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Bounce-Averaged Velocity of Trapped Particle Drift in Toroidal Helical Systems

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Abstract

The normal to a magnetic surface component of the bounce-averaged velocity of trapped particle drift is investigated. We define this component as $v_{an}=\delta r_n/\tau_b$ where δr_n is an excursion of the trapped particle across the magnetic surface during one bounce time τ_b. A method of computing v_{an} is considered in case of a stellarator magnetic field given in real-space coordinates, without transforming the field to magnetic coordinates. We also consider an evaluation of neoclassical transport for the $1/\nu$ transport regime.

Keywords:
stellarator, neoclassical transport, trapped particle, second adiabatic invariant, real-space coordinates

1. Introduction

The quantity v_{an} is of great importance for confinement properties of non-axisymmetric toroidal devices at low collisionality (long mean free path regime). This quantity is directly related to the variation on a magnetic surface of the second adiabatic invariant $J_{\parallel}=\oint v_{\parallel}\mathrm{d}l$, which enters into the general equations of neoclassical transport theory for the contribution from the magnetic field asymmetry to the total particle and energy fluxes across the magnetic surfaces (see, *e.g.*, [1,2]). We present this variation of $J_{\parallel}$ in the coordinate system ψ, θ_0, φ, where a magnetic field line is the intersection of the ψ=const. and θ_0=const. surfaces, ψ=const. is the magnetic surface equation. From [3] we can write

$$\frac{\partial J_{\parallel}}{\partial \theta_0}=\frac{eB}{mc}\sqrt{\frac{g}{g_{33}}}\,\delta\psi\,, \qquad (1)$$

where g_{ik} is the metric tensor of the coordinate system ψ, θ_0, φ, g=Det(g_{ik}), $\delta\psi$ is the increment in ψ due to the excursion of a trapped particle across the magnetic surface during the bounce period. We shall compute $\delta\psi$ by the method of integration along the magnetic field lines for a given stellarator magnetic field in Cartesian coordinates. For this calculation guiding center drift equations are used in the limit of small gyroradius. After calculating $\delta\psi$ one can find $\partial J_{\parallel}/\partial\theta_0$ (Eq. (1)) as well as δr_n and v_{an}:

$$\delta r_n=\delta\psi/|\nabla\psi_i|\,, \qquad v_{an}=\delta r_n/\tau_b\,, \qquad (2)$$

where $\nabla\psi_i$ is the $\nabla\psi$ value at the initial point of integration.

2. Main Equations

For arbitrary steady-state magnetic and electric fields the guiding center drift equations can be written as (see, *e.g.*, [4])

$$\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t}=\frac{\boldsymbol{B}}{B}v_{\parallel}+\boldsymbol{v}_{\mathrm{d}}\,, \qquad (3)$$

$$\frac{\mathrm{d}v_{\parallel}}{\mathrm{d}t}=-\left(\frac{1}{2}J_{\perp}\nabla B-\frac{e}{m}\boldsymbol{E}\right)\cdot(\boldsymbol{h}+\rho_{\parallel}\nabla\times\boldsymbol{h})/D, \qquad (4)$$

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where $\boldsymbol{v}_d$ is the drift velocity perpendicular to a magnetic line, $\rho_\parallel = mcv_\parallel/eB$, $\boldsymbol{h} = \boldsymbol{B}/B$, $D = 1 + \rho_\parallel \boldsymbol{B}\cdot(\nabla\times\boldsymbol{B})/B^2$, $J_\perp/v_\perp^2/B$. We assume (as is common in calculating $J_\parallel$) that the Larmor radius of a particle and the electric field are rather small, and consequently, the trapped particle excursion across a magnetic field line during a bounce time is insignificant. Therefore, as a first approximation, $\boldsymbol{v}_d$ may be neglected in (3) and Eqs.(3) and (4) can be solved by the method of integration along a magnetic field line. In this case the quantity $\boldsymbol{v}_d$ [4] can be used for calculating the rate of change of , ψ, $\mathrm{d}\psi/\mathrm{d}t = \boldsymbol{v}_d\cdot\nabla\psi$. As a result, we arrive at the set of equations containing Eq. (4) and the equations

$$\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t} = \frac{\boldsymbol{B}}{B}v_\parallel \tag{5}$$

$$\frac{\mathrm{d}\psi}{\mathrm{d}t} = \boldsymbol{v}_d\cdot\nabla\psi. \tag{6}$$

