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Roles of Bumpy Field for Collisionless Particle Confinement in Helical Axis Heliotrons

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Abstract

Roles of bumpy field for collisionless particle confinement in helical axis heliotrons are investigated by the model magnetic field in the Boozer coordinates. The mod-B_{min} contours can be shifted in the major radius direction with the control of the bumpy field. The area of the closed mod-B_{min} contour is a good measure to evaluate global collisionless particle confinement when the radial variation of the bumpy field is relatively small. The negative ratio of bumpy/helicity gives the largest area of closed mod- B_{min} contours for the positive ratio of toroidicity/helicity. The radial variation of the bumpy field gives rise to toroidally localized mod-B_{min} contours, which is significantly effective to improve collisionless particle confinement.

Keywords:

helical axis heliotron, bumpy field, mod-B_{min} contour, collisionless particle confinement

1. Introduction

The new concept of heliotron configurations, helical axis heliotron[1], has been proposed. The concept arises to remove the limitation of planar axis heliotrons for realizing good particle confinement and good MHD stability simultaneously. The main requirements to realize this compatibility are the formation of vacuum magnetic well in the entire plasma region and the reduction of ripple transport by the control of magnetic field ripple. In order to realize high beta plasma confinement in helical axis heliotrons, it is also essential to improve particle confinement properties. Since the bumpy field typically appears in helical axis heliotrons, it is essential to understand its roles on plasma confinement properties. As a first step, roles of bumpy field on collisionless particle confinement are investigated. This paper is organized as follows. Section 2 describes characteristics of mod-B_{min} contours in helical axis heliotrons with emphasizing roles of the bumpy field. The toroidal localization of mod-B_{min} contours by the radially increasing bumpy field and its favorable effects on collisionless particle confinement are also explained. Brief summary and discussions are given in Section 3.

2. Mod-B_{min} Structure and Collisionless Particle Confinement

Studying particle confinement properties in magnetic configurations usually requires time consuming orbit following calculation. Therefore, it may not be suitable to use it in the design phase of the magnetic configurations, during which a lot of configurations are examined. However, if one considers only deeply trapped particles, it becomes possible to predict their trajectories by the structure of mod-B_{min} contours. The Boozer coordinates (ψ, θ_B, ζ_B) [2] are utilized in this paper, where psi is the normalized toroidal flux function and θ_B (ζ_B) the poloidal (toroidal) angle. Magnetic

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surface cross sections projected on the poloidal cross section are concentric circles with the center at $\psi=0$.

Since the bumpy field typically appears in helical axis heliotrons, the model magnetic field is considered to include the bumpy field based on the multiple helicity approach [3] as

$$B/B_0 = 1 - \epsilon_T - \epsilon_H \cos(\eta + \alpha), \tag{1}$$

where $\epsilon_T = \epsilon_t \cos\theta_B$, $\epsilon_H = \sqrt{\epsilon_h^2 + 2\epsilon_h\,\epsilon_b \cos\theta_B + \epsilon_b{}^2}$ and $\eta = L\theta_B - M\zeta_B$. The phase angle α is determined from $\cos\alpha = (\epsilon_h + \epsilon_b \cos\theta_B)/\epsilon_H$. The ϵ_b denotes the bumpy field component. The ϵ_t and ϵ_h are assumed to be proportional to (r/a) as typically valid in helical axis heliotrons[1]. For convenience, ϵ_t is written as $\epsilon_t = \epsilon_{ta}\,(r/a)$ and ϵ_h as $\epsilon_h = \epsilon_{ha}\,(r/a)$, where ϵ_{ta} (ϵ_{ha}) is the toroidicity (helicity) at the plasma edge, respectively. It is also assumed that ϵ_{ha} and ϵ_{ta} are positive by the definition of the model magnetic field (1). The bumpy field is assumed to be radially constant here for simplicity. The roles of radial variation of the bumpy field will be mentioned later.

