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Physics Design of a High β Quasi-Axisymmetric Stellarator

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Abstract

We have found a new regime of quasi-axisymmetric stellarators which are ballooning stable at high β. Initial studies of the regime are reported. Our results suggest an approach that may be useful more generally for improving the ballooning properties of stellarators.

Keywords:
ballooning stability, quasi-symmetric stellarator, neoclassical transport, three-dimensional equilibrium, quasi-axisymmetry

1. Introduction

It was shown by Nuehrenberg, Lotz and Gori that it is possible to construct three-dimensional magnetic fields in which the nonaxisymmetry generates rotational transform, yet the field strength is nearly axisymmetric in Boozer coordinates (quasi-axisymmetry)[1]. We have been studying the feasibility of a high β, low aspect ratio, quasi-axisymmetric stellarator. A first step has been the analysis of the properties of the MHH2 stellarator[2]. This has been followed by the investigation of a new approach for constructing quasi-axisymmetric stellarators which are ballooning stable at high β. A previous paper reported many of our conclusions concerning the MHH2 device[3,4]. In this paper we will report on some additional studies of the MHH2 device and will report preliminary results of our investigation of the new approach for increasing the ballooning β limit.

2. MHH2 Transport

We have performed a numerical and analytic assessment of the transport to be expected in variants of quasi-axially symmetric stellarators, including a TFTR-sized ($R=3$ m, $B=2$ T) MHH2 [2] (here called MHH2-T), a second of the same size called NHH2-T, closely related to the MHH2 configuration, and, more recently, a PBX-sized ($R=1.5$ m, $B=1.4$ T) MHH2, here called MHH2-P. NHH2 is a mathematical construct, having fields and other parameters precisely the same as in MHH2, but with the sign of m reversed in each Fourier amplitude B_{mn} in magnetic field strength B. This makes NHH2 conform better with assumptions made in existing theories of stellarator transport, and so provides a bridge between MHH2 and existing theory.

In Fig. 1 is shown a comparison[3] of numerical results for MHH2-T and NHH2-T, and of these results with analytic theory. The 2 curves with symbols are numerical results, and the 4 heavier curves without symbols are analytic ones. The 3 lower analytic curves show each of the 3 transport 'branches' (viz., symmetric neoclassical (sym), 'banana drift' branch (bd), and 'superbanana' branch' (sb)), and the top curve is their sum. One notes that while being of comparable size, the

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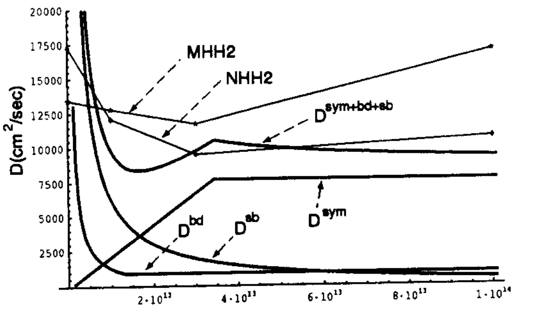

Fig. 1 Comparison of numerical results for MHH2-T and NHH2-T, and of these results with analytic theory.

scaling of diffusion coefficient D with central density n_{e0} is somewhat different for MHH2, and is not as close as NHH2 to the analytic prediction. This is expected, because NHH2 adheres better to the assumption in most analytic theory that the toroidal connection length L_t is large compared with the length L_r across a ripple well.

At a value of radial electric field E_r making the ion and electron particle fluxes equal, one finds for these parameters ion and electron energy confinement times $\tau_{Ei}=44$ msec, $\tau_{Ee}=68$ msec for NHH2-T. Using the empirical 'International Stellarator Scaling'[5] as an estimate of the expected anomalous transport, one finds $\tau_E^{ISS95}=21$ msec, comparable with the neoclassical values just given.

