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Magnetic Islands and Drift Surface Resonances in Helias Configurations

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Abstract

This work is devoted to the study of magnetic islands and resonances of drift surfaces which can take place in the magnetic field of a Helias configuration and which affect the particle confinement properties. It is shown that due to external magnetic field perturbation the large size islands, the overlapping of resonances and stochasticity are possible in the magnetic configuration with the $\beta(0)=0.03$.

Keywords:

HELIAS magnetic configuration, magnetic perturbation, resonance, magnetic islands, drift surface islands, resonance overlapping, stochasticity

1. Introduction

The stochasticity of particle motion is one of the most interesting and important aspects of plasma confinement in the toroidal fusion devices [1]. Our goal here is to consider consistently the resonance behaviour of the magnetic force lines in the vacuum configuration and in the configuration modified by the plasma pressure (finite beta case). Our main attention is focussed to the high energy particles, particularly alpha-particles with the initial energy of 3.5 MeV. The isolated resonances in the magnetic surfaces, the overlapping of the resonances and the possible stochastic processes in the particle motion under the effect of magnetic perturbations in Helias magnetic configuration [2] is the subject of our study.

Perturbations considered in this work do not violate the drift approximation. The attention here is focussed to perturbations which have the same periodicity along the torus as the magnetic field (period number $M=5$) or multiples of it.

2. Basic Equations

We use the guiding center equations in canonical variables [3,4] which allow us to consider the effect of the electro-magnetic field on the particle motion in the magnetic configuration both in vacuum and in finite plasma pressure cases. We use the toroidal magnetic flux ψ as radial variable, ϑ and ζ as angular variables along the minor and major circumferences of torus, respectively, P_ϑ and P_ζ as momenta of particle motion, and $\rho_\parallel$ as the parallel gyroradius. All the quantities with a dimension of an energy are normalized to $m\omega_o^2 R_o$, where ω_o is the gyrofrequency on axis; all distances and lengths are normalized to the major radius of the magnetic axis R_o, the magnetic field B is normalized to its value on axis. The magnetic field perturbation is described in the following form $\delta \boldsymbol{B} = \nabla \times (\alpha \boldsymbol{B})$. Φ is the electric potential. The variable $\rho_\parallel$ is connected with parallel canonical momentum ρ_c by $\rho_c = \rho_\parallel + \alpha$. The equations of motion are the following

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$$\dot{\psi} = \left(g\dot{P}_{\vartheta} - I\dot{P}_{\zeta} \right) \frac{1}{\gamma}$$

$$\dot{\vartheta} = \left[(\mu + \rho_{\parallel}^2 B) \frac{\partial B}{\partial \psi} g + \rho_{\parallel} B^2 (\iota - \rho_c g') - \rho_{\parallel} B^2 \frac{\partial \alpha}{\partial \psi} g + g \frac{\partial \Phi}{\partial \psi} \right] \frac{1}{\gamma}$$

$$\dot{\zeta} = \left[-(\mu + \rho_{\parallel}^2 B) \frac{\partial B}{\partial \psi} I + \rho_{\parallel} B^2 (1 + \rho_c I') + \rho_{\parallel} B^2 \frac{\partial \alpha}{\partial \psi} I - I \frac{\partial \Phi}{\partial \psi} \right] \frac{1}{\gamma}$$

$$\dot{\rho}_{\parallel} = \left[(1 + \rho_c I')\dot{P}_{\zeta} + (\iota - \rho_c g')\dot{P}_{\vartheta} \right] \frac{1}{\gamma} - \left(\frac{\partial \alpha}{\partial \psi} \dot{\psi} + \frac{\partial \alpha}{\partial \vartheta} \dot{\vartheta} + \frac{\partial \alpha}{\partial \zeta} \dot{\zeta} + \frac{\partial \alpha}{\partial t} \right)$$

$$\dot{P}_{\vartheta} = -(\mu + \rho_{\parallel}^2 B) \frac{\partial B}{\partial \vartheta} + \rho_{\parallel} B^2 \frac{\partial \alpha}{\partial \vartheta} - \frac{\partial \Phi}{\partial \vartheta}$$

$$\dot{P}_{\zeta} = -(\mu + \rho_{\parallel}^2 B) \frac{\partial B}{\partial \zeta} + \rho_{\parallel} B^2 \frac{\partial \alpha}{\partial \zeta} - \frac{\partial \Phi}{\partial \zeta}$$

with $\gamma = \rho_o(gI' - Ig') + g + I\iota$.

