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# Density Control Problems in Large Stellarators with Neoclassical Particle Transport

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**Abstract**

With respect to the particle flux, the off-diagonal term in the neoclassical transport matrix becomes crucial in the stellarator long-mean-free-path (lmfp) regime. Central heating with peaked temperature profiles can make an active density profile control by central particle refuelling mandatory. The neoclassical particle confinement can exceed significantly the energy confinement at the outer radii. As a consequence, the required central refuelling may be larger than the neoclassical particle fluxes at outer radii leading to the loss of the global density control.

**Keywords:**
neoclassical transport, high-mirror advanced stellarators, W7-X, density profile control, central refuelling, particle transport barrier

## 1. Introduction

Within neoclassical theory, the particle and energy fluxes are linked together. The energy dependence of the $\nabla B$ drift in the stellarator *lmfp* regime, where the neoclasscial transport is dominated by the ripple trapped particles[1], results in signifcant non-diagonal terms in the transport matrix. Central power deposition and, as a consequence, peaked temperature profiles will lead to significant outward particle fluxes. To avoid very hollow density profiles or even pressure profiles (a positive pressure gradient will lead to strong MHD instability), central particle sources (*e.g.*, central refuelling by pellets) are needed. In larger stellarators, gas puffing as well as wall recycling will only affect the region close to the plasma edge. Outside of the power deposition region at lower temperature, the neoclassical particle fluxes may be in conflict with the central refuelling rate needed to control the inner density profile. The appearance of a particle transport barrier can lead to the loss of the global density control. In particular, the effect of different stellarator magnetic-field configurations on the expected density control problems is analyzed in this paper.

## 2. Mono-Energetic Transport Coefficients

In the simplest approximation, a classical stellarator can be described by the superposition of a toroidal and a helical magnetic field, with magnitude $B/B_0 = 1 - \epsilon\cos\theta - \epsilon_h\cos(m\theta - N_p\phi)$, where $\theta$ [$\phi$] is the poloidal [toroidal] angle, $\epsilon$ [$\epsilon_h$] is the poloidal [helical] modulation of $B$ ($\epsilon$ need not be equal to $\epsilon_t = r/R$, the inverse aspect ratio) and $m$ [$N_p$] is the helical field multipolarity [period number]. In the *lmfp* regime particles localized in the helical ripple are generally expected to provide the dominant contribution to the overall neoclassical transport. Within the traditional analytic theory (e.g., see Refs. [1,2]) the form this contribution takes is determined by the relative magnitudes of the effective collision frequency, $\nu_{\mathrm{eff}} = \nu/2\epsilon_h$, and the $E \times B$ precessional frequency, $\Omega_E = E_r/rB_0$. For the simple model field of a classical stellarator, the results may be summarized by the "mono-energetic" diffusion coefficients in the $1/\nu$, the $\sqrt{\nu}$ and the $\nu$

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collisionality regimes

$$D_{1/\nu} = \frac{4}{9\pi}\left(v_d \frac{\epsilon}{\epsilon_t}\right)^2 \frac{(2\epsilon_h)^{3/2}}{\nu} \tag{1}$$

$$D_{\sqrt{\nu}} = \frac{4\sqrt{2}}{9\pi}\left(v_d \frac{\epsilon}{\epsilon_t}\right)^2 \frac{\nu^{1/2}}{|\Omega_E|^{3/2}} \tag{2}$$

$$D_\nu = \left(\frac{v_d}{\Omega_E}\frac{\epsilon}{\epsilon_t}\right)^2 \frac{\nu}{2\mathscr{F}_{bl}} \tag{3}$$

which are predicted analytically for $\nu_{eff} \gg \Omega_E$, $\nu_{eff} \lesssim \Omega_E$ and $\nu_{eff} \ll \Omega_E$, respectively. In these expressions, $v_d = mv^2/2qB_0R$ is the $\nabla B$-drift velocity of mono-energetic particles (the averaged radial component of the $\nabla B$-drift is decreased by the factor $\epsilon/\epsilon_t$ corresponding to the reduction of the averaged toroidal curvature in an advanced stellarator), and $\mathscr{F}_{bl} = \sqrt{\epsilon + 2\epsilon_h} - \sqrt{2\epsilon_h}$.