In going from the TFTR to the PBX-sized machine, one finds a considerable reduction in confinement times. At $n_{e0}=0.3\times10^{20}/\mathrm{m}^3$, the numerical diffusion coefficient falls by about a factor of 9, $D^{num}(MHH2-P)/D^{num}(MHH2-T)=9.05$, and thus, using $\tau_E\simeq a^2/(4D)$, $\tau_E^{num}(MHH2-P)/\tau_E^{num}(MHH2-T)=0.027$. Using the '$1/\nu$' superbanana expression $D_{-1}\propto 1/(BR)^2$ to analytically estimate the neoclassical transport, confinement in the smaller machine is degraded by an amount consistent with D^{num}: $D_{-1}(MHH2-P)/D_{-1}(MHH2-T)=8.16$. Finally, the ISS95-scaling predicts almost the same diminution in confinement time: $\tau_E^{ISS95}(MHH2-P)/\tau_E^{ISS95}(MHH2-T)=0.028$. Thus, one expects the smaller machine to again have the neoclassical confinement time about twice that predicted by ISS95 scaling, which should be about $\tau_E^{ISS95}(MHH2-P)=0.028$ (21 msec)$=0.59$ msec.

3. MHH2 Equilibrium Flux Surfaces

Stellarators are generally designed to have good vacuum flux surfaces. As pressure is added, the three-dimensional pressure-driven currents contain Fourier components which resonate with rational surfaces in the plasma, producing magnetic islands and destroying flux surfaces. Here we report new calculations with a broader pressure profile, $p=p(0)(1-s)$. The broader profile is found to give a higher equilibrium β limit.

Our calculations of equilibrium flux surfaces have been performed using the PIES code.[6] It calculates three-dimensional MHD equilibria without making any assumptions about the integrity of the flux surfaces. Although PIES has a free-boundary capability, the calculations described here are for a fixed boundary.

For the vacuum case, $\beta=0.0$, small islands are present at the $\iota=4/7$ and $\iota=8/13$ surfaces. The islands are sufficiently small that they can be expected to have no significant effect on plasma confinement. When β is raised to 4%, Fig. 2 shows that the 4/7 islands get large and the surfaces become quite rippled, indicating the presence of many small island chains. Nevertheless, there continues to be flux surfaces across much of the cross section. This is to be contrasted with the results for the more peaked pressure profile[4], where it was found that when β is raised to 2.1% surfaces are lost across a substantial fraction of the minor radius. The equilibrium β limit for the broader pressure profile is over twice that for the peaked profile.

The sensitivity of the equilibrium β limit to the pressure profile suggests that for appropriate pressure profiles the equilibrium β limit could be raised still further. The ballooning β limit is, however, considerably lower.[4] This is the critical issue, and we turn to it next.

4. Ballooning Stability

Ballooning stability calculations for MHH2 indicate that the β limit is below 1.5%. We regard this as the critical issue for MHH2. As we have discussed in

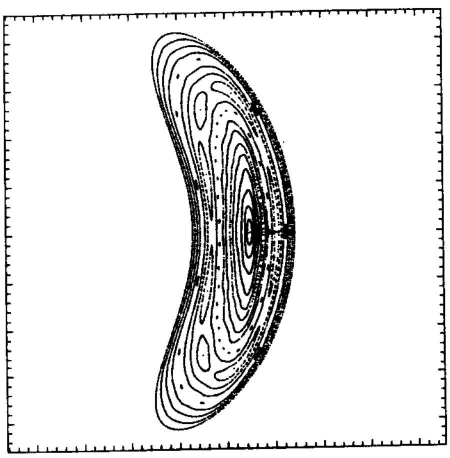

Fig. 2 Poincare plot calculated by the PIES code for the broad pressure profile, $\beta=4\%$.

this paper, the equilibrium β limit is considerably higher, and the confinement properties are adequate, at least for thermal particles. Ballooning β limits in stellarators generally tend to be low compared to those for tokamaks, typically on the order of 2%. The exception to this is W7-X, where ballooning codes predict a β limit of at least 5%.

In addition to the generic ballooning difficulties in stellarators, which appear to be exacerbated in quasi-symmetry, we need to contend with the effect of the aspect ratio. Studies of quasi-helical and linked mirror configurations have indicated that the MHD stability tends to be best at large aspect ratio.[7] We are interested in low aspect ratio quasi-axisymmetric devices.