The rotational transform ι is measured in units of 2π. The functions g, I, B^2 must by taken from the plasma equilibrium code.

3. Resonances in Helias Configuration

The particle motion and the resonance effects in the Helias configuration are already studied in number of papers [5-7]. We focus our attention to the effect of perturbing magnetic field $\delta \boldsymbol{B} = \nabla \times (\alpha \boldsymbol{B})$ on the particle trajectories with the aim to control the particle motion and their transport.

We considered the possible resonance phenomena in Helias configuration in the so-called vacuum approximation, *i.e.* for the magnetic force lines without plasma, and for the magnetic force lines in the finite beta case, *i.e.* configuration changed by the finite plasma pressure. The magnetic fields both for the vacuum and finite beta cases are taken from [8].

3.1 Analytical form of perturbation

The perturbation has the following form

$$\alpha_{mn} = \alpha_1 (r/a_{mn})^m [(a_{\rm pl} - r)/(a_{\rm pl} - a_{mn})]^p \sin(n\zeta - m\vartheta - \omega t + \delta),$$

where quantities a_{mn} and p create maximum with value α_1 at $r/a_{\rm pl} = a_{mn}/a_{\rm pl} = m/(m+p)$; $a_{\rm pl}$ is the plasma radius. The perturbation enters the system of equations via its spatial and time derivatives. Here and in the system of equations the radial variable r and the flux variable ψ are connected by the following expression: $r = R_o(2\psi)^{1/2}$

3.2 Possible resonances

For the Helias configuration (with the $M=5$ magnetic field periods along the torus) three main resonances with the wave numbers along the major n and minor m circumferences ($n=1$, $m=1$; $n=5$, $m=5$; $n=10$, $m=11$) were studied. Such kind of resonances may be regarded as a result of the perturbations induced from outside the plasma or induced from inside the plasma as MHD activity. In this work only time-independent magnetic perturbations ($\omega=0$) are considered.

3.3 m = n = 1 resonance at the edge of plasma

A perturbation with mode numbers $n=1$ and $m=1$ creates a large island on a magnetic surface where the rotational transform is $\iota=0.94$ at $r_0=180$ cm (Fig. 1). It occurs in the vacuum case (Fig. 1(a)) and in the finite beta case with $\beta(0)=0.03$ (Fig. 1(b)). The starting point of the external magnetic force line is at $r_0=180$ cm and for internal one in Fig. 1B is at $r_0=180$ cm. The perturbation has the wave numbers $n=1$, $m=1$, $\delta=0$ and the amplitude $\alpha_1=3\times10^{-4}$ (see Fig. 1(a)) and $\alpha_1=1\times10^{-4}$ (Fig. 1(b)). On Fig. 1(a) one can see the projection of field line on the poloidal plane and the Poincare plots. The ratio of the magnetic perturbation to the magnetic field on the circular axis of the torus is $\delta B/B_0=4.5\times10^{-5}$ for the magnetic surface with $r_0=120$ cm and $\delta B/B_0=7\times10^{-5}$ for one with $r_0=180$ cm.