For classical stellarator configurations, which can be reasonably well represented by the simple model field, the predictions of the bounce-averaged analytic theory in Eqs. 1 to 2 have been compared with the numerical solution of the mono-energetic drift kinetic equation by using the DKES code[3,4]; good agreement was found[5]. Additionally, the analytic approach has also been confirmed by Monte-Carlo simulations[2,6].

For a more complex magnetic field geometry, however, the traditional analytic approach cannot be used. The magnetic field strength on flux surfaces is represented by the $m$, $n$ Fourier modes, $\beta_{mn}(r)$, with respect to the poloidal, $\theta$, and toroidal angle, $\phi$, in magnetic (Boozer) co-ordinates. The coefficients of the simple model field for classical stellarators are given by $\epsilon \equiv -\beta_{10}$ and $\epsilon_h \equiv \beta_{m1}$; see above. For a given Fourier spectrum, $\beta_{mn}(r)$, the DKES code estimates the mono-energetic transport matrix as a function of $\nu/v$ and $E_r/v$. Neglecting the Ware pinch term, the particle and energy transport is determined completely by the mono-energetic transport coefficient.

DKES computations of this mono-energetic transport coefficient in the *lmfp* regime for quite different stellarator configurations (even in the case of a fairly broad $\beta_{mn}$ Fourier spectrum as is necessary to describe the W7-AS configurations) showed that the $1/\nu$, the $\sqrt{\nu}$ and the $\nu$ collisionality regimes are clearly present [5]. Furthermore, the different dependencies on the radial electric field for each regime could also be identified corresponding to the traditional analytic theory; see Eqs. (1) to (3). Consequently, these analytical transport coefficients were used to fit the DKES results for the different regimes, *i.e.*, numerical parameters only depending on radius have been obtained to fit the "analytical" transport coefficients to the DKES data. Additionally, the different regimes are smoothly connected (using $D^{-1} = D^{-1}_{1/\nu} + D^{-1}_{\sqrt{\nu}} + D^{-1}_{\nu}$).

## 3. Thermal Neoclassical Transport Matrix

For each particle species $\alpha$, the "thermal" transport matrix, $D^{\alpha}_{jk}$, is obtained by convolution with respect to the Maxwellian based on the mono-energetic transport coefficient. In the energy convolution both $\nu(v)/v$ and $E_r/v$ vary significantly leading to a mixing of the mono-energetic transport regimes. The neoclassical particle and energy flux densities, $\Gamma_\alpha$ and $Q_\alpha$, are given by

$$\Gamma_\alpha = -n_\alpha \cdot \left\{ D^{\alpha}_{11}\left(\frac{n'_\alpha}{n_\alpha} - \frac{q_\alpha E_r}{T_\alpha}\right) + D^{\alpha}_{12}\frac{T'_\alpha}{T_\alpha} \right\} \tag{4}$$

$$Q_\alpha = -n_\alpha T_\alpha \cdot \left\{ D^{\alpha}_{21}\left(\frac{n'_\alpha}{n_\alpha} - \frac{q_\alpha E_r}{T_\alpha}\right) + D^{\alpha}_{22}\frac{T'_\alpha}{T_\alpha} \right\} \tag{5}$$

with $\alpha$ = e, i (impurity fluxes are omitted here), and $q_\alpha$ being the particle charge. The so-called "convective term", $\frac{3}{2}T_\alpha\Gamma_\alpha$, is included in this neoclassical definition of $Q_\alpha$. For the total energy flux density, an additional $T_\alpha\Gamma_\alpha$ contribution is obtained from the term $\underline{\nabla} \cdot \underline{\underline{\Pi}} \cdot \underline{v}$ (with the approximation $\underline{\underline{\Pi}} \simeq nT\,\underline{\underline{1}}$ for the pressure tensor, and $\Gamma = n_r v_r$), and taken into account in the energy balance.