Our approach to improving the ballooning stability properties is suggested by a closer look at the energy principle for ballooning instability,

$$\delta W = \frac{1}{2\mu}\int \mathrm{d}\boldsymbol{r}\,[k_\perp^2\,|\,\boldsymbol{b}\cdot\nabla X|^2 - \frac{2\mu}{B^2}(\boldsymbol{b}\times\boldsymbol{k}_\perp\cdot\nabla p)(\boldsymbol{b}\times\boldsymbol{k}_\perp\cdot\kappa)\,|X|^2].$$

Ballooning stability is determined by the sign of this expression. The first term is stabilizing, corresponding to the energy required for field line bending. The coefficient in front of this term contains the local shear, so that the stabilizing effect becomes small where the local shear is small. The second term gives the potentially destabilizing effect of the pressure gradient. Its sign is determined by the curvature, κ. In regions of good curvature, this term is also stabilizing. In regions of bad curvature, the term is destabilizing, and can potentially drive ballooning instabilities.

In a quasi-axisymmetric stellarator, the curvature is axisymmetric in Boozer coordinates. The isosurfaces of local shear, on the other hand, generally rotate helically as a function of the toroidal angle. Because of this, the low shear region generally intersects the bad curvature region. This is to be contrasted with the situation in tokamaks, where the shear and curvature isosurfaces are both axisymmetric. Second stable tokamak equilibria generally contain a zero shear isosurface, which does not cause any difficulties because it does not intersect the region of bad curvature.

These observations suggest an approach for obtaining ballooning stability at high β in a quasi-axisymmetric device. We would like to prevent the local shear isosurfaces from winding around helically. For this purpose, we introduce a strong $n=0$ component to the shaping which dominates the local shear. We use studies for TPX and Aries as a guide in choosing a stabilizing shape.

Figures 3—6 illustrate a test of this approach in which we have used an Aries equilibrium[8] to determine the $n=0$ Fourier components of the outer flux surface. We add nonaxisymmetric Fourier components that raise ι by about 30% at the edge, preserving quasi-axisymmetry as we do so. For this purpose, we use the optimization code developed by Nuehrenberg *et al.* We fix the $n=0$ Fourier components as well as the $n=1$, $m=1$ Fourier component, and we allow the optimization code to adjust the remaining Fourier coefficients. The shape of the outer flux surface is shown in Fig. 3. The aspect ratio, calculated using the usual stellarator convention, is 2.1. For our three-dimensional equilibrium and stability calculations, we retain the pressure and current profiles used in the Aries equilibrium. The ι and pressure profiles are shown in Figs. 4 and 5. About 90% of the current in the Aries equilibrium is bootstrap driven.

Figure 6 shows the ballooning eigenvalues calculated for β=6.5%. At this value of β, the equilibrium is ballooning unstable, with a narrow region of instability near the edge. If we lower the pressure, maintaining the same profile, we find that the configuration becomes ballooning stable everywhere when β is lowered to 5.5%. Alternatively, the radial dependence of the eigenvalues suggests that we can stabilize the mode by locally decreasing the pressure gradient in the narrow region of instability. We can argue that the ballooning mode itself will lead to such a flattening, or we could force the pressure gradient to be lower in that region by incompletely symmetrizing the ripple there. (Also there is nothing special about the Aries pressure profile. It was chosen to be optimal for Aries, and may not be optimal for the nonaxisymmetric case.) The solid line in Fig. 5 shows the modified pressure profile, corresponding to a β of 6.2%. The configuration is now stable

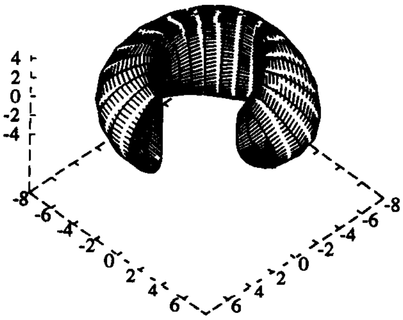

Fig. 3 Shape of the outer flux surface in our high β quasi-axisymmetric stellarator.

