J. Plasma Fusion Res. SERIES, Vol.1 (1998) 472–475

Stellarator Transport Simulation Using δf Monte Carlo Algorithms

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(Received: 30 September 1997/Accepted: 12 January 1998)

Abstract

Several versions of Monte Carlo codes based on δf methods are under development for studying transport in nonaxisymmetric toroidal plasmas. This paper reports initial progress of the development. The δf simulations of neoclassical diffusion and bootstrap current in a stellarator are presented together with the results of benchmark using a tokamak geometry. Comparisons with conventional Monte Carlo methods are also made.

Keywords:

δf method, Monte Carlo simulation, stellarator, Hamiltonian guiding-center equation, neoclassical transport, bootstrap current

1. Introduction

The so-called δf methods are numerical techniques which become important in particle simulation of plasma instabilities and turbulence. Recently similar techniques have been applied successfully also to neoclassical transport simulations of tokamaks [1, 2]. Since stellarators have much complex geometry often hard to treat analytically, application of these new techniques to stellarators would be attractive. In this work, we examine applicability of δf algorithms to three-dimensional stellarator configurations. The longer-range goal of this work is to develop efficient computational tools capable of investigating transport related issues in nonaxisymmetric toroidal plasmas.

2. δf Schemes

Here, we briefly outline the δf schemes: The drift-kinetic equation for a guiding center distribution function (in the usual notations) is given by

$$\frac{\partial f}{\partial t} + (v_{\parallel}\hat{\boldsymbol{b}} + \boldsymbol{v}_{\mathrm{d}}) \cdot \nabla f + a_{\parallel}\frac{\partial f}{\partial v_{\parallel}} = C(f) \tag{1}$$

Collision operator C used at present is of Lorentz type.

$$C = \frac{\nu}{2}\frac{\partial}{\partial \lambda}(1-\lambda^2)\frac{\partial}{\partial \lambda} \tag{2}$$

with ν the collision frequency and with $\lambda(=v_{\parallel}/v)$ the pitch of particles.

In δf methods, distribution function is divided into a known equilibrium and a perturbation (*i.e.*, δf). Putting $f = f_0 + \delta f$, where f_0 is a local Maxwellian, the zeroth-order equation becomes

$$\frac{\partial f_0}{\partial t} + v_{\parallel}\hat{\boldsymbol{b}} \cdot \nabla f_0 + a_{\parallel}\frac{\partial f_0}{\partial v_{\parallel}} = C(f_0) \tag{3}$$

which is trivially satisfied by the Maxwellian f_0. The first-order equation can be written as

$$\frac{\partial \delta f}{\partial t} + (v_{\parallel}\hat{\boldsymbol{b}} + \sigma \boldsymbol{v}_{\mathrm{d}}) \cdot \nabla \delta f + a_{\parallel}\frac{\partial \delta f}{\partial v_{\parallel}} - C(\delta f) = \boldsymbol{v}_{\mathrm{d}} \cdot \boldsymbol{\kappa} f_0 \tag{4}$$

The RHS represents the source term that causes distortion of the distribution function due to the radial drift,

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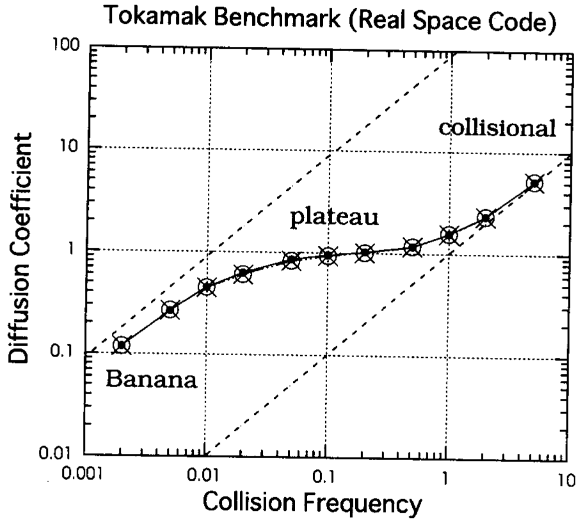

Fig. 1 D versus ν for the model tokamak ($\epsilon_t = 0.05$). Solid and open circles represent D from the linearized and nonlinear δf schemes; crosses represent D from traditional Monte Carlo method.