The radial electric field is determined by the roots of $Z_i\Gamma_i = \Gamma_e$; additional non-ambipolar particle fluxes are disregarded. As the $D^{\alpha}_{jk}$ depend on $E_r$, multiple roots of the ambipolarity condition may exist, *e.g.*, see Refs. [7,8]. The "ion root" with negative $E_r$ for $T_e \sim T_i$ is predicted for all collisionalities. As at higher densities only the "ion root" can exist, the following analysis considers only this case.

Under the assumption of very "extended" mono-energetic transport regimes, the mixing due to the energy convolution can be neglected. For these "pure" regimes, the thermal transport matrix can be easily estimated. With the normalisation $\delta_{jk} = D_{jk}/D_{11}$ and $\Delta(r)$ describing the additional radial dependence as, *e.g.*, obtained from numerical fits of the "analytical" coefficients given by Eqs. (1) to (3) to the DKES results, the thermal transport matrix is given for the different regimes by

$$D_{1/\nu} = \Delta_{1/\nu}\frac{1}{n}T^{7/2};$$
$$\delta_{12} = \frac{7}{2},\ \delta_{21} = 5,\ \delta_{22} = \frac{45}{2} \tag{6}$$

$$D_{\sqrt{\nu}} = \Delta_{\sqrt{\nu}} \sqrt{n}\, T^{5/4} \left(\frac{r}{E_r}\right)^{3/2};$$

$$\delta_{12} = \frac{5}{4},\ \delta_{21} = \frac{11}{4},\ \delta_{22} = \frac{99}{16} \tag{7}$$

$$D_\nu = \Delta_\nu n T^{1/2} \left(\frac{r}{E_r}\right)^2;$$

$$\delta_{12} = \frac{1}{2},\ \delta_{21} = 2,\ \delta_{22} = 3 \tag{8}$$

$$D_{\rm pl} = \Delta_{\rm pl} T^{3/2};$$

$$\delta_{12} = \frac{3}{2},\ \delta_{21} = 3,\ \delta_{22} = \frac{15}{2}. \tag{9}$$

Here, the plateau regime scaling is included, the other axisymmetric transport regimes can be disregarded. Note in this context, that the transport matrix in the $\nu$ regime scales differently from the tokamak "banana" regime ($\delta_{12} = -\frac{1}{2}$, $\delta_{21} = 1$, $\delta_{22} = \frac{1}{2}$). Furthermore, the "effective" helical ripple, $\langle\epsilon_h\rangle$, the generalisation of $\epsilon_h$ for non-classical stellarator configurations, is defined by $\Delta_{1/\nu} \propto \langle\epsilon_h\rangle^{3/2}$.

## 4. Link of Particle and Energy Fluxes

The central particle sources needed to control the density profile (*e.g.*, refuelling by pellets) are related to the power deposition by the neoclassical link of particle and energy fluxes. With the assumption of "pure" transport regimes, this link will be estimated in the following. For simplicity, $T \equiv T_e \simeq T_i$ is assumed (which is the case at higher density), and only the combined energy balance is considered. For this case, the collisional power transfer as well as the $\Gamma_a E_r$ terms in the electron and ion energy balances cancel. Furthermore, $n \equiv n_e = Z_i n_i$ = const., *i.e.*, external density profile control by central particle sources is assumed. Then, the ambipolarity condition ($Z_i\Gamma_i = \Gamma_e$) is given by

$$(D^e_{11} + Z_i D^i_{11}) \frac{eE_r}{T} = (D^i_{12} - D^e_{12}) \frac{T'}{T}. \tag{10}$$

In order to analyze the neoclassical link of the particle and energy fluxes, the radial electric field from the ambipolarity condition is discussed for two limits: $D^e_{jk} \ll D^i_{jk}$ and $D^e_{jk} \approx D^i_{jk}$. The first approach is valid at the outer radii with lower temperature independent of the specific magnetic configuration, while the second one becomes important only in the central region for configurations with a significant $1/\nu$ regime, *e.g.*, for the W7-X "high-mirror" configuration.

