J. Plasma Fusion Res. SERIES, Vol.1 (1998) 495–498

Study of Kinetic Ion-Temperature-Gradient-Driven Instabilities in Toroidally Rotating Plasmas

SUGAMA Hideo* and HORTON Wendell[1]
National Institute for Fusion Science, Toki 509-5292, Japan
[1]*Institute for Fusion Studies, The University of Texas at Austin, Austin, Texas 78712, USA*

(Received: 30 September 1997/Accepted: 22 October 1997)

Abstract

Ion-temperature-gradient-driven instabilities (ITG modes) in toroidally rotating plasmas are studied. The gyrokinetic equation for ions, the adiabatic assumption for electrons and the charge neutrality condition are used with the ballooning representation to derive the kinetic integral ITG eigenmode equation. Solving the integral eigenmode equation numerically, effects of the sheared rotation on the linear growth rate and the mode structure are investigated. In the presence of the large sheared toroidal rotation, the radial wavenumber in the ballooning representation depends on time through the ballooning angle $\theta_k(t) = \theta_{k0} - (\partial V^\zeta/\partial q)t$ where $\partial V^\zeta/\partial q$ denotes the derivative of the toroidal angular velocity with respect to the safety factor. Then, the growth rate of the mode is also regarded as a function of the time through the ballooning angle as $\gamma = \gamma(\theta_k(t))$. For $\theta_k(t) > \pi/2$, the mode is stabilized. The significant reduction of the time-averaged growth rate due to the sheared toroidal rotation is expected.

Keywords:

ITG modes, sheared toroidal rotation, ballooning representation

1. Introduction

In most magnetically confined plasmas, observed particle and heat fluxes are dominated by anomalous transport [1]. The anomalous transport is considered to be due to turbulent fluctuations driven by various plasma instabilities. Recently, as a means to the confinement improvement such as H-modes [2] and internal transport barriers [3] found in tokamaks, turbulence suppression by background sheared flows (or sheared radial electric fields) has been attracting considerable attention. In this work, we investigate effects of toroidal flow shear on ion-temperature-gradient (ITG) driven modes [4, 5, 6, 7], which are micro-instabilities causing the anomalous transport in a core plasma. We consider a toroidally rotating axisymmetric system, in which the magnetic field and the toroidal flow are written as $\boldsymbol{B} = I\nabla\zeta + \nabla\zeta \times \nabla\Psi$ and $\boldsymbol{V}_0 = V_0\hat{\boldsymbol{\zeta}} = -Rc(\partial\Phi_0/\partial\Psi)\hat{\boldsymbol{\zeta}}$, respectively. Here, θ and ζ denote the poloidal and toroidal angles, respectively, and Ψ represents the poloidal flux. The background electrostatic potential is denoted by Φ_0.

By the ballooning mode representation for toroidally rotating system [8, 9], an arbitrary fluctuation field $\hat{F}$ is written in the WKB (or eikonal) form as

$$\hat{F}(r, \theta, \zeta, t) = \hat{F}(r, \theta, t)\exp[iS(r, \theta, \zeta, t)] \tag{1}$$

where the eikonal S satisfies $\boldsymbol{B}\cdot\nabla S = 0$, and $(\partial/\partial t + \boldsymbol{V}_0\cdot\nabla)S = 0$. Then, for a toroidal mode number n, the eikonal S is given by $S = -n\,[\zeta - q(r)\theta + \int\theta_{k0}(q)dq - V^\zeta t]$ where $V^\zeta = V_0/R = -c\partial\Phi_0/\partial\Psi$ represents the toroidal angular velocity and $\int\theta_{k0}(q)dq$ appears as an integral constant. The safety factor q is also available for a radial coordinate instead of r. The perpendicular

Corresponding author's e-mail: sugama@nifs.ac.jp

wavenumber vector is given by

$$k_\perp \equiv \nabla S = -n[\nabla\zeta - q\nabla\theta + \{\theta_k(t) - \theta\}\nabla q]\,. \quad (2)$$

We should note that the radial wavenumber component contains time dependence through the time-dependent ballooning angle defined by

$$\theta_k(t) \equiv \theta_{k0} - (\partial V^\zeta/\partial q)t\,. \quad (3)$$

In the next section, the kinetic integral ITG mode equation for the toroidally rotating system is derived by using the ballooning representation.

