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Basic Study for Innovative Concepts of Stellarator Configurations

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Abstract

Basic roles of several essential plasma boundary modulations on magnetohydrodynamic (MHD) equilibria are investigated. The appropriate combination of principle helical modulations to eliminate bumpy field is explained to realize quasi-axisymmetric (QAS) and quasi-helically symmetric (QHS) configurations. The triangular modulation is effectively utilized to form the magnetic well. The bumpy modulations of plasma boundary is essential to reduce the toroidicity in the magnetic field, which can lead to QHS configurations. Based on these roles of plasma boundary modulations, the classification of QAS and QHS configurations is considered. The possibility of quasi-bumpy symmetric (QBS) configuration is also mentioned.

Keywords:
plasma boundary shape, MHD equilibria, helical modulation, triangular modulation, bumpy modulation, bumpy field, symmetric stellarator configurations (QAS, QHS and QBS)

1. Introduction

Stellarator configurations have a large flexibility to optimize confinement properties utilizing their three dimensionality. Magnetic configurations can be controlled by the plasma boundary modulations because MHD equilibria can be specified as a boundary value problem. Several concepts to improve confinement properties in stellarators have been proposed based on this approach. In quasi-helically symmetric (QHS) configurations, the essential point to improve reflected particle confinement is to eliminate the toroidicity in the magnetic field in the Boozer coordinates [1]. However, the bootstrap current is expected to flow even in the QHS configurations [2]. It is particularly dangerous in low shear stellarators, where the rotational transform has to be carefully adjusted to avoid low order rational surfaces. For the reduction of bootstrap current, the appropriate combination of helical, toroidal and bumpy field components is employed in the W7-X [3]. Another concept is the quasi-axisymmetric (QAS) configuration [4]. It has a similar magnetic field structure as in tokamaks in the Boozer coordinates.

The plasma boundary shape are different among QAS, QHS and W7-X based on different desired physical criteria. It is essential to grasp basic roles of plasma boundary modulations to understand approaches to these configurations and to consider more innovative future stellarator configurations.

In section 2, basic roles of several plasma boundary modulations on MHD equilibria are described. Section 3 is devoted to describe the realization and classification of symmetric stellarator configurations. Finally, summary will be given in section 4.

2. Magnetic Configuration Control with Plasma Boundary Modulations

In order to clarify basic roles of plasma boundary modulations, several effects on MHD equilibria have been studied by the fixed boundary version of VMEC

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[5]. The plasma boundary can be Fourier decomposed in the cylindrical coordinates (R, ϕ, Z) as

$$R(s, \theta_V, \zeta_V) = \sum_{mn} R_{mn}(s) \cos(m\theta_V - nM\zeta_V),$$

$$Z(s, \theta_V, \zeta_V) = \sum_{mn} Z_{mn}(s) \sin(m\theta_V - nM\zeta_V),$$

where s is the magnetic surface label and θ_V (ζ_V) is the poloidal (toroidal) angle and m (n) is the poloidal (toroidal) mode number in the VMEC coordinates. Here, M is the field period number. The left handed coordinate system is employed for boundary harmonics.

We choose the exact axisymmetric configuration described by R_{00} = 2.0 m, R_{10} = 0.4 m, Z_{00} = 0.0 m and Z_{10} = 0.6 m as the basic configuration. The ratio of the magnetic axis (R_{axis}) to the center of mass of magnetic surface cross section in the major radius direction ($R_{\mathrm{C.M.}}$) at four poloidal cross sections with the interval of 1/4 toroidal period is also considered as the measure of magnetic well. In this configuration, the magnetic well is evaluated to be about 5%. The magnetic well is defined by $(V'(0) - V'(\psi_T))/V'(0)$, where V is the volume enclosed by the magnetic surface corresponding to the toroidal flux function ψ_T and the prime denotes the derivative with respect to ψ_T.