**The limit $D^e_{jk} \ll D^i_{jk}$:**

The ambipolar $E_r$ can be estimated from $\Gamma_i \simeq 0$, and the ambipolar particle flux density, $\Gamma$, is approximately given by the electron flux density with this $E_r$ taken into account,

$$\Gamma = \Gamma_e \simeq -nD^e_{11} \left(\frac{1}{Z_i}\delta^i_{12} + \delta^e_{12}\right)\frac{T'}{T}. \tag{11}$$

The total neoclassical energy flux density, $Q_{\rm tot} = Q_e + Q_i + 2\Gamma T$ (with the additional $\Gamma T$ from the pressure tensor term taken into account), is related to the particle flux density by

$$\Xi = \frac{Q_{\rm tot}}{\Gamma T} = 2 + \xi_e + \xi_i \frac{D^i_{11}}{D^e_{11}} \tag{12}$$

with $$\xi_e = \frac{\delta^e_{22} + \frac{1}{Z_i}\delta^i_{12}\,\delta^e_{21}}{\frac{1}{Z_i}\delta^i_{12} + \delta^e_{12}}$$

and $$\xi_i = \frac{\delta^i_{22} - \delta^i_{12}\,\delta^i_{21}}{\frac{1}{Z_i}\delta^i_{12} + \delta^e_{12}}$$

The electrons are assumed to be in the $1/\nu$ regime, and $Z_i = 1$. Then, $\xi_e = 6.25$ and $\xi_i = 0.5$ for the ion $\nu$ regime, $\xi_e = 6.05$ and $\xi_i = 0.58$ for the ion $\sqrt{\nu}$ regime, and $\xi_e = 6$ and $\xi_i = 0.6$ for the ion plateau regime. Consequently, these ratios turn out to be fairly equivalent for the different ion transport regimes. Most important is the ratio of the ion to the electron transport coefficient, $D^i_{11}/D^e_{11} < \sqrt{m_i/m_e}$, which determines the fraction of the ion energy flux. For the electrons at the beginning of the $1/\nu$ regime, the ion energy flux dominates, and $\Gamma T$ is much less than $Q_{\rm tot}$.

**The limit $D^e_{jk} \approx D^i_{jk}$:**

The electron and ion transport coefficients can be comparable in magnitude if the temperature is sufficiently high, and if the electron $1/\nu$ regime is significant. These conditions can be realized in the central region in configurations with a fairly large toroidal mirror term. Then, the ambipolar radial electric field in Eq. (10) becomes small in magnitude, and is implicitely given by $D^i_{12} \approx D^e_{12}$. Neglecting $E_r/T$ as a driving term for these conditions, the ambipolar particle flux density, $\Gamma$, is linked to the total neoclassical energy flux density

$$\Xi = \frac{Q_{\rm tot}}{\Gamma T} = 2 + \frac{\delta^e_{22}}{\delta^e_{12}} + \frac{\delta^i_{22}}{\delta^i_{12}}. \tag{13}$$

Since the ratios $\delta_{22}/\delta_{12}$ corresponding to the different transport regimes are rather similar ($1/\nu$: 6.43, $\nu$: 6,

$\sqrt{\nu}$: 4.95, and plateau regime: 5), the electron and ion energy fluxes are also comparable. For example, for 10 MW central heating at $T(0) = 5$ keV, a particle source of about $10^{21}$/s is needed.