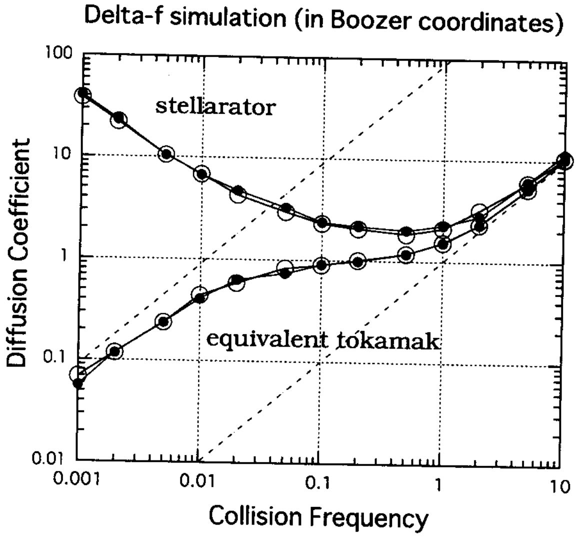

Fig. 2 D versus ν for the model stellarator and for the equivalent tokamak ($\epsilon_t = 0.05$). Solid and open circles represent D obtained from the linearized and the nonlinear δf schemes.

with κ the inverse of density gradient scale length. Two types of δf schemes, Dimits-Lee scheme ("partially linearized algorithm" [3]) and Parker-Lee scheme ("nonlinear characteristic method" [4]), have been implemented in our codes. In Eq.(4) (and in the following), $\sigma=0$ for the "linearized" scheme and $\sigma=1$ for the "nonlinear" scheme. By introducing the parameter σ, one can conveniently realize the two δf schemes in a single code.

Characteristics (trajectories) of j-th markers are solved in conjunction with equations for time-varying weights ω_j of markers. Time evolution of weights is described by

$$\frac{\mathrm{d}\omega_j}{\mathrm{d}t} = (1 - \sigma\omega_j)\boldsymbol{v}_\mathrm{d} \cdot \boldsymbol{\kappa} \tag{5}$$

As the initial condition we put $\omega_j=0$ because $f(t=0)=f_0$. A fourth order Runge-Kutta formula is used to integrate the system of equations. The codes are vectorized with index j of markers so as to exploit near maximum efficiency of vector processors.

The distribution function is expressed as

$$f=f_0 + \delta f = \sum_j (1 + \omega_j)\delta(X - X_j(t)) \tag{6}$$

where delta function, $\delta(X - X_j(t))$, is a five-dimensional guiding center Klimontovich density which evolves along the trajectories of markers. Taking appropriate moment of δf, rather than of total-f, one obtains desired quantities such as diffusion flux, bootstrap current j_b, and Pfirsch-Schlüter current $j_\mathrm{P.S}$. Since markers are sampled only form the perturbed part which directly contributes to the transport, one expects reduction of sampling noise or equivalently computational cost, which has been the major reason that limits the usefulness of Monte Carlo simulations.

3. Simulation Results

Two versions of δf codes, one in real-space coordinates and the other in magnetic coordinates, have been written. For the purpose of benchmark, simple tokamak geometry was first examined [5] using the real-space version. Figure 1 shows the diffusion coefficient D for electrons in the model tokamak obtained with the "linearized" and "nonlinear" δf schemes as well as with the traditional, test particle random walk procedures. Diffusion coefficients are normalized by the plateau value. In the collisional and the collisionless limits, results for D tend to theoretical linear dependencies on ν. The three different methods gave results that are in excellent agreement. In addition, Fig.1 agrees with previous work by Lotz-Nührenberg [6] where traditional Monte Carlo method is used to measure local diffusion coefficient in a similar tokamak model.