It is noted that

$$\frac{B_{min}(x,y)}{B_0} = 1 - \epsilon_{ta}\frac{x}{a} - \sqrt{\epsilon_{ha}^2 \frac{x^2+y^2}{a^2} + 2\epsilon_{ha}\epsilon_b \frac{x}{a} + \epsilon_b^2} \tag{2}$$

for the model magnetic field (1), where $x = r\cos\theta_B$ and $y = r\sin\theta_B$. When $\epsilon_{ha} \neq \epsilon_{ta}$, deeply trapped particle trajectory following the value of B_{min} is obtained as

$$(x - X_{dtp})^2 + e^2 y^2 = \rho_{dtp}^2, \tag{3}$$

where

$$X_{dtp} \equiv -\frac{\epsilon_{ha}\epsilon_b + \epsilon_{ta}\left(1 - \dfrac{B_{min}}{B_0}\right)}{\epsilon_{ha}^2 - \epsilon_{ta}^2}\,a,$$

$$e^2 \equiv \frac{\epsilon_{ha}^2}{\epsilon_{ha}^2 - \epsilon_{ta}^2}, \tag{4}$$

$$\rho_{dtp}^2 \equiv \frac{\left[\left(1 - \dfrac{B_{min}}{B_0}\right)\epsilon_{ha} + \epsilon_b\epsilon_{ta}\right]^2}{(\epsilon_{ha}^2 - \epsilon_{ta}^2)^2}\,a^2.$$

The Eq.(3) denotes that mod-B_{min} contours are elliptic with the elongation e when $\epsilon_{ha} > \epsilon_{ta}$. In this paper, the cases of $\epsilon_{ha} > \epsilon_{ta}$ are considered as typically valid for helical axis heliotrons[1], where mod- B_{min} contours can close. The X_{dtp} depends on $\epsilon_{ha}\epsilon_b$, which implies that the center of mod-B_{min} contours can be shifted by the bumpy field control. Theore, helical axis heliotrons have a larger flexibilty to control the mod-B_{min} structure than conventional planar axis heliotrons, where the inward magnetic axis shift is typically necessary to improve trapped particle confinement[4].

In conventional heliotrons, the area of closed mod-B_{min} contours is one of the measures to evaluate collisionless particle confinement. Here, it is examined whether this is also valid in helical axis heliotrons or not. The fraction of the area of the outermost closed mod-B_{min} contours to that of the plasma boundary is obtained as:

$$f = \frac{\pi\rho_{dtp}\left(\dfrac{\rho_{dtp}}{e}\right)}{\pi a^2} = \frac{\left[\left(1 - \dfrac{B^*_{min}}{B_0}\right)\epsilon_{ha} + \epsilon_b\epsilon_{ta}\right]^2}{\epsilon_{ha}(\epsilon_{ha}^2 - \epsilon_{ta}^2)^{3/2}}, \tag{5}$$

where $\pi\rho_{dtp}(\rho_{dtp}/e)$ is the area of the outermost closed contour with the field strength B_{min} of B^*_{min}. Figure 1 shows the dependence of f on ϵ_b/ϵ_{ha} for several values of $\epsilon_{ta}/\epsilon_{ha}$. The maximum value of f for each $\epsilon_{ta}/\epsilon_{ha}$ case is obtained at a single point of negative ϵ_b/ϵ_{ha} with closed contour of $B^*_{min}/B_0 = 1 - \epsilon_{ha}$.

To investigate relations between the value of f and collisionless particle confinement efficiency, collisionless particle orbits are followed by solving the guiding center equations in the Boozer coordinates. For reference, three configurations A ($f=1$), B ($f=0.866$) and C ($f=0.661$) shown in Fig.1 are considered. It is noted that A is the helically symmetric configuration. Collisionless protons are followed with the average magnetic field strength of 1 T on the magnetic axis. They are initially launched from magnetic surfaces located at $r/a=0.25$, 0.5 and 0.75 with a uniform distribution in the pitch angle of the velocity space (15 points), in the poloidal (10 points) and in the toroidal (10 points) angles on each magnetic surface. The total number of followed particles is 4500. The proton temperature profile is