2. Kinetic ITG Mode Equation

Here, we assume that normalized fluctuating quantities and their characteristic wavenumbers and frequencies obey the gyrokinetic ordering: $\hat{f}_a/f_a \sim e_a\hat{\phi}/T_a \sim k_\parallel/k_\perp \sim (\partial/\partial t + V_0\cdot\nabla)/\Omega_a \sim \rho_a/L$ where the subscript a denotes the particle species, and the ordering parameter ρ_a/L is given by the ratio of the thermal gyroradius $\rho_a \equiv v_{Ta}/\Omega_a$ [$v_{Ta} \equiv (2T_a/m_a)^{1/2}$: the thermal velocity, $\Omega_a \equiv e_aB/(m_ac)$: the gyrofrequency] to the equilibrium scale length L. The fluctuating distribution function is divided into adiabatic and non-adiabatic parts as $\hat{f}_a(k_\perp) = -f_{aM}e_a\hat{\phi}(k_\perp)/T_a + \hat{h}_a(k_\perp)e^{iL_a(k_\perp)}$ where $L_a(k_\perp) \equiv k_\perp\cdot(v' \times b)/\Omega_a$ ($v' \equiv v - V_0$). The gyrokinetic equation for the toroidally rotating system [10, 11] with the large aspect ratio $R/r \gg 1$ is written in the linear and collisionless form as

$$\left[\frac{(V_0 + v'_\parallel)}{Rq}\frac{\partial}{\partial\theta} + \frac{\partial}{\partial t} + i(\omega_E + \omega_{Da})\right]\hat{h}_a(\theta, t)$$

$$= f_{a0}J_0(\alpha)\left[\frac{V_0}{Rq}\frac{\partial}{\partial\theta} + \frac{\partial}{\partial t} + i(\omega_E + \omega_{*Ta})\right]\left(\frac{e_a\hat{\phi}(\theta, t)}{T_a}\right) \quad (4)$$

where $\alpha = k_\perp v'_\perp/\Omega_a$ represents the finite gyroradius effect, $\omega_E = k_\perp\cdot V_0 = -nV^\zeta$ is the Doppler shift due to the toroidal rotation, $\omega_{Da} = \epsilon_n\omega_{*a}(m_a/T_a)[\frac{1}{2}(v'_\perp)^2 + (V_0 + v'_\parallel)^2][\cos\theta + \hat{h}\{\theta - \theta_k(t)\}\sin\theta]$ is the magnetic (∇B and curvature) drift frequency, and $\omega_{*Ta} = \omega_{*a}[1 + \eta_a\{m_a(v')^2/2T_a - \frac{3}{2}\} - (m_av'_\parallel/T_a)(L_n\mathrm{d}V_0/\mathrm{d}r)]$ [$\omega_{*a} = k_\theta cT_a/(e_aB\,L_n)$: the diamagnetic drift frequency, $L_n = -(\mathrm{dln}\,n/\mathrm{d}r)^{-1}$: the density gradient scale length]. In Eq. (4), the differential operator $\partial/\partial\theta$ is taken along the magnetic field line. Note that the effect of the toroidal flow shear $\mathrm{d}V_0/\mathrm{d}r$ is included in ω_{Da} through $\theta_k(t)$ [see Eq. (3)] and in ω_{*Ta}.