Having established benchmark using the real-space code, we applied the second, magnetic coordinates code to a model stellarator. In this version of the δf codes, markers are pushed using Hamiltonian guiding-

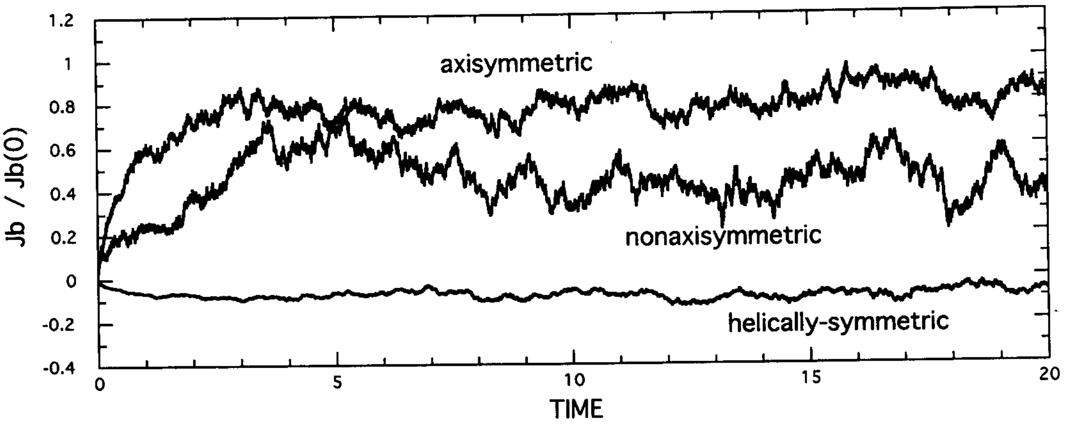

Fig. 3 Bootstrap current versus collision times in the low collisionality regime ($\nu^*=0.08$). j_b is normalized by axisymmetric collisionless value $j_b(0)$.

center equations which can be derived from drift Lagrangian [7]. A model nonaxisymmetric B-field spectrum for a stellarator used in this paper is:

$$\frac{B(\psi,\theta,\phi)}{B_0}=1-\epsilon_t\cos\theta+\epsilon_h\cos(\ell\theta-m\phi) \quad (7)$$

where (ψ, θ, ϕ) are Boozer's coordinates. Parameters specified are $\ell=2$, $m=19$; amplitudes ϵ_t, ϵ_h, and the rotational transform ι are chosen approximately similar to the values of Heliotron E.

Figure 2 shows the diffusion coefficient D for the model nonaxisymmetric B-field. Also shown is equivalent tokamak's D obtained simply by omitting the ϵ_h term from the B-field spectrum. Note that D for $\epsilon_h=0$ reproduced the earlier benchmarks with the real-space version (Fig. 1). For nonzero ϵ_h, D becomes larger than that of the axisymmetric counterpart in the plateau regime, and shows $1/\nu$ dependence in the low collisionality regime.

Figure 3 shows the time evolution of ion bootstrap current j_b for (i) axisymmetric ($\epsilon_h=0$), (ii) nonaxisymmetric, and (iii) helically symmetric ($\epsilon_t=0$) configurations. After transient periods for about 3–5 collision times, j_b signals show stationary phase. Although substantial fluctuation sometimes persists in the stationary phase (especially in nonaxisymmetric runs), time average of j_b over the stationary phase usually gives reliable answers. The low noise feature of the δf schemes may be understood form Fig. 4, where time averaged j_b's evaluated from total-f and δf are compared. The conventional total-f scheme yields noise level greater than unity; therefore, it allows no accurate evaluation of j_b.