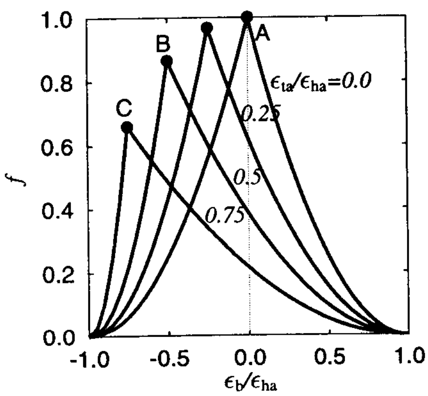

Fig. 1 The fraction of the area of the outermost closed mod-B_{min} contour to the plasma cross section, f, as a function of ϵ_b/ϵ_{ha} for several values of $\epsilon_{ta}/\epsilon_{ha}$.

assumed as: $T_i(r/a) = 1.0[1 - (r/a)^2]$ keV. A 1 keV proton has $\rho_i/a \sim 2.4\times10^{-2}$, where ρ_i is the Larmor radius, and it corresponds to 70 keV proton in the W7-X with a=0.53 m and B_0=2.5 T[5]. The particles are followed for 2 ms or until crossing the plasma boundary. It is noted that a 1 keV proton only with parallel velocity makes about 80 circuits of a torus during 2 ms. The radial electric field is not taken into account in this calculation. The loss rates are 0, 19.7 and 27.2% for ***A***, ***B*** and ***C***, respectively. The loss rate is increased as f is decreased. Theore, it is concluded that the area of closed mod-B_{min} contours can be utilized to measure global collisionless particle confinement also in helical axis heliotrons when the radial variation of the bumpy field is relatively small. It is useful to improve or optimize collisionless particle confinement by comparing several helical axis heliotron configurations.

The bumpy field has been assumed to be radially constant to make the model magnetic field analytically tractable. However, it is typical to have a radial variation in helical axis heliotrons for finite β equilibria[1]. Theore, roles of radial variation of the bumpy field are examined with fixing $\epsilon_{ha}=0.2$ and $\epsilon_{ta}/\epsilon_{ha}=0.5$. The bumpy field in the model magnetic field (1) is revised as:

$$\epsilon_b \rightarrow \epsilon_b\,[1 + \delta_b(r/a)^2], \qquad (6)$$

where ϵ_b is the bumpy field component on the magnetic axis, and this value is assumed to be the same as in (1).

Figures 2 show the change of mod-B_{min} contours as δ_b is increased. Figure 2(a) shows the outermost closed mod-B_{min} contour projected on the poloidal cross section and 2(b) its vertical ($(r/a)\sin\theta_B$) trajectory along the toroidal direction for δ_b=0.0, 0.5 and 2.0 cases. It is noted that δ_b=0.0 case corresponds to the configuration ***B*** mentioned above. In δ_b=0.0 case, the outermost closed mod-B_{min} contour connects $\zeta_B=0$ and $\zeta_B=(1/2)(2\pi/M)$. As δ_b is increased, the magnetic field strength around $\phi=(1/2)(2\pi/M)$ decreases due to the larger bumpy field around the plasma edge. This arises the significant localization of closed mod-B_{min} contours around $\zeta_B=(1/2)(2\pi/M)$ as seen in Fig. 2(b) (cf., δ_b=2.0).

The effects of the localization of closed mod-B_{min} contours on collisionless particle confinement are examined by comparing the loss rates for different values of δ_b. They are obtained by following 4500 protons under the same assumptions and conditions as above. The obtained loss rates are 19.7, 12.8 and 1.9% for δ_b=0.0, 0.5 and 2.0 cases, respectively. As shown in Fig. 2(a), the closed mod-B_{min} area is the largest for δ_b=0.0 among three configurations, and almost the same for δ_b=0.5 and 2.0 cases. However, the loss rate reduces significantly as δ_b is increased. This implies that the area of the outermost closed mod-B_{min} contours becomes an insufficient measure for global collisionless particle confinement when the radial variation of the bumpy field becomes remarkable. Theore, it is necessary to follow the particle orbit to compare several helical axis heliotron configurations in such a circumstance. It is also concluded that the toroidal localization of closed mod-B_{min} contours around the region where the bumpy ripple is predominant is effective to improve collisionless particle confinement in helical axis heliotrons.