In order to obtain a closed system of linear equations, we use the gyrokinetic equation (4) for the ions ($a = i$), the Boltzmann relation for the electrons ($a = e$), and the charge neutrality: the latter two conditions are written as

$$\int \mathrm{d}^3v\,\hat{f}_i = \hat{n}_i = \hat{n}_e = n_0(e\hat{\phi}/T_e) \quad (5)$$

Then, the linear behavior of the ITG mode is described by Eqs. (4) and (5). The main destabilizing sources of the ITG mode are given from the ion temperature gradient denoted by $\eta_i \equiv \mathrm{dln}\,T_i/\mathrm{dln}\,n$ in ω_{*Ti} and the magnetic curvature included in ω_{Di}.

In the case of no rotation ($V_0 = 0$), we have $\theta_k(t) = \theta_{k0}$ from Eq. (3) and no explicit temporal dependence appears in the linear coefficients of Eqs. (4) and (5). Thus, we can do Fourier transform easily with respect to the time and replace $\partial/\partial t$ by $-i\omega$. Then, Eqs. (4) and (5) reduce to the eigenvalue problem with ω as a complex-valued eigenfrequency. Romanelli [5] and Dong *et al.* [6] solved this problem to obtain the kinetic dispersion relation of the ITG mode for $V_0 = 0$ and $\theta_k = 0$. However, when the toroidal flow shear exists, Eqs. (4) and (5) should be solved as an initial value problem because of the explicit temporal dependence of the ballooning angle $\theta_k(t)$. In order to solve them more easily, we here assume that the temporal variation of $\theta_k(t)$ is much slower than the characteristic frequency of the mode observed in the rotating frame: $\partial V^\zeta/\partial q \sim V_0/(Rq) \ll (\partial/\partial t + i\omega_E) \sim \omega_{*e}$. We write the temporal dependence of the fluctuations as $\hat{\phi}(t) \propto \exp(-i\omega_E t - i\int\omega(t)\mathrm{d}t)\,\hat{\varphi}(t)$ where $\omega(t) \sim \omega_{*e} \gg (\mathrm{d}\omega(t)/\mathrm{d}t)/\omega(t) \sim (\mathrm{d}\hat{\varphi}(t)/\mathrm{d}t)/\hat{\varphi}(t) \sim V_0/(Rq)$. Then, $\omega(t) = \omega[\theta_k(t)]$, and $\hat{\varphi}(t) = \hat{\varphi}[\theta_k(t)]$ are regarded as a pair of an eigenvalue and an eigenfunction which depend on the time through $\theta_k(t)$. The stability of the mode should be judged from the average growth rate defined by $\gamma_{ave} = \lim_{T\to+\infty} T^{-1}\int_0^T \gamma(t)\,\mathrm{d}t = (2\pi)^{-1}\int_0^{2\pi}\gamma(\theta_k)\,\mathrm{d}\theta_k$ where $\gamma(t) = \mathrm{Im}\,\omega(t)$.

Using the approximation described above, we obtained from Eqs. (4) and (5) the integral ITG mode equation which is given by

$$\left(1 + \frac{T_e}{T_i}\right)\hat{\phi}(k) = \int_{-\infty}^{+\infty}\frac{\mathrm{d}k'}{\sqrt{2\pi}}K(k, k')\hat{\phi}(k') \quad (6)$$

with

$$K(k, k') = -i\int_{-\infty}^{0}\omega_{*e}\mathrm{d}\tau\frac{\sqrt{2}e^{-i(\omega-\omega_E)\tau}}{\sqrt{a}(1 + a)\sqrt{\lambda}}e^{-(k-k')^2/4\lambda}$$

$$\times\left[\frac{\omega}{\omega_{*e}}\tau_e + 1 - \frac{3}{2}\eta_i + \frac{\eta_i(k - k')^2}{4a\lambda} + \frac{2\eta_i}{(1 + a)}\right.$$

$$\left(1 - \frac{k_\perp^2 + k'^2_\perp}{2(1 + a)\tau_e} + \frac{k_\perp k'_\perp}{(1 + a)\tau_e}\frac{I_1}{I_0}\right) - \frac{\tau_e q(k - k')}{\epsilon_n\hat{s}\omega_{*e}\tau}$$