It is noted that MHD equilibrium properties can also be varied by changing geometrical aspect ratio (A_p) and the field period number (M). In other words, MHD equilibrium properties are different in the configurations with the same plasma boundary shape with different values of A_p and/or M. In the rest of this section, $M = 2$ cases are considered to investigate roles of plasma boundary modulations. It is noted that the following descriptions are well valid for $M = 2$ case, and should be carefully considered to apply to the cases with other values of M.

2.1 Principle helical modulations: R_{11}, Z_{11}, $R_{1,-1}$ and $Z_{1,-1}$

Principle helical modulations are firstly considered. Especially, R_{11} and Z_{11} appear in several magnetic configurations.

When R_{11} and Z_{11} are applied on the basic configuration independently, the bumpy field component, B_{01}, is significantly enhanced due to the magnetic flux conservation, which enhances the magnetic field strength where the area of magnetic surface cross section becomes small. The area is the smallest at $\phi = 0$ for the negative R_{11} case; on the other hand, it is the case at $\phi = (1/2)(2\pi/M)$ for the positive Z_{11} case. Therefore, the sign of B_{01} is opposite each other between these cases.

The variation of area of magnetic surface cross sections along the toroidal direction greatly governs the amplitude of bumpy field as long as the magnetic axis excursion is not remarkable. Therefore, it is important to make the area of magnetic surface cross section at $\phi = 0$ and $\phi = (1/2)(2\pi/M)$ almost the same to eliminate B_{01} for QAS and QHS configurations. The condition to realize this requirement in the presence of principle helical modulations leads to the relation,

$$E_{\mathrm{RZ}} \times (R_{11} + R_{1,-1}) \sim -(Z_{11} + Z_{1,-1}), \quad (1)$$

where E_{RZ} is defined by Z_{10}/R_{10}. The magnetic configuration with $R_{11}/R_{10} = -0.5$, $Z_{11}/R_{10} = 0.75$ and $E_{\mathrm{RZ}} = 1.5$ ("Conf. 1") almost satisfies eq.(1), which has relatively small B_{01} compared to the cases where R_{11} and Z_{11} are changed independently.

2.2 Triangular modulations: R_{21} and Z_{21}

When $R_{21}/R_{10} = 0.25$ is applied on the basic configuration, magnetic surface cross sections are changed to triangular ones. The $R_{\mathrm{axis}}/R_{\mathrm{C.M.}}$ is significantly enhanced around $\phi \sim 0$, where the magnetic surface cross section is deformed to the outward pointing triangular cross section. The inward pointing triangular one appears around $\phi \sim (1/2)(2\pi/M)$, and $R_{\mathrm{axis}}/R_{\mathrm{C.M.}}$ is below 1 there. However, the average $R_{\mathrm{axis}}/R_{\mathrm{C.M.}}$ is still above 1, which implies the deep magnetic well about 6.3%. The triangular modulation Z_{21} has almost the same effects as in R_{21} case when $Z_{21}/R_{10} = 0.25$ is applied on the basic configuration. However, in this case, the outward pointing triangular cross section appears around $\phi \sim (1/2)(2\pi/M)$, which is relatively effective to increase $R_{\mathrm{axis}}/R_{\mathrm{C.M.}}$ to enhance magnetic well.

The magnetic configuration with both R_{21}/R_{10} = 0.25 and Z_{21}/R_{10} = 0.25 simultaneously on the "Conf.1" is mentioned ("Conf.2"). In this case, the magnetic surface cross section is indented at $\phi = 0$ due to R_{21} and becomes outward pointing triangular cross section around $\phi \sim (1/2)(2\pi/M)$ due to Z_{21}. This variation of magnetic surface cross sections is familiar in the W7-X, QAS and QHS configurations.

