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Net Current Effects on the HINT Computation

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Abstract

A revised scheme of the HINT computation for calculating stellarator equilibria with a net toroidal current is presented. To check the validity of the scheme, it is applied to LHD equilibria with a net current.

Keywords:

HINT computation, 3-D MHD equilibrium, stellarator, relaxation, net current

1. Introduction

The HINT computation is developed to study three dimensional MHD equilibria in toroidal helical systems. This calculation is based on the time-dependent relaxation technique using small values of resistivity and viscosity, which was developed by Park *et al.* [1]. The original HINT code was proposed in Ref.[2]. First targets of the computational study were finite beta stellarator equilibria with **no** net current. It was founded on fact that stellarators have the possibility of net current-free steady operation. One of important advantages of the HINT computation is that it does not need to assume existence of nested flux surfaces in equilibria. Thus, the HINT code enabled us to investigate quantitatively 1) the deterioration of magnetic surfaces and 2) the formation and self-healing of magnetic islands in the finite beta plasma (Refs.[3, 4]).

In the original HINT code we assume that no net current exists, and we cannot study effects of net current on stellarator equilibria. Here, the net current means the Ohmic current, the bootstrap current, and/or the Ohkawa current. The net current can change drastically the rotational transform, and maybe it affects the magnetic surface breaking and the formation/self-healing of magnetic islands. Thus, next target should be investigation of effects of net current. The HINT computation has a potential to overcome this trial. To apply the HINT computational method to equilibria with a net current, we need to revise this computation.

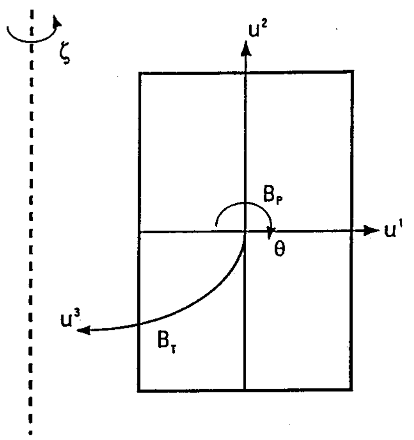

Fig. 1 Directions of the toroidal and poloidal magnetic field.

2. Relaxation Equations

We find MHD equilibria starting from an arbitrary nonequilibrium initial plasma and field configuration by means of a time-dependent relaxation method with small values of resistivity η and viscosity ν. Calculations are performed in the following two steps. The first step is the relaxation process of pressure along field lines.

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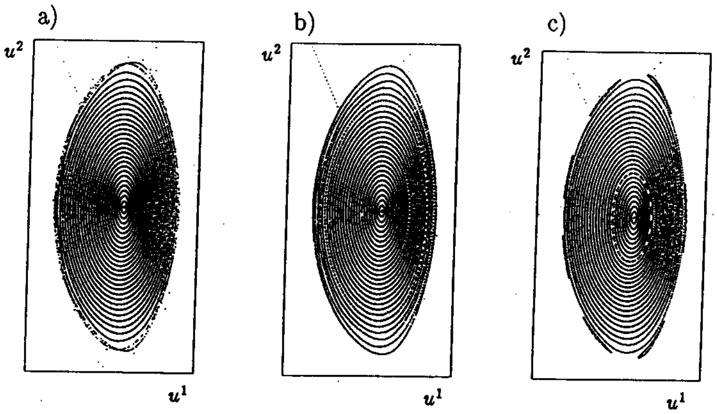

Fig. 2 Poincaré plots of field lines at the toroidal angle ζ = 0 for LHD equilibria with β_0 = 1.4% and a) currentless, b) J_{net} = −50 [kA], c) J_{net} = +50 [kA].