$$\left.\times\left(\frac{L_n}{c_s}\frac{\mathrm{d}V_0}{\mathrm{d}r}\right)\right]\Gamma_0(k_\perp, k'_\perp) \quad (7)$$

where $\tau_e = T_e/T_i$, $\lambda = (\omega_{*e}\tau)^2(\hat{s}\epsilon_n/q)^2/\tau_e a$, $(\theta - \theta_k) = k/\hat{s}k_\theta$, $(\theta' - \theta_k) = k'/\hat{s}k_\theta$, $a = 1 + i2\epsilon_n\tau_e^{-1}\omega_{*e}\tau[(\hat{s}+1)(\sin\theta - \sin\theta') - \hat{s}\{(\theta - \theta_k)\cos\theta - (\theta' - \theta_k)\cos\theta'\}]/(\theta - \theta')$, $\Gamma_0 = I_0(k_\perp k'_\perp/[(1+a)\tau_e])$, $k_\perp^2 = k_\theta^2 + k^2$, and $k'^2_\perp = k_\theta^2 + k'^2$. Here, the wavenumber variables k_θ, k, and k' are normalized by ρ_s^{-1} ($\rho_s = \sqrt{2T_e/m_i}/\Omega_i$). Note that the kernel defined by Eq. (7) contains the flow shear effects through the explicit term ($\propto \mathrm{d}V_0/\mathrm{d}r$) and $\theta_k = \theta_{k0} - (\partial V^\zeta/\partial q)t$. If we put $V_0 = \mathrm{d}V_0/\mathrm{d}r = 0$, the integral ITG mode equation (6) with the kernel (7) reduces to the same one as given by Dong *et al.* [6]. In the next section, Eq. (6) is numerically solved to obtain the growth rate and the eigenfunction of the ITG mode.

3. Numerical Results

The normalized growth rates γ/ω_{*e} obtained by numerically solving Eq. (6) with Eq. (7) are shown in Fig. 1 (a) and (b) for $\eta_i = 2$, $k_\theta = 0.75$, $\epsilon_n = 0.2$, $q = 2$, $T_e/T_i = 1$, and $\hat{s} = r\mathrm{dln}\,q/\mathrm{d}r = 1$. Figure 1 (a) shows the normalized growth rate as a function of the normalized toroidal flow shear $(L_n/L_E)(V_0/c_s)$ [$c_s \equiv (2T_e/m_i)^{1/2}$, $L_E \equiv -V_0/(\mathrm{d}V_0/\mathrm{d}r)$] for $\theta_k = 0$. It is found that, when the ballooning angle is fixed as $\theta_k = 0$, the growth rate increases with increasing the toroidal flow shear. This behavior of the growth rate is similar to the destabilization of the quasitoroidal ITG mode due to the parallel flow shear [7]. In Eq. (4) where the large aspect ratio approximation is used, the perpendicular flow component of the toroidal flow is of $O(r/R)$ of its parallel component and is neglected. Thus, the toroidal flow shear dependence of the toroidal ITG mode for $\theta_k = 0$ shows the similar tendency to the parallel flow shear dependence of the quasitoroidal ITG mode. If the perpendicular component of the toroidal flow is taken into account, the growth rate is expected to be reduced due to the perpendicular flow shear stabilization [7]. For the case of Fig. 1 (a), the dependence of the real frequency of the toroidal flow shear is weaker than that of the growth rate [$(\omega_r - \omega_E)/\omega_{e*} = -0.41$ for $(L_n/L_E)(V_0/c_s) = 0$ and -0.48 for $(L_n/L_E)(V_0/c_s) = 1.5$]. For the same case, the eigenfunction is localized in the bad curvature region around $\theta = 0$ ($\theta = 0$ corresponds to the outermost point in the toroid) while the toroidal flow shear makes the eigenfunction asymmetric in θ.