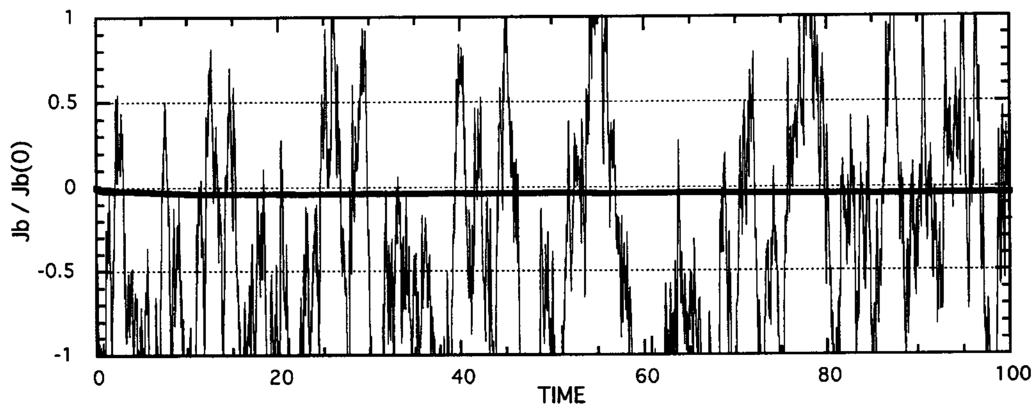

Fig. 4 Bootstrap current (time averaged) versus collision times. Highly oscillating curve was evaluated from total-f while heavy solid curve was evaluated from δf. Calculated for the nonaxisymmetric field in the collisional regime ($\nu^*=20.1$).

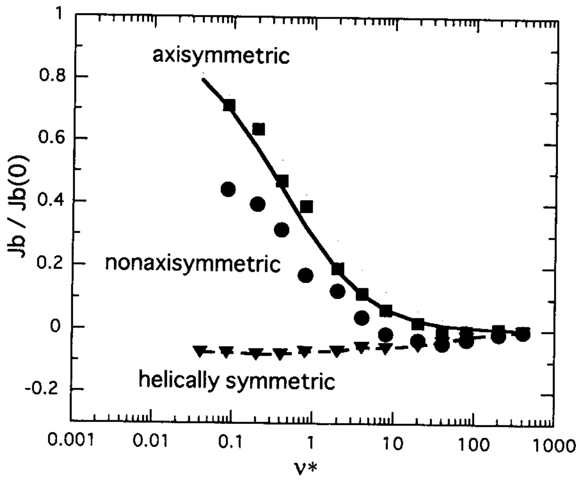

Fig. 5 Bootstrap current (time averaged) versus ν^*. Squares (axisymmetric field), circles (nonaxisymmetric field), and triangles (helically symmetric field). Solid line depicts tokamak scaling by Sasinowski and Boozer [2].

The δf scheme, by contrast, yields much less noise with the same number of markers (typically 1000) used in our simulations.

Figure 5 summarizes $\nu^*(\equiv\sqrt{2}\nu R_0/(\epsilon_t^{3/2}\iota v_{\rm th}))$ dependence of j_b. Axisymmetric results roughly fit to tokamak scaling proposed in Ref.[2]. We note the reversal of j_b in the helically symmetric runs. The bootstrap current j_b in nonaxisymmetric field lies in between those in the axisymmetric and helically symmetric ones. Bootstrap current is nearly canceled in the plateau regime and slightly opposite in more collisional regime. For the Heliotron E like parameters studied here, magnitude of collisionless j_b in the helically symmetric limit is smaller than that of equivalent tokamak by a factor, $(\epsilon_h/\epsilon_t)^{1/2}\cdot|\iota\ell/(\iota\ell-m)|\sim 0.07$, and is consistent with the analytical estimate.

Finally, Pfirsch-Schlüter currents $j_{\rm P.S.}$, calculation of which serves as useful diagnostics of the internal consistency of simulation codes, were evaluated using the δf methods; $j_{\rm P.S.}$ due to toroidicity, ϵ_t, and generalized $j_{\rm P.S.}$ due to helical ripple, ϵ_h, both agreed with theoretical estimates.

4. Summary

Prototypes of δf Monte Carlo simulation codes were developed for neoclassical transport studies of stellarators. The simulation results from real-space version as well as magnetic coordinates version of the δf codes were benchmarked with analytical and previous numerical results. Evaluated diffusion coefficient, bootstrap current, and Pfirsch-Schlüter current for a model stellarator showed theoretically expected transport properties. So far, the δf codes run for single species (ions or electrons). Future directions of this work will be exploration of wider class of stellarators and improvements in collision operator, in particular incorporation of momentum-conserving term that is required for multi-species transport simulations.

Acknowledgments

Much of the simulations presented here were performed using computers of National Institute of Fusion Science.

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