For calculating $\nabla\psi$ we use the equations [5]

$$\frac{\mathrm{d}P}{\mathrm{d}t} = -\frac{v_\parallel}{B}\left(\frac{\partial B^1}{\partial\xi_1}P + \frac{\partial B^2}{\partial\xi_1}Q + \frac{\partial B^3}{\partial\xi_1}G\right),$$

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = -\frac{v_\parallel}{B}\left(\frac{\partial B^1}{\partial\xi_2}P + \frac{\partial B^2}{\partial\xi_2}Q + \frac{\partial B^3}{\partial\xi_2}G\right),$$

$$\frac{\mathrm{d}G}{\mathrm{d}t} = -\frac{v_\parallel}{B}\left(\frac{\partial B^1}{\partial\xi_3}P + \frac{\partial B^2}{\partial\xi_3}Q + \frac{\partial B^3}{\partial\xi_3}G\right),$$

where B^1, B^2 and B^3 are contravariant components of $\boldsymbol{B}$ in a normal curvilinear system of coordinates (ξ_1, ξ_2, ξ_3), $P = \partial\psi/\partial\xi_1$, $Q = \partial\psi/\partial\xi_2$, $G = \partial\psi/\partial\xi_3$, the multiplier $v_\parallel/B$ corresponds to the integration variable t.

Solving Eqs. (4)–(6), we shall find the ψ value for each moment of time t. For $t = \tau_b$ we obtain

$$\delta\psi = \psi(\tau_b) \qquad (\psi(0) = 0) \tag{7}$$

This quantity has to be used in Eqs. (1) and (2).

The initial conditions which are necessary for solving Eqs. (4)–(6) can be found in consequence of the preceding computation of the magnetic surface of interest. After this computation, within the part of this surface corresponding to one magnetic field period, we choose a number of magnetic field lines for which we wish to calculate $\delta\psi$. For each of these lines, the point corresponding to a minimum B value is taken as the initial point of integration in Eqs. (4)–(6). The initial values of $\nabla\psi$ at these points are determined using the method [5] concurrently with the magnetic surface computation. Solving Eqs. (4)–(6) under these initial conditions, one can obtain the $\delta\psi$ distribution as a function of the initial value of the particle longitudinal velocity $v_{\parallel i}$ and the position of the magnetic line segment under consideration (this position corresponds to a certain value of θ_0).

It is convenient to present the calculation results in a normalized form as a dependence of η on γ, where the parameters η and γ are related to v_{an} and $v_{\parallel i}$ by

$$\eta = v_{an}R/(v_{\perp 0}r_{L0}), \qquad \gamma = v_{\parallel i}/v_{\perp 0} \tag{8}$$

Here $v_{\perp 0} = \sqrt{J_\perp B_0}$, $r_{L0} = mcv_{\perp 0}/eB_0$, B_0 is the average longitudinal magnetic field, and R is the major radius of the torus. In particular, for the model of a standard stellarator [1] the amplitude of the η distribution over a magnetic surface equals $\eta_m = 0.5$.

3. Application to Quasi-Helical Symmetry

To exemplify the above technique, we present the results of the v_{an} calculations for the magnetic field which is appropriate for a zero-beta variant of the toroidal quasi-helically symmetric stellarator [6]. For quasi-helical symmetry the particle drift was formerly investigated only in magnetic coordinates (see, *e.g.*, [7]). We consider a real-space model of the configuration [6] with the magnetic field expressed through a superposition of toroidal harmonic functions containing the associated Legendre functions ($0 \le n \le 12$, $m = m_p M$, $|M| \le 12$, n and m are the poloidal and toroidal harmonic numbers, m_p is the number of periods along the torus, $m_p = 6$). The decomposition coefficients were found by minimizing the magnetic field component, which is normal to the given boundary magnetic surface [6].