When the flow shear exists, the growth rate and real frequency are functions of the time through $\theta_k(t) \equiv \theta_{k0}(q) - (\partial V^\zeta/\partial q)t$. Figure 1 (b) shows the growth rate as a function of $\theta_k(t)$ for $\mathrm{d}V_0/\mathrm{d}r = -V_0/L_E \to 0$ where the other parameters are the same as in Fig. 1 (a). We find that the growth rate approaches to zero at $\theta_k(t) \simeq \pi/2$ and that the ITG mode is stabilized for

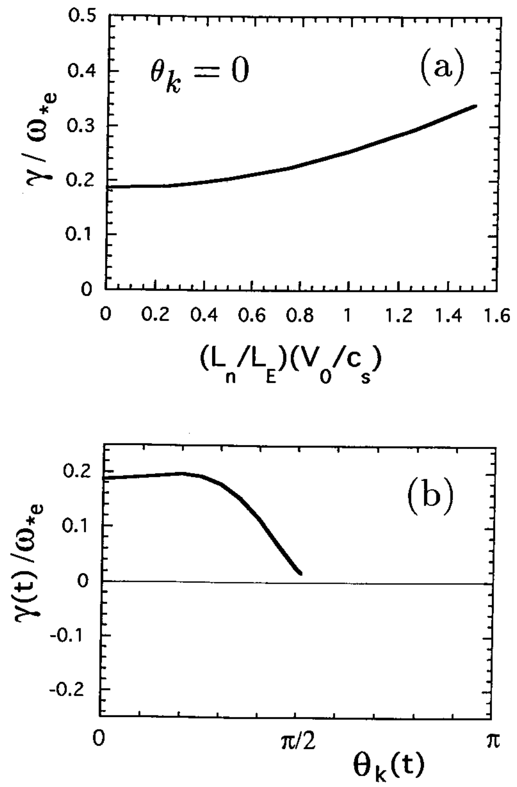

Fig. 1 The normalized growth rate γ/ω_{e*} of the ITG mode as a function of the normalized toroidal flow shear $(L_n/L_E)(V_0/c_s)$ for $\theta_k = 0$ (a) and as a function of $\theta_k(t)$ for $\mathrm{d}V_0/\mathrm{d}r = -V_0/L_E \to 0$ (b). Here $\eta_i = 2$, $k_\theta = 0.75$, $\epsilon_n = 0.2$, $q = 2$, $T_e/T_i = 1$, and $\hat{s} = r\mathrm{dln}\,q/\mathrm{d}r = 1$.

$\theta_k(t) > \pi/2$. Here the growth rate for the stable case is not shown and we need to improve our numerical code to calculate the negative growth rate. Figure 2 shows the spatial structure of the eigenfunction $\hat{\phi}(r, \theta, \zeta)$ corresponding to the case of $\theta_k(t) = 0.4\pi$ in Fig. 1 (b), which is obtained by the ballooning transform of $\hat{\phi}(k)$. The eigenfunction is localized around $\theta \simeq \theta_k = 0.4\pi$ but it is partly contained in the good curvature region $|\theta| > \pi/2$, which causes the reduction of the growth rate. Thus, with changing the ballooning angle $\theta_k(t)$, the eigenfunction moves poloidally along the magnetic field line and the average growth rate $\gamma_{\mathrm{ave}} = \oint \gamma[\theta_k(t)]\mathrm{d}\theta_k(t)/2\pi$ is expected to be significantly reduced.

4. Conclusion

In this work, effects of the toroidal flow shear on the toroidal ITG modes are studied by numerically solving the kinetic integral solution. When the toroidal flow shear exists, the ballooning angle depends on the time as $\theta_k(t) = \theta_{k0} - (\partial V^\zeta/\partial q)t$. Then, the growth rate