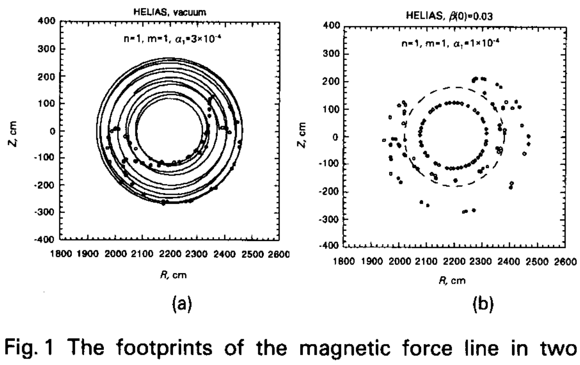


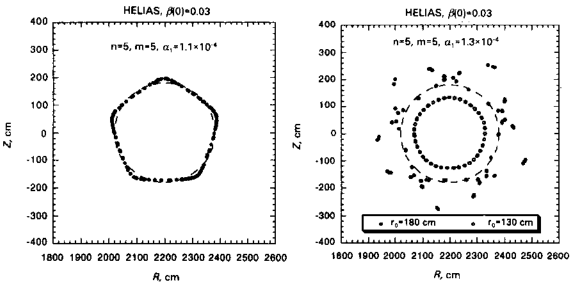
Fig. 1 The footprints of the magnetic force line in two meridional cross sections (at the beginning and at the half of the magnetic field period) for the vacuum case (a) and for the finite beta ($\beta(0)=0.03$) case (b).

3.4 m = n = 5 resonance at the edge of plasma

Under the perturbation with $n=5$, $m=5$ the family of 5 large islands was obtained (Fig. 2). In the finite beta case ($\beta(0)=0.03$) this structure arises above a threshold perturbation larger then $\delta B/B_0=2\times10^{-5}$. One can see the magnetic surface under the perturbation below (Fig. 2(a)), and above (Fig. 2(b)) this threshold. The starting point of the magnetic force line is at $r_0=180$ cm. The perturbation has the "wave" numbers $n=5$, $m=5$, $\delta=0$ and the amplitude $\alpha_1=1.1\times10^{-4}$(a) and $\alpha_1=1.3\times10^{-4}$(b).

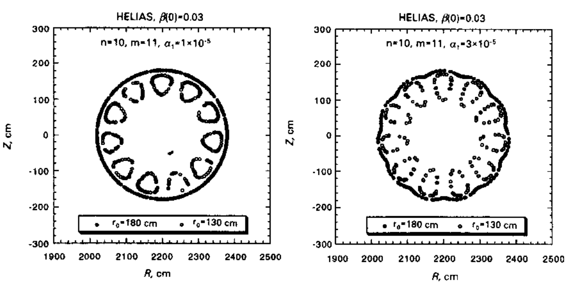
3.5 Island structure inside the plasma

On the magnetic surface at $r_0=130$ cm with the rotational transform $\iota=0.909$ an isolated resonance arises due to the single perturbation with $n=10$, $m=11$ (Fig. 3(a), 3(b)). The starting point of the external magnetic force line is at $r_0=180$ cm. The perturbation has the amplitudes $\alpha_1=1\times10^{-5}$ in Fig. 3(a) and $\alpha_1=3\times10^{-5}$ in Fig. 3(b).



(a) (b)

Fig. 2 The footprints of the magnetic force line in the finite beta ($\beta(0)=0.03$) case before (a) and after bifurcation (b).



(a) (b)

Fig. 3 The footprints of the magnetic force line in one meridional cross section for the finite beta ($\beta(0)=0.03$) case.

3.6 Overlapping of two resonances

The superposition of two resonances in finite beta case ($\beta(0)=0.03$) (Fig. 4, solid lines) with the maximum amplitudes $\delta B_{5,5}/B_0=4\times10^{-5}$ ($n=m=5$, $\alpha_1=1.3\times10^{-4}$) and $\delta B_{10,11}/B_0=1\times10^{-5}$ ($m=10$, $m=11$, $\alpha_1=1\times10^{-5}$) leads to distortion of 5 and 11 islands on the surfaces with the radius $r_0=180$ cm and with $r_0=130$ cm (Fig. 5(a)). Increasing of the amplitude for the 10/11 perturbation to $\alpha_1=3\times10^{-5}$ (Fig. 4, dashed line) leads to an overlapping of the 5 islands (Fig. 2(b)) and 11 islands (Fig. 3(b)) and formation of a stochastic layer (Fig. 5(b)).