Figure 1 shows the boundary surface [6] and two magnetic surfaces calculated for the magnetic field

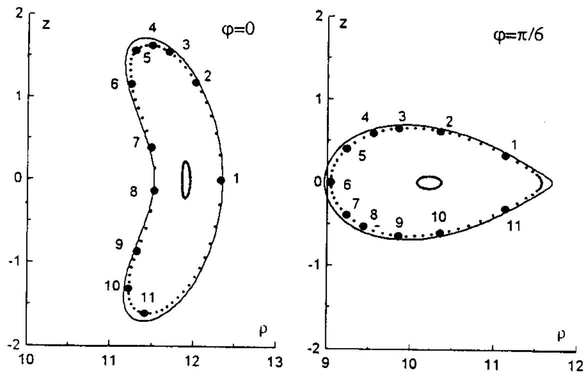

Fig. 1 Boundary surface [6] (solid line) and magnetic surfaces, corresponding to ρ_0= 12.33 (near boundary, t= 1.475) and ρ_0= 11.95 (inner, t= 1.41); the dimensions are given in the same relative units as in [6] (ρ_0 is the initial point of integration in the φ=0 cross-section for z= 0).

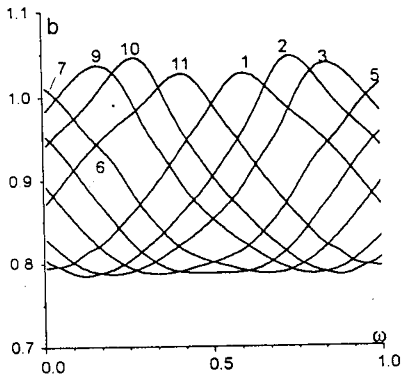

Fig. 2 Distribution of B over the length of one magnetic field period for magnetic lines, numbered in Fig. 1 for the surface ρ_0= 12.33 (b =B/B_0, ω= $m_p\varphi/2\pi$).

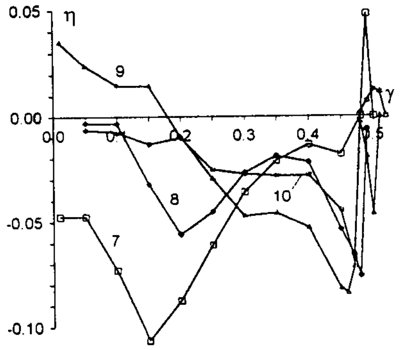

Fig. 3 Graphs of η for the magnetic surface ρ_0= 12.33 (η= $v_{an}R/v_{\perp 0}r_{L0}$, γ= $v_{\|i}/v_{\perp 0}$, the curves are numbered according to numbers of magnetic lines in Fig. 1); for field lines 2, 3, 4, 5 the η values are approximately equal in magnitude but opposite in sign to those in the graphs for field lines 10, 9, 8, 7; for field lines 1, 6, 11 $\eta\approx 0$ independent of γ; for $\gamma > 0.5$ $\eta \approx 0$; γ values of $\gamma\approx 0.5$ correspond to the transition from particles trapped within one magnetic field period to particles that are untrapped or trapped within two or more field periods.

model used (the magnetic lines for which v_{an} was calculated are indicated by numbers). In Fig. 2 the B/B_0 distributions are presented for the specified lines. Fig. 3 shows the results of v_{an} calculations for the near-boundary magnetic surface. On the average, for this surface the maximum value of η turns out to be of $\eta_m \approx 0.05$ in contrast to $\eta_m \approx 0.5$ for the standard stellarator [1]. For the inner magnetic surface (ρ_0=11.95) the maximum value of η has been found to be of $\eta_m \approx 0.25$. Note, that the results for the near-boundary magnetic surface are in reasonable agreement with the v_{an} estimates based on the results of [7] for quasi-helical symmetry.