2.3 Principle bumpy modulations: R_{01} and Z_{01}

When $R_{01}/R_{10} = 0.5$ is applied on the basic configuration, magnetic axis shifts more outward around $\phi \sim (1/2)(2\pi/M)$, and also the average $R_{\mathrm{axis}}/R_{\mathrm{C.M.}}$ is greatly enhanced. This average value is significantly larger than that in the basic configuration and the magnetic well becomes deeper to about 8.5%. The $B_{10}/(a/R_{\mathrm{maj}})$ is reduced to 0.89.

When both radial and vertical bumpy modulations R_{01} and Z_{01} are applied on the "Conf.2" with $R_{01}/R_{10} = 0.5$ and $Z_{01}/R_{10} = -0.25$, the magnetic axis excursion becomes remarkable ("Conf.3"). It is noted that $B_{10}/(a/R_{maj})$ is reduced to 0.61 in this configuration. In other words, the decoupling of magnetic field structure from the real torus geometry is successfully achieved. If much larger bumpy modulations of plasma boundary are applied, magnetic configurations without B_{10} would be possible. This role can be effectively utilized to reduce B_{10} to realize QHS and QBS configurations.

To show the modulations of plasma boundary shape through the mentioned Fourier harmonics clearly, the magnetic surface cross sections at $\phi = 0$, $(1/4)(2\pi/M)$ and $(1/2)(2\pi/M)$ are shown in Figs.1 for (a) "Conf.1", (b) "Conf.2" and (c) "Conf.3".

3. Plasma Boundary Modulations for Realizing Symmetric Stellarator Configurations

Based on the roles of plasma boundary modulations described in section 2, the realization and classification of symmetric stellarator configurations are considered in this section.

The essential point to realize a QAS configuration (*ex.*, Fig.2(a)) is that the fine suppression of B_{01} with keeping helical fields sufficiently small. For the formation of vacuum magnetic well, efforts are made to realize the outward pointing triangular cross section around $\phi \sim (1/2)(2\pi/M)$ and tear-drop shaped cross section around $\phi \sim (1/4)(2\pi/M)$. It is also recognized that the configurations with smaller M and/or A_p are suitable to realize QAS configurations due to the relatively larger toroidicity compared to helical fields.

On the other hand, the reduction of B_{10} is indispensable factor to realize QHS configurations (*ex.*, Fig.2(b)). This role is effectively played by the bumpy modulations of plasma boundary shape. The large magnetic axis excursion is applied to reduce the toroidicity in the magnetic field to about 1% in $M = 4$ systems, which can be recognized by comparing Fig.2(b) with Fig.2(a). The requirements to form the magnetic well and to realize a pure magnetic field structure lead to the appropriate combination of triangular modulations. This application causes the indented cross section around $\phi \sim 0$ and the outward pointing triangular cross section around $\phi \sim (1/2)(2\pi/M)$. The relatively larger

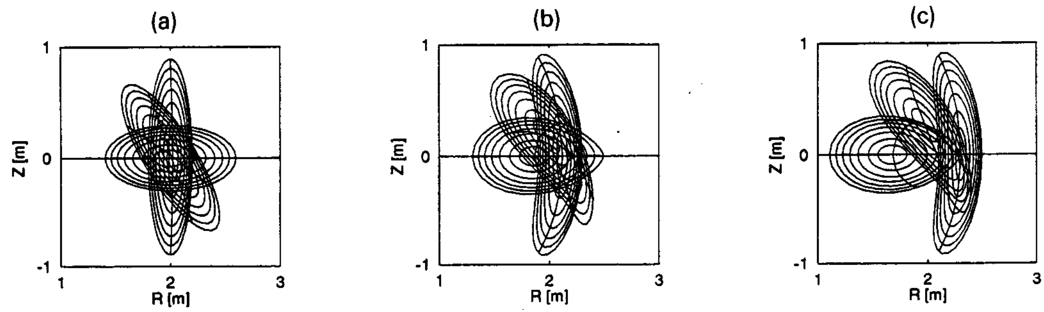

Fig. 1: Magnetic surface cross sections at $\phi = 0$, $(1/4)(2\pi/M)$ and $(1/2)(2\pi/M)$ for (a) "Conf.1", (b) "Conf.2" and (c) "Conf.3".

