
[4] D. Pfirsch and E. Rebhan, Nucl. Fusion **14**, 547 (1974).
[5] K. Shinya and H. Ninomiya, JAERI-M 9278 (1981).
[6] Li Fangzhu *et al.*, Chinese Journal of Plasma Phys. and Nucl. Fusion **9**, 12 (1989).
[7] V.S. Mukhovatov and V.D. Shafranov, Nucl. Fusion **11**, 605 (1971).
[8] M.O. Galvão *et al.*, Comments Plasma Phys. Controlled Fusion **15**, 219(1993).