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Evaluation of Disturbed Magnetic Surfaces with Fractal Dimensions

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Abstract

For collapsed magnetic surfaces that are produced by overlapping of two magnetic islands, the fractal dimension can effectively estimate the degradation of them. The fractal dimensions of cross sections of regular magnetic surfaces and clear magnetic islands are nearly 1, while that of a collapsed magnetic surface is more than 1.2 in the present study. The Lyapunov exponents and dimensions are also calculated, which suggest the behavior of the field line of the collapsed magnetic surface is chaos.

Keywords:

stellarator, heliotron, torsatron, magnetic island, fractal dimension

1. Introduction

It is important for nuclear fusion researches to evaluate the degree of degradation of magnetic bottles in which plasma is confined. The orbits of magnetic field lines are known to be described by the equations of Hamiltonian form as are the orbits of particle motions[1,2], which some times show chaotic behavior. The fractal dimension is one of the candidate for the measure, because the shapes of the magnetic islands found in the disturbed magnetic surfaces seem to have self-similarity[3]. The Poincare map of chaotic field lines should be fractal and its dimension should be non-integer, while the fractal dimension of the cross section of clear magnetic surfaces should be 1. Therefore fractal dimension may distinguish between the two.

2. Fractal Dimension (FD) Method

In stellarators, the island appears at low rational surfaces when helical symmetry is disturbed by toroidicity[4]. A Heliotron device with coil parameters given in Table 1 is chosen as a model for this research because low rational surfaces exist at the periphery. Magnetic surfaces are calculated by integrating the field line equation in cylindrical coordinates with respect to the toroidal angle for a minimum of twenty times around the torous and plotting the intersection of the field lines with a given plane at a troidal angle θ=constant and the corresponding symmetric planes[5]. The calculations of the integration are carried out with the 6th order Runge-Kutta method. Figure 1 sketches the cross sections of magnetic surfaces near the boundary.

Table I Helical device parameters

Helical coils
Multipolarity L = 2, Helical coil period M = 7, Major radius R_0 = 2.1 m, Minor radius CA = 0.31–0.34
Vertical field coils
Radius R_{out} = 2.6 m, R_{in} = 1.6 m, Z position = ± 0.5 m, Current ratio I_h/I_{vout} = −2.7 m, I_{vin}/I_{vout} = 0.956

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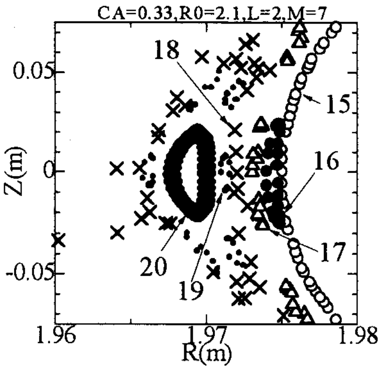

Fig. 1 The Poincaré map of a collapsed magnetic surface and magnetic islands.

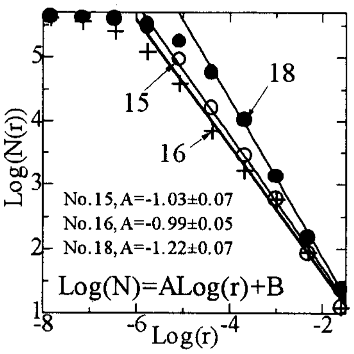

Fig. 2 The fractal dimensions for various Poincare maps (the clean magnetic surface (No.15 in Fig. 1), the magnetic island (No.16). the collapsed magnetic surface (No.18)).

For the estimation of the fractal dimension of the Poincare map, we applied the box-counting method to numerically calculated magnetic surfaces. In the box-counting method, a cross section of a magnetic surface is put onto a regular mesh with mesh size r, and count the number $N(r)$ of grid boxes which contain some of the cross section. If $N(r) \propto r^{-A}$, then A is a fractal dimension of the cross section. The fractal dimensions of some magnetic surfaces are shown in Fig. 2. The fractal dimensions of the cross section of the clear magnetic surface (No.15 in Fig. 1) and the magnetic island (No.16) are nearly 1 as we expect, while that of the collapsed magnetic surface (No.18) is more than 1.2. The fractal dimension of the Poincaré map of magnetic field lines between No.17 and No.18 gradually increases from 1.0 to 1.2. This suggests that the degree of the collapsed magnetic surfaces may be evaluated with a fractal dimension. The collapse of the outer magnetic surfaces is observed around the magnetic islands. The result agrees with theoretical prediction for the collapse of magnetic surfaces[5], saying that as the irregular magnetic field increases, the magnetic islands grow in size and overlap each other, then the magnetic surfaces there collapse.

We also estimate the Lyapunov exponents and Lyapunov dimension[6] in confirmation of the behavior of magnetic field lines. The Lyapunov dimension is

$$D_{KY} = j + \left(\sum_{i=1}^{j} \lambda_i\right)/|\lambda_{j+1}| \quad \left(\sum_{i=1}^{j} \lambda_i \geq 0,\ \sum_{i=1}^{j} \lambda_i < 0\right)$$

where λ_j is the Lyapunov exponents. If the behavior of a magnetic field line is chaotic, one of the Lyapunov exponents should be positive and the fractal dimension is non-integer. The results show that the maximum Lyapunov exponent of No.18 is positive, while that of No.15 is zero. The sign of Lyapunov exponents of No.18 and No.15 are (+,0,−) and (0,0,−). We should, however, notice that the difference between the Lyapunov exponents λ_1 and λ_2 for No.18 is small. The Lyapunov dimensions of No.18 and 15 are 2.15 ± 0.12

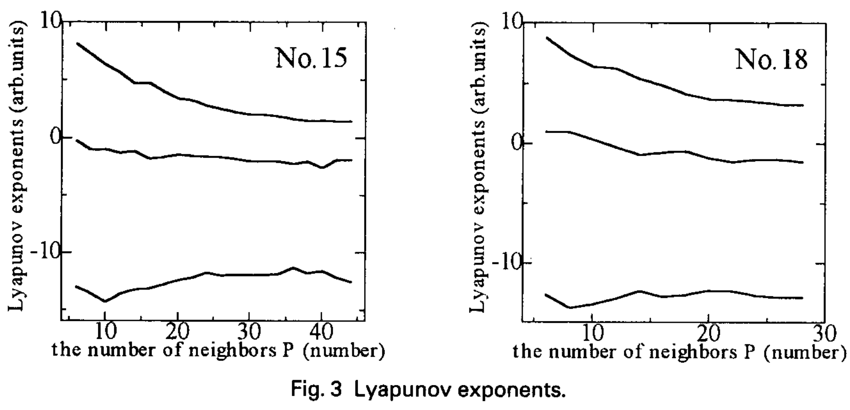

Fig. 3 Lyapunov exponents.

and 2.03 ± 0.14. The box-counting dimension is the dimension of the cross section of a magnetic surface, while the Lyapunov dimension is the dimension of a magnetic surface. Therefore the calculated Lyapunov and box-counting dimensions are not inconsistent. The sign ((0,0,−)) and fractal integer dimension of No.15 suggest that the behavior of the field line is quasi-periodic as expected from the clear magnetic surface and island.

3. Conclusion

As far as our calculations, a fractal dimension can distinguish collapsed magnetic surfaces and clear ones. We also show that the field line of the collapsed magnetic surface has a positive Lyapunov exponent and non-integer fractal dimension. It means that the behavior of the field line is chaos.

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