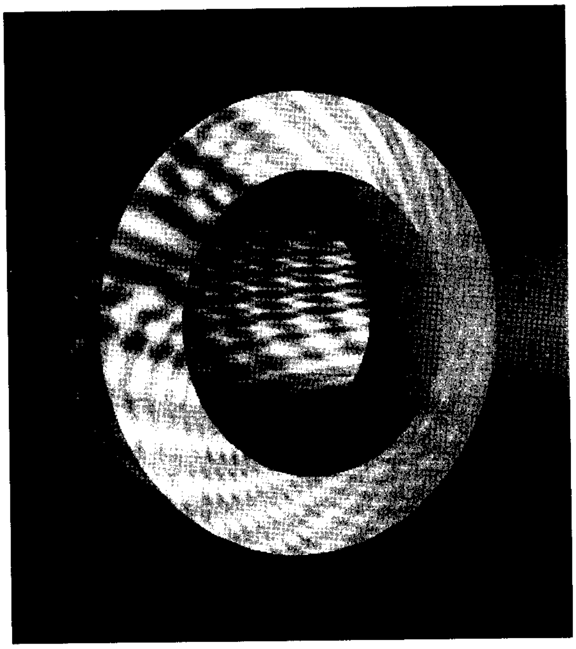

Fig. 2 The spatial structure of the eigenfunction $\hat{\phi}(r, \theta, \zeta)$ corresponding to the case of $\theta_k(t) = 0.4\pi$ in Fig. 1 (b).

and real frequency of the modes also depend on the time through the ballooning angle $\theta_k(t)$. For $\theta_k(t) = 0$ fixed at the bad curvature region, the growth rate increases as the toroidal flow shear increases. The poloidal symmetry of the mode structure is broken by the flow shear. As $\theta_k(t)$ increases, the mode structure moves from the bad to good curvature region. For $\theta_k(t) > \pi/2$, the mode is stabilized. The significant reduction of the average growth rate $\gamma_{ave} = \oint \gamma[\theta_k(t)] d\theta_k(t)/2\pi$ is expected especially for weak toroidal flow shear. Here, because of the large aspect ratio approximation, the perpendicular flow component of the toroidal flow is neglected and the toroidal flow shear for $\theta_k = 0$ shows similar effects to the parallel flow shear destabilization. The perpendicular component of the toroidal flow shear, if included, can reduce the growth rate. However, in this paper, the stabilizing effect by the toroidal flow shear through the rotating ballooning angle is more emphasized. Combined effects of the (parallel and perpendicular) shear flows, the time-dependent ballooning angle, and the negative magnetic shear are to be studied as a future task. Also, for stellarators with quasi-symmetry, the sheared flow in the symmetric direction, which can be produced more easily than in the other directions, is expected to stabilize microinstabilities like the ITG mode.

Acknowledgments

The authors acknowledge useful discussions with Dr. J. Q. Dong. A part of this work was done during the authors visited the National Centre for Theoretical Physics at the Australian National University. This work is supported in part by the U.S. Department of Energy Grant DE-FG05-80ET-53088.

References

[1] J. W. Connor and H. R. Wilson, Plasma Phys. Control. Fusion **36**, 719 (1994).
[2] ASDEX Team, Nucl. Fusion **29**, 1959 (1989).
[3] Y. Koide *et al.*, Plasma Phys. Control. Fusion **38**, 1011 (1996).
[4] W. Horton, M. Wakatani and J. A. Wooton, *AIP Conference Proceedings 284, U.S.-Japan Workshop on Ion Temperature Gradient-Driven Turbulent Transport*, New York: AIP, 1994, ch. 1, p. 3.
[5] F. Romanelli, Phys. Fluids B **1**, 1018 (1989).
[6] J. Q. Dong, W. Horton and J. Y. Kim, Phys. Fluids B **4**, 1867 (1992).
[7] J. Q. Dong and W. Horton, Phys. Fluids B **5**, 1581 (1993).
[8] R. L. Dewar and A. H. Glasser, Phys. Fluids **26**, 3038 (1983).
[9] W. A. Cooper, Plasma Phys. Control. Fusion **30**, 1805 (1988).
[10] M. Artun and W. M. Tang, Phys. Plasmas **1**, 2682 (1994).
[11] H. Sugama and W. Horton, Phys. Plasmas **4**, 405 (1997).