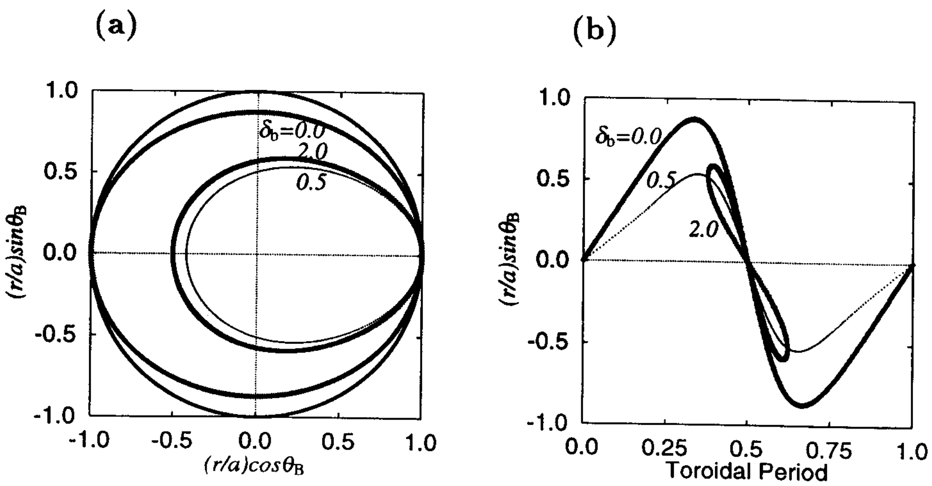

Fig. 2 (a) The outermost closed mod-B_{min} contour projected on the poloidal cross section and (b) its vertical trajectory along the toroidal direction, for δ=0, 0.5 and 2.0 cases.

3. Summary and Discussion

Roles of bumpy field on collisionless particle confinement in helical axis heliotrons have been considered based on the mod-B_{min} structure in the Boozer coordinates.

Mod-B_{min} contours projected on the poloidal cross section become elliptic shape depending on the ratio of toroidicity/helicity and those centers can be shifted with the bumpy field control. This implies that there is a possibility for B_{min} contours to coincide with magnetic surfaces without inward magnetic axis shift. This property is favorable for compatibility between good particle confinement and good MHD properties because the vacuum magnetic well can be formed without inward magnetic axis shift. The appropriate range of bumpy/helicity ratio to close mod-B_{min} contours is shifted into negative value as the ratio of toroidicity/helicity is increased. According to the orbit following calculation, collisionless particle loss rate is decreased as the area of closed mod-B_{min} contours is increased. Theore, the area of closed mod-B_{min} contours can be utilized as the measure to evaluate global collisionless particle confinement when the radial variation of the bumpy field is relatively small. This measure is useful to grasp the approach to improve collisionless particle confinement in helical axis heliotrons especially in the initial design phase.

Mod-B_{min} contours become toroidally localized when the bumpy field amplitude increases toward the plasma edge. This is due to the formation of local minimum of the magnetic field strength around the toroidal angle where the bumpy field contributes to weaken the field strength. The orbit following calculation shows significant improvement of the collisionless particle confinement in the presence of toroidally localized mod-B_{min} contours.

The MHD properties of particular configurations discussed in this paper have not been mentioned because the magnetic configurations are based on the model magnetic field. However, a helical axis heliotron configuration, which has a radially constant bumpy field at zero beta and radially increasing bumpy field at finite beta equilibria, has been studied in Ref.[1]. It shows that the configuration is Mercier stable up to the volume average beta value of 4.5%. The improvement of collisionless particle confinement for finite beta equilibria compared to the vacuum case has been obtained. It is partly successful in satisfying the compatibility between good particle confinement and good MHD properties. Further improvement of collisionless particle confinement should be pursued taking account of the compatibility with the MHD stability, in order to make a helical axis heliotron concept more attractive.

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