4. Neoclassical Transport Evaluation

The results of the $\partial J_\| / \partial \theta_0$ calculations (1) can be directly used in evaluating the neoclassical fluxes associated with the magnetic field asymmetry for stellarators. Using Eqs. (2.1) and (2.2) of Ref. [8], where the variables ξ_1, ξ_2, ξ_3 correspond to the variables ψ, θ_0, φ in the present paper, one can obtain the equations for the particle and energy fluxes across a magnetic surface, sF_n and sF_T, for the $1/\nu$ transport regime (for a magnetic field period):

$$sF_n = -\frac{c^2\pi}{\nu_j e_j^2 m_j}\int\frac{d\theta_0}{B}\sqrt{\frac{g_{33}}{g}}\int_0^\infty \frac{dJ_\perp}{A_j v^2}\frac{\partial f_j^{(0)}}{\partial\psi} \times \int_{w_{\min}}^{w_{\max}}\frac{dw}{J_\|(w)}\left(\int_{w_{\min}}^{w} dw'\,\frac{\partial J_\|}{\partial\theta_0}\right)^2 \tag{9}$$

(sF_T differs from (9) by the factor $mJ_\perp B_0/2$ in the integrand). Here $f_j^{(0)}$ is the Maxwellian distribution for j-type particles, w is the particle energy, $w_{\min} = w_{\min}(J_\perp, \psi, \theta_0)$ and $w_{\max} = w_{\max}(J_\perp, \psi, \theta_0)$ are the minimum and maximum energies which are attainable by the trapped particles. The Coulomb frequency ν_j and the quantity A_j are the same as in [1] and [8]. The $J_\|$ value can be obtained by integrating the equation $dJ_\|(t)/dt = v_\|^2$ (concurrently with Eqs. (4), (5)). The variable θ_0 can be defined in such a way that $B\sqrt{g/g_{33}}=1$ and $d\theta_0=(\boldsymbol{B}\cdot\nabla\varphi)dl_p/|\nabla\varphi\times\nabla\psi|$. Here l_p is an arc length of the curve which is the intersection of the magnetic surface and the φ=const surface. With the use of these relationships the neoclassical transport fluxes, sF_n and sF_T, can be calculated if $\partial J_\|/\partial\theta_0$ and $J_\|$ are obtained as functions of $J_\perp$ and w (or $J_\perp$ and $v_{\|i}$).

It follows from (9) that the neoclassical transport coefficients are qualitatively proportional to a square of $\partial J_\|/\partial\theta_0$ (or to a square of η as it follows from (1), (2), (8)). The maximum value of η for the near-boundary magnetic surface (see Figs. 1, 2, 3) is approximately by a factor of ten less than η_m for the standard stellarator model. Thus for this surface one would expect that the contribution caused by the magnetic field asymmetry to the neoclassical transport coefficients would be a hundred times less than the corresponding contribution for the standard model of a stellarator [1] with equivalent sizes.

It is necessary to make the following remark. To exemplify the proposed method, we have considered 'good' closed magnetic surfaces. Now we shall briefly discuss the possibility of applying this method for magnetic islands formed at rational surfaces and for ergodic regions. The magnetic surface function ψ which enters into Eq. (6), can be defined as a single-valued function for the regions with closed magnetic surfaces as well as with island surfaces. Therefore, the technique proposed in this paper is completely applicable for calculations within magnetic islands formed at rational surfaces. However, for ergodic regions with stochastic behavior of magnetic field lines the magnetic surface function cannot be defined as a single-valued function. Therefore, in the strict sense the proposed method can not be applied directly within ergodic regions. For these regions the proposed method can be used only for approximate estimations in case of a low stochasticity for rather small intervals of the preceding field line integration (while the stochasticity does not strongly manifest itself).

5. Conclusions

If one has a method of computing the magnetic field and its spatial derivatives, then the technique proposed in this paper can be used for evaluating neoclassical losses across the magnetic surfaces. This technique allows one to calculate the trapped particle excursion across the magnetic surface during one bounce time, and also neoclassical transport fluxes across the magnetic surface, in the magnetic field given in real-space coordinates without transforming the field to magnetic coordinates. The present method can be used for the plasma confinement analysis in real stellarator magnetic configurations such as "Uragan", "Wendelstein", "Heliotron", "LHD" and others.

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