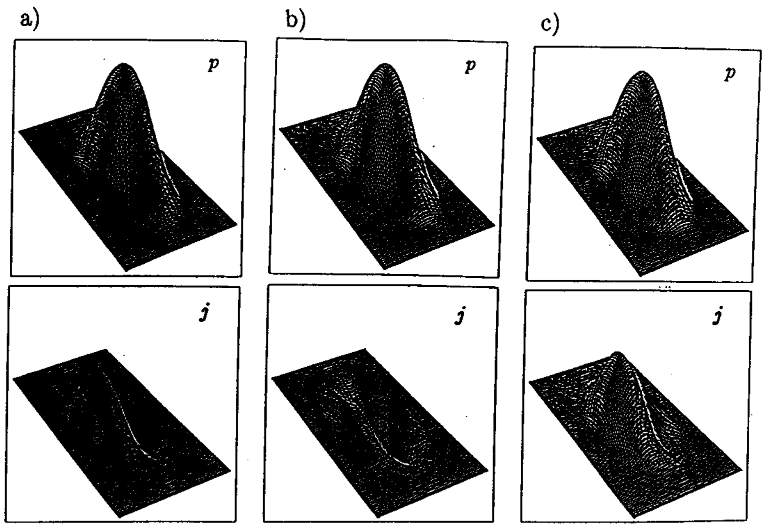

Fig. 3 The pressure and the current at the toroidal angle ζ = 0 for LHD equilibria with β_0 = 1.4% and a) currentless, b) J_{net} = −50 [kA], c) J_{net} = +50 [kA].

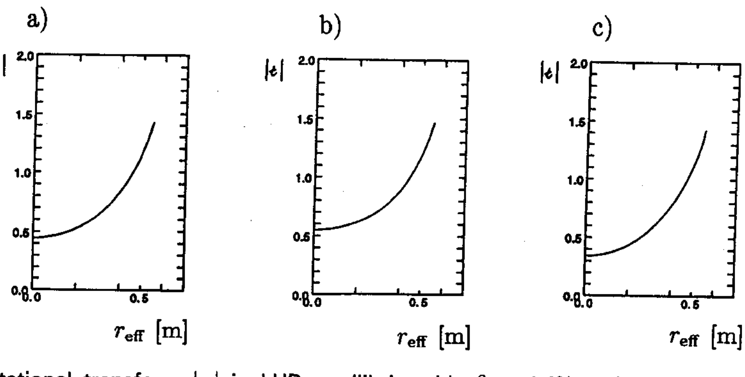

Fig. 4 Profiles of the rotational transform $|\iota|$ in LHD equilibria with β_0 = 1.4% and a) currentless, b) J_{net} = −50 [kA], c) J_{net} = +50 [kA].

The relaxation of pressure is expected to be so slow as compared to other relaxation processes. To speed up the relaxation, we solve the artificial sound wave equation under a fixed magnetic field until $\boldsymbol{B}\cdot\nabla p=0$ is satisfied.

$$\frac{\partial p}{\partial t}=\boldsymbol{B}\cdot\nabla v_s, \tag{1}$$

$$\frac{\partial v_s}{\partial t}=\boldsymbol{B}\cdot\nabla p \qquad (\boldsymbol{B}\text{ fixed}), \tag{2}$$

where v_s is the artificial sound wave velocity. The second step is the relaxation process of magnetic field under a fixed pressure frofile.

$$\rho_m\frac{\partial \boldsymbol{v}}{\partial t}=-\nabla p+\boldsymbol{j}\times\boldsymbol{B}+\nu\nabla^2\boldsymbol{v}, \tag{3}$$

$$\frac{\partial \boldsymbol{B}}{\partial t}=-\nabla\times\boldsymbol{E}=\nabla\times(\boldsymbol{v}\times\boldsymbol{B}-\eta\boldsymbol{j}), \tag{4}$$

$$\mu_0\boldsymbol{j}=\nabla\times\boldsymbol{B} \qquad (p\text{ fixed}). \tag{5}$$

The Equation (3) gives the equilibrium condition $\boldsymbol{j}\times\boldsymbol{B}=\nabla p$, when both the time variation of momentum (the left-hand-side) and the viscosity term are zero. In the steady state, Eq.(4) is reduced to the Ohm's law;

$$\boldsymbol{u}\times\boldsymbol{B}-\eta\boldsymbol{j}=-\boldsymbol{E}=\nabla\phi, \tag{6}$$

where ϕ is a scalar potential. If the scalar potential is a single-valued function, we have

$$\langle\boldsymbol{E}\cdot\boldsymbol{B}\rangle=-\langle\nabla\phi\cdot\boldsymbol{B}\rangle=0=\eta\langle\boldsymbol{j}\cdot\boldsymbol{B}\rangle, \tag{7}$$

where $\langle\ \rangle$ means the flux surface average. Thus, we obtain stellarator equilibria **without** a net current.