## 5. Need for Central Refueling

The link of the neoclassical energy and particle fluxes in the bulk part of the plasma leads to a requirement for particle sources to control the central density profile. The assumed purely neoclassical particle flux density defines the particle source profile, $S_p(r) = r^{-1}\mathrm{d}(r\Gamma)/\mathrm{d}r$. In the combined energy balance, additional losses are taken into account, $P^*(r) = r^{-1}\mathrm{d}(rQ_{tot})/\mathrm{d}r$, with $P^* = P_h - P_l - r^{-1}\mathrm{d}(rQ_{an})/\mathrm{d}r$ where $P_h(r)$ is the heating power density profile, $P_l(r)$ describes radiative losses, and $Q_{an} = -n\chi_{an}T'$ is an additional energy flux density due to an "anomalous" heat diffusivity, $\chi_{an}$. Then, the particle source profile needed to control the density profile ($n$ = const.) is given by

$$S_p = \frac{P^*}{T\Xi} - \left(\frac{T'}{T} + \frac{\Xi'}{\Xi}\right)\Gamma. \qquad (14)$$

For conditions where $D^i_{jk} \approx D^e_{jk}$ holds, $\Xi' = 0$, see Eq. (13) Neglecting the additional $T'$ term the profile of the particle sources, $S_p(r)$, must be quite similar to the power deposition profile, $P_h(r)$, if the additional energy losses are negligible which is reasonable for peaked central heating. These estimates are independent of the specific magnetic configuration as long as the condition $D^e_{jk} \approx D^i_{jk}$ holds (which is, however, violated at outer $r$ with lower $T$).

The other limit, $D^e_{jk} \ll D^i_{jk}$, is rather similar, but the additional $r$ and $T$ dependence of the ratio $D^i_{11}/D^e_{11}$ in Eq. (12) must be analyzed. With $\Gamma_i \simeq 0$, $eE_r \approx 1.25\ T'$ is obtained from Eq. (10) for the ion $\sqrt{\nu}$ regime leading to $D^i_{11} \propto T^{5/4}\ (r/T')^{3/2}$ where an additional $r$ dependence in $\Delta_{\sqrt{\nu}}$ is neglected. $D^e_{11}$ in the $1/\nu$ regime reflects the radial dependence of the helical ripple. Corresponding to Eqs. (1) and (6), the electron transport coefficients scale with $T^{7/2}\langle\epsilon_h\rangle^{3/2}$ where $\langle\epsilon_h\rangle$ is the "effective" helical ripple. For high-mirror advanced stellarators such as W7-X, $\langle\epsilon_h\rangle$ is mainly determined by the toroidal mirror term, *i.e.*, $\langle\epsilon_h\rangle$ is roughly constant in the inner region leading to a dominant $1/\nu$ regime (*i.e.*, Eq. (13) is fulfiled). For configurations with only a small toroidal mirror term, $\langle\epsilon_h\rangle$ has the same dependence on $r$ as the leading helical Fourier mode: $\langle\epsilon_h\rangle \propto \beta_{11} \propto r$ for $m = 1$ configurations (*e.g.*, see Ref. [9] for the W-7X "low-mirror" configuration), and $\langle\epsilon_h\rangle \propto \beta_{21} \propto r^2$ for $m = 2$. Then,

$$\frac{D^i_{11}}{D^e_{11}} \propto \left(\frac{r}{T'\langle\epsilon_h\rangle}\right)^{\frac{3}{2}} T^{-\frac{9}{4}} \qquad (15)$$

is obtained. For configurations with $m \geq 1$, $\Xi \to \infty$ for $r \to 0$ leads to smaller $S_p$ close to the axis compared to a "high-mirror" configuration.

Consequently, for a peaked central power deposition profile a central particle refuelling is also needed in the $D^e_{jk} \ll D^i_{jk}$ case. The required particle source strength is comparable with that needed for $D^e_{jk} \approx D^i_{jk}$. Furthermore, the particle source profile needed to control the central densities turns out to be broader for $m \geq 1$ configurations (due to $\Xi' < 0$).