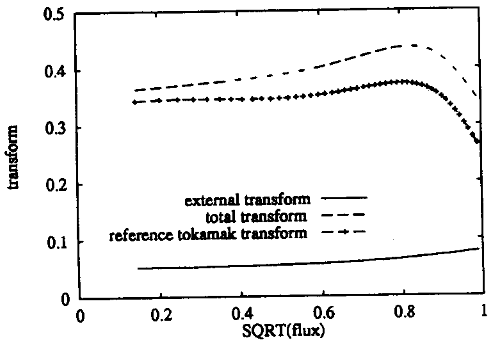

Fig. 4 Rotational transform profiles for Aries equilibrium and for quasi-axisymmetric stellarator equlibrium at β=6.5%.

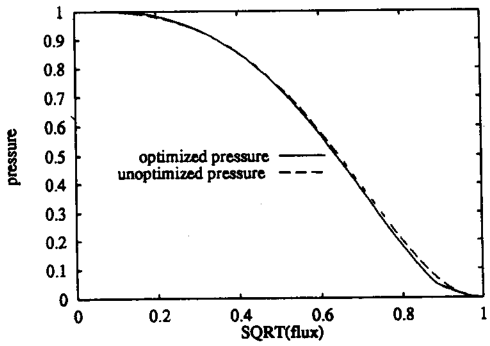

Fig. 5 Pressure profiles before and after flattening.

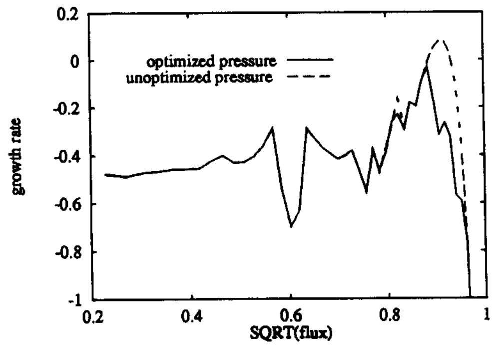

Fig. 6 Ballooning eigenvalues.

everywhere. We can expect that this method can be used to raise β still further, raising the overall level of the pressure gradient while flattening in a narrow region near the edge.

5. Summary and Discussion

An assessment of the transport, equilibrium and stability properties of MHH2 suggests that ballooning stability is the critical issue that must be addressed if quasi-axisymmetric configurations are to be attractive. Ballooning stability is an issue for stellarators in general, with W7-X the prominent exception. We have suggested a method for improving the ballooning stability of stellarators by aligning the isosurfaces of curvature and local shear. This can be accomplished for a quasi-axisymmetric stellarator by imposing a strong $n=0$ component on the shaping. Tokamak stability calculations can be used as a guide in designing a stabilizing shape. Our calculations show that the approach does work. The method opens up a new regime of low aspect ratio, high β quasi-axisymmetric configurations. There is much work that remains to be done in terms of exploring the range of possible configurations within this regime, and designing an optimum configuration.

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References

[1] J. Nuehrenberg, W. Lotz and S. Gori, *Theory of Fusion Plasmas*, eds., E. Sindoni, F. Troyon and J. Vaclavik, SIF, Bologna, 1994.

[2] P.R. Garabedian, Phys. Plasmas **3**, 2483 (1996).

[3] H.E. Mynick, Plasma Physics Reports **23**, 547 (1997).

[4] A. Reiman, L.P. Ku, D. Monticello, C. Nuehrenberg, and W. Cooper, Plasma Physics Reports **23** (1997).

[5] U. Stroth, Murakami, R.A. Dory, H. Yamada, F. Sano and T. Obiki, National Institute for Fusion Science Report, NIFS-375 (1995).

[6] A. Reiman and H. Greenside, Comput. Phys. Commun. **43**, 157 (1986).

[7] G. Grieger, C. Beidler, E. Harmeyer, J. Junker, J. Kisslinger, W. Lotz, P. Merkel, A. Montvai, J. Nuehrenberg, F. Rau, A. Schlueter, H. Wobig and R. Zille, *Plasma Physics and Controlled Nuclear Fusion Research, Proceedings of the 12th International Conference*, Nice, 1988 (IAEA, Vienna, 1989), Vol. **2**, p.369.

[8] S. Jardin, C. Kessel, C. Bathke, D. Ehst, T. Mau, F. Najmabadi and T. Petrie, Fusion Engineering and Design **38**, 27 (1997).