Next, we consider stellarator equilibria with a net current. The scalar potential ϕ is a multi-valued function, and we have

$$\eta\langle\boldsymbol{j}\cdot\boldsymbol{B}\rangle=-\langle\nabla\phi\cdot\boldsymbol{B}\rangle\neq 0. \tag{8}$$

To satisfy $\langle\boldsymbol{E}\cdot\boldsymbol{B}\rangle=0$, we should revise Eq.(4) as follows.

$$\begin{aligned}\frac{\partial \boldsymbol{B}}{\partial t}&=-\nabla\times\boldsymbol{E}\\&=\nabla\times\left(\boldsymbol{v}\times\boldsymbol{B}-\eta\left\{\boldsymbol{j}-\boldsymbol{B}\frac{\langle\boldsymbol{j}\cdot\boldsymbol{B}\rangle_{\mathrm{net}}}{\langle B^2\rangle}\right\}\right),\end{aligned} \tag{9}$$

where

$$\begin{aligned}\langle\boldsymbol{j}\cdot\boldsymbol{B}\rangle_{\mathrm{net}}&=\langle\boldsymbol{j}\cdot\boldsymbol{B}\rangle_{\mathrm{Ohmic}}\\&\quad+\langle\boldsymbol{j}\cdot\boldsymbol{B}\rangle_{\mathrm{bootstrap}}+\langle\boldsymbol{j}\cdot\boldsymbol{B}\rangle_{\mathrm{Ohkawa}}.\end{aligned} \tag{10}$$

This equation means any current except the Ohmic, the bootstrap, and the Ohkawa currents is decreased by resistivity. Thus, in the steady state, we can obtain stellarator equilibria **with** a net current;

$$\langle\boldsymbol{E}\cdot\boldsymbol{B}\rangle=0=\eta\{\langle\boldsymbol{j}\cdot\boldsymbol{B}\rangle-\langle\boldsymbol{j}\cdot\boldsymbol{B}\rangle_{\mathrm{net}}\}. \tag{11}$$

3. Conclusion

We apply our new scheme to LHD equilibria and solve the modified relaxation equations which we proposed above. To check the new scheme, we use the following artificial current. We assume that the net current is proportional to the pressure profile;

$$\langle\boldsymbol{j}\cdot\boldsymbol{B}\rangle_{\mathrm{net}}/\langle B^2\rangle\propto p, \tag{12}$$

where the pressure p is calculated as $p\propto(1-\psi/\psi_{\mathrm{edge}})^{2.5}$ for the case of currentless. And we choose values of the total net current J_{net} as follows.

$$J_{\mathrm{net}}=\int \mathrm{d}\boldsymbol{S}_{\mathrm{T}}\cdot\boldsymbol{j}_{\mathrm{net}}=\pm 50\ [\mathrm{kA}]. \tag{13}$$

Directions of the toroidal and poloidal magnetic field are shown in Fig.1, where B_{T} is the toroidal field ($B_{\mathrm{T}}=4$[T] at the magnetic axis) and $\boldsymbol{B}_{\mathrm{P}}$ is the poloidal field. If we add −50 [kA] of the net current into equilibria, the poloidal magnetic field is increased by the net current. It means that the rotational transform ι is increased. While, if we add +50 [kA] of the net current, ι is decreased. As shown in Figs. 2–4, we have results of the modified HINT computation. In Fig.4, we can see that ι is changed by adding the net current. Error of force balance R_f in calculations is less than 10^{-5}, thus the new computational method is correctly working, where R_{f} is defined as

$$R_{\mathrm{f}}=\frac{\int \mathrm{d}\tau\,(\nabla p-\boldsymbol{j}\times\boldsymbol{B})^2}{\int \mathrm{d}\tau\,\{(\nabla p)^2+(\boldsymbol{j}\times\boldsymbol{B})^2\}}. \tag{14}$$

We are planning to calculate equilibria with the bootstrap current and the Ohkawa current.

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