3.7 Drift surface resonances

Drift rotational transform differs from magnetic rotational transform (Fig. 6(a)), therefore drift surface

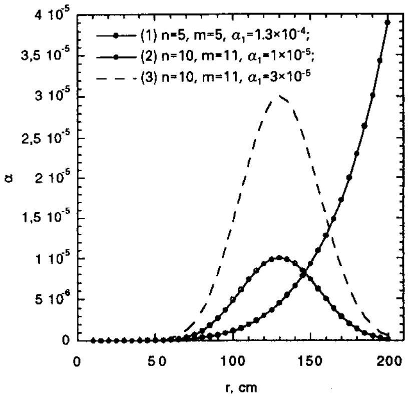
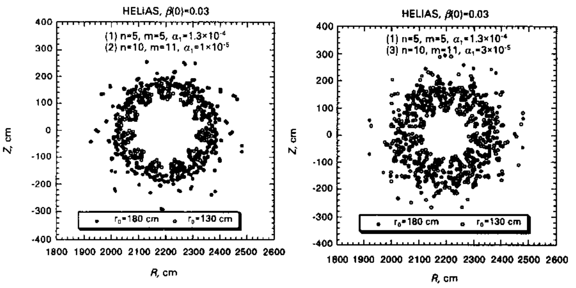
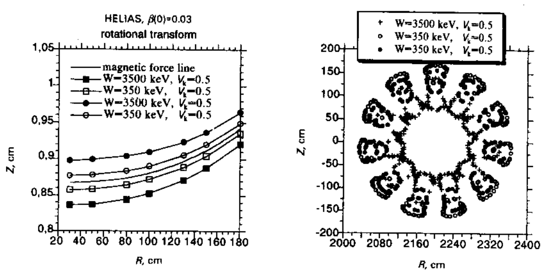


Fig. 4 The radial profiles of the magnetic perturbations (1) $n=5$, $m=5$, $\alpha_1=1.3\times10^{-4}$; (2) $n=10$, $m=11$, $\alpha_1=1\times10^{-5}$; (3) $n=10$, $m=11$, $\alpha_1=3\times10^{-5}$



(a) (b)

Fig. 5 The superposition of two perturbations. (a) The meridional cross-sections of the magnetic surfaces at the beginning of each 10th period of the magnetic field under the superposition of perturbations (1) and (2). (b) The same as (a) but under the superposition of perturbations (1) and (3).



(a) (b)

Fig. 6 Rotational transform (a) and drift surface resonances (b) of the particles with different values of energy W and velocity parameter $V_k = V_{\|}/V$.

resonances are considerably dependent on the value of energy W and velocity parameter $V_k = V_{\|}/V$. On Fig. 6(b) one can see the drift surface resonances due to the single perturbation with $n=10$, $m=11$ and $\alpha_1 = 1\times10^{-5}$; the trajectories of α-particles with $W=3{,}500$ keV and $V_k = -0.5$ have no island structure.

4. Summary

In Helias configuration both in vacuum and in the finite beta cases the magnetic islands were obtained for the surfaces at the periphery of plasma ($r_0 = 180$ cm) and inside the plasma ($r_0 = 180$ cm) and inside the plasma ($r_0 = 130$ cm) under the magnetic field perturbation with the "wave" numbers $n=1$, $m=1$; $n=5$, $m=5$; $n=10$, $m=11$ and the amplitudes $\delta B/B_0 = 1\times10^{-5} - 3\times10^{-4}$. The over lapping of 5/5 and 10/11 resonances and formation of the stochastic layer, caused by the superposition of two perturbations, were observed. It was shown that drift surface resonances are considerably dependent on the value of energy W and velocity parameter $V_k = V_{\|}/V$.

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