## 6. Neoclassical Particle Transport Barrier

The particle flux at outer radii (*i.e.*, outside of the power deposition region) must be at least as large as the central particle source. Otherwise the global density control may be lost, *i.e.*, the outer density increases. In this context, the outermost particle sources from recycling are disregarded; shielding is assumed to dominate at the outermost radii where the strong density gradient is established.

An outer particle transport barrier may be formally defined by $S_p \leq 0$. Additional energy losses or an enhanced energy flux tends to establish such a scenario. In principle, the temperature profile, which differs significantly for the magnetic configurations under investigation, must be estimated. In Eq. (14), however, the main effect of the magnetic configuration on the appearance of this crucial particle transport barrier is directly included in the $\Xi'$ term. With the $\sqrt{\nu}$ regime scaling of $D^i_{11}$ and the $1/\nu$ regime scaling of $D^e_{11}$, and neglecting the radial dependence of $T'$ (in Eq. (15)) leads to

$$\Xi' = -\xi_i \frac{D^i_{11}}{D^e_{11}} \left\{ \frac{9}{4}\frac{T'}{T} + \frac{3}{2}\left(\ln\frac{\langle\epsilon_h\rangle}{r}\right)' \right\}. \qquad (16)$$

Here, $\Xi' > 0$ amplifies the transport barrier problem, *i.e.*, $S_p$ is decreased. Since $\xi_i D^i_{11}/D^e_{11} > 0.5\ \Xi$ (the relative fraction of the ion energy flux; compare Eq. (12)), the $T'/T$ term in Eq. (14) is overcompensated by the $T'/T$ contribution from $\Xi'/\Xi$ in Eq. (16) resulting in a negative contribution to $S_p$. The other term ($\propto$(ln $(\langle\epsilon_h\rangle/r))'$) contains the magnetic configuration effect in $\langle\epsilon_h\rangle$ which can lead to a negative or a positive contribution to $S_p$: (ln $(\langle\epsilon_h\rangle/r))' = -1/r$ for a high-mirror configuration ($\langle\epsilon_h\rangle \simeq$ const.); $= 0$ for an $m = 1$ configuration without toroidal mirror ($\langle\epsilon_h\rangle \propto r$); and $= 1/r$ for an $m = 2$ configuration ($\langle\epsilon_h\rangle \propto r^2$). Within this simple analysis, an $m = 2$ or even an $m = 1$ configuration with only small toroidal mirror terms are less susceptible to loss of the global density control than an $m = 0$ configuration. In the "high-mirror" W7-X configuration, $\langle\epsilon_h\rangle$ even decreases with $r$; see Ref. [9]. This kind of "over-optimisation", *i.e.*, the neoclassical confinement is mainly improved at the outer radii, leads to the most dangerous scenario in this context.

Consequently, $S_p < 0$ outside of the power deposition region is in contradiction to the assumption of a stationary and flat density profile, and the formal "particle sink" needed is equivalent to an increase of the density ($\partial n/\partial t > 0$) at these outer radii. With increasing density, the temperature is reduced and, for example, the radiative losses are increased, leading to even more negative $S_p$. The $D^e_{11}$ coefficient related to the positive density gradient is too small to compensate the non-diagonal temperature gradient driven flux. In this way, the neoclassical particle transport barrier can lead to the loss of the global density control.

## 7. Conclusion

In large stellarators central particle refuelling will be mandatory. To avoid hollow density and pressure profiles or rapidly increasing density, the particle source strength must be nearly proportional to the heating power. Especially for the W7-X "high mirror" configuration, the particle and power deposition profiles must be quite similar. A particle transport barrier must be avoided, otherwise the global density control is lost. In W7-X, these effects can be studied by varying the toroidal mirror term. For example, lowering the mirror term slightly degrades the particle confinement at outer radii which may help to prevent the loss of the global density control. Especially for the most dangerous W7-X "high-mirror" scenario, a "density limit" for stationary operation is predicted. At higher densities, the global density control is lost. This "density limit" increases with heating power, but decreases with radiative losses and additional "anomalous" energy fluxes.

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