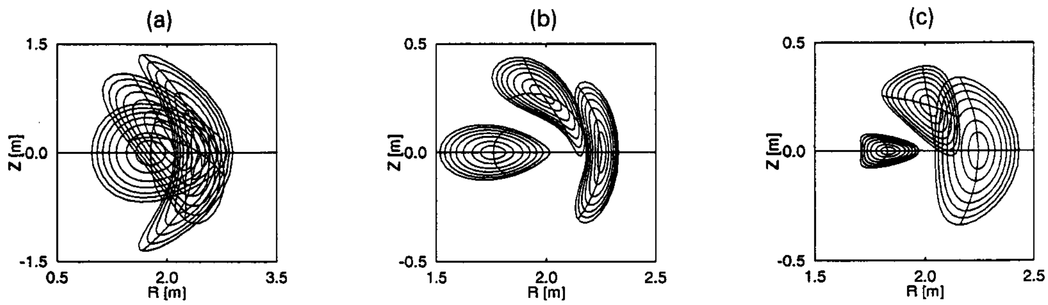

Fig. 2: Magnetic surface cross sections at $\phi = 0$, $(1/4)(2\pi/M)$ and $(1/2)(2\pi/M)$ for (a) QAS, (b) QHS and (c) QBS configurations.

helical field contributions in larger M cases allow to reduce the magnetic axis excursion to suppress B_{10}. The configurations with larger A_p are also suitable to realize QHS configurations due to the relatively smaller toroidicity. The difference of A_p between QAS and QHS is easily seen in Figs.2(a) and 2(b).

The possibility of a quasi-bumpy (or poloidally) symmetric (QBS) configuration is also considered. The breaking of eq.(1) gives the large B_{01} with either sign depending on the variation of area of magnetic surface cross sections. By utilizing this, the magnetic field with B_{01} dominantly is possible. Figure 2(c) shows the magnetic surface cross sections at $\phi = 0$, $(1/4)(2\pi/M)$ and $(1/2)(2\pi/M)$ for a QBS-like configuration. These cross sections are obtained from a QHS configuration (Fig.2(b)) just by changing the principle helical modulation R_{11}. It is noted that this configuration has a finite rotational transform, and therefore, it is distinguishable from the so-called bumpy torus. The detailed confinement properties are now under investigations.

4. Summary

The effects of plasma boundary modulations on MHD equilibria are investigated. Based on them, the realization and classification of symmetric stellarator configurations are demonstrated.

The bumpy field can be controlled by considering the variation of the area of magnetic surface cross sections in the toroidal direction. The satisfaction of Eq.(1) is effective to suppress the bumpy field, which is a necessary condition for QAS and QHS configurations. The magnetic surface cross sections with the bean shape at the beginning of the field period and the outward pointing triangular shape at the half period arise from the appropriate combination of helical and triangular modulations, which is effective to form the vacuum magnetic well. The QAS and QHS configurations with the vacuum magnetic well are therefore similar from the viewpoint of the shape of magnetic surface cross sections. The significant differences between them are (1) aspect ratio (A_p), (2) field period number (M) and (3) magnetic axis excursion (or the bumpy modulation of the plasma boundary shape). The smaller (larger) A_p and/or M easily lead to QAS (QHS) configurations due to its inherent larger toroidicity (helicity). The magnetic axis excursion is an essential point to reduce the toroidicity in the magnetic field, which leads to QHS and QBS configurations. The QBS configurations can be obtained rather easily by breaking Eq.(1) because the variation of the area of magnetic surface cross sections in the toroidal direction induces a relatively large bumpy field. The third symmetry, bumpy (or poloidal) symmetry, would lead to an innovative concept for the stellarator optimization.

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