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Statistical Analysis of the Helical Magnetic Fields

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Abstract

The magnetic field lines of helical devices behave like a dynamic system with periodic external forces. Their properties are studied through calculation of Lyapunov numbers and fractal dimensions. The position of the outermost magnetic surface corresponds to onset of the field line chaos (that is the positive Lyapunov number).

Keywords:
outermost magnetic surface, helical coil, separatrix layer, magnetic island, Lyapunov number, fractal dimension

1. Introduction

The outermost magnetic surface (OMS), which defines the boundary between nested magnetic surfaces and the separatrix layer or so-called "divertor trace", is one of the key parameters for the coil winding of the helical experimental devices. Many Simulation codes for plasma equilibrium and stability analysis need informations of OMS, but there is no common criterion to find its location. It has been studied mainly by means of the Poincare map of vacuum field lines. Quantitative comparison for the different coil geometry, however, is difficult with this method.

In this work, we restrict our attention to the $\ell = 2$ heliotron/torsatron coil geometry. Field line data are obtained mainly with the KMAGN code. Original code has been modified to trace the field line not along the toroidal direction but along the line itself. We trace field lines numerically with the length of several thousands meter. If we start to trace the field lines inside the OMS, they move mainly in the toroidal direction and several thousands punctured points on the toroidal cross section form one smooth curve. But, outside OMS, field lines move in the radial direction or in the poloidal direction, sometimes move backwards, and at last escape from our calculation region, which extends $(3 \sim 5) \times$ "coil minor radius" from the minor axis. In this case, field lines are stretched, folded, and nested many times[1] and randomly scattered punctured points are obtained. In other words, field lines have the chaotic property and we can use the determination tools of chaos onset to define the OMS itself.

In the section 2, we calculate Lyapunov number to detect the onset of fields line chaos. In the section 3, we calculate the fractal dimension. The section 4 is the conclusion.

2. Lyapunov Number

One of most promising tools for detecting the onset of chaos is the Lyapunov number of the reconstructed attractor. Various algorithm to calculate the Lyapunov number has been proposed [2, 3]. Wolf's method counts the expansion of the distance between the reference orbit in the phase space and neighborhood point [2]. After about 1,000 step along the orbit, usually, maximum Lyapunov number (λ_1) converges to constant value. On the contrary, Sano's group calculates the linear operator matrix acting on tangent vectors, with the least-square method [3]. This procedure gives us all Lyapunov numbers (positive, zero, and negative one), but much more data (about 5 times larger

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than for Wolf's procedure) is needed to converge Lyapunov numbers. So, in this section, we present results with Wolf's one.

In the Figure 1, examples of the Wolf's algorithm calculation are shown for three field lines with the different start points. One starts inside of OMS (R = 2.480[m]) and its Lyapunov number converges to zero rapidly in a few hundreds step. Another line starts outside of OMS (R = 2.495[m]). Lyapunov number becomes large positive value and converges to ~6.8 in about 500 step. Between these two cases, Lyapunov number initially shows a large spike and converges more slowly to a positive value.

The Lyapunov number profile near OMS is shown in the Figure 2. For $R<2.48$[m], non-positive Lyapunov number is obtained and the field line forms magnetic surface. Just outside of OMS, there exists natural island. The minimum value at $r = 2.49$[m] in the figure corresponds to this island in the "chaos sea". Though

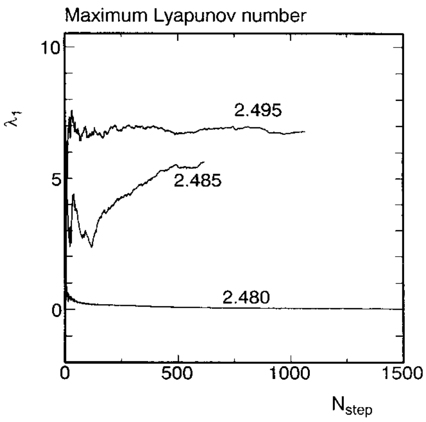

Fig. 1 Convergence of Lyapunov number with Wolf' algorithm. Start position of lines are also shown in the figure.

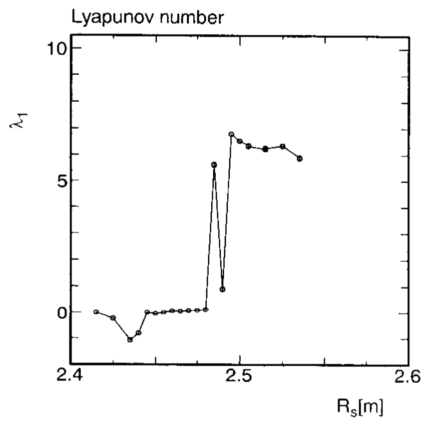

Fig. 2 Profile of fractal dimention near OMS.

the origin of another dip (R~2.435[m]) is not clear yet, the position of OMS is clearly determined. Between different coil configurations, the positions of OMS are distinguished.

3. Fractal Dimension

There are many different definitions of the "so-called " fractal dimension. Though almost all definitions are too difficult to be used with Poincare map data or time series data, capacity dimension (D_0) and correlation dimension (D_2) are relatively easy to calculate. Information dimension (D_1) is also obtained from Lyapunov spectrum [2]. But this is not included in this work.

We divide toroidal cross section into the square cell with the size of r. Then we count the number $N(r)$ of cells which contain at least one data point of Poincare map in it. If the data size N of Poincare map is sufficiently large, at the limit $r \to 0$, $N(r) \sim r^{-D_0}$. So from the relation between r and $N(r)$, $-D_0$ can be obtained with the least square method.

In the case of D_2, Grassberger's paper gives us a calculation method with the correlation integral[4].

$$C(r) = \frac{1}{MN} \sum_{j=1}^{M} \sum_{i=1}^{N} H(r - |x_i - x_j|)$$

where $H(x)$ is the Heaviside step function. (M can be set much smaller value than the data size N to save the computational time.) If N is sufficiently large, at the limit $r \to 0$, $C(r) \sim r^{D_2}$.

In order to determine D_0 or D_2, the choice of the r range for the least square fitting is important. Because of finite data size, $N(r)$ and $C(r)$ at small r become smaller than the expected powers-law. If we neglect this fact, D_0 would be underestimated and D_2 would be overestimated. So correlation dimension D_2 is preferred in order to distinguish inside of OMS (D~1.0) and outside ($D>1.0$).

One example of $C(r)$ is the Figure 3. From the definition, $C(r)$ is expected to saturate to the 1.0 for large r. But in this case, field line starts on a rational surface (magnetic surface with rational rotation transform) and $C(r)$ firstly saturates to the lower value (~0.11) at $r>0.005$[m]. (Saturation to 1.0 is omitted in this figure.) As shown by the dot line, the "apparent" correlation dimension deduced form whole range data becomes low value (~0.234). This corresponds to the fact that the punctured plot of the rational surface does not form a closed line but consists a few discrete dots. If the r range is restricted to small value (we watch the small structure around one island, the solid line in the figure), correlation dimension becomes 1.119 (>1.0).

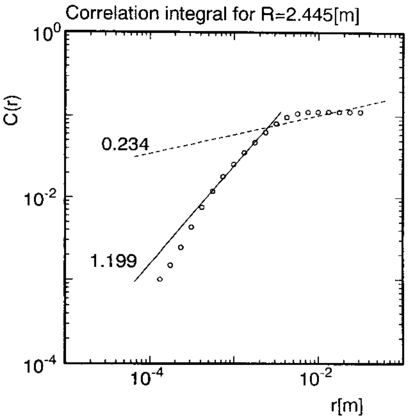

Fig. 3 Correlation integral for the rational surface. Deduced correlation dimensions are also shown.

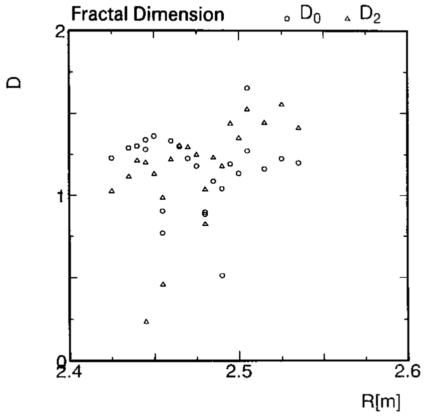

Fig. 4 Profile of fractal dimention near OMS.

The fractal dimension profile near OMS is shown in the Figure 4. From fractal theory, D_0 must be larger than D_2. But, as mentioned above, finiteness of data violates this rule. Because of scattered data, the position of OMS is not shown as clearly as λ_1. Magnetic islands on rational surfaces ($R = 2.445, 2.455, 2.490$[m] in the Fig. 4) are, however, shown more clearly with fractal dimension ($D < 1.0$).

4. Conclusion

We applied two algorithms for Lyapunov numbers calculation of dynamic systems to magnetic field data and compared their results. Though Sano's algorithm gives us all Lyapunov numbers to calculate the information dimension D_1, necessary data size is large. On the contrary, Wolf's algorithm can calculate positive (and usually only maximum) Lyapunov number but necessary data size is smaller.

The profile of maximum Lyapunov number λ_1 is examined. Field lines inside OMS have non-positive λ_1. Outside of OMS, λ_1 is large positive value and shows the minimum value near natural islands. For different coil configurations, different λ_1 profiles are obtained and the positions of OMS are distinguished between these configurations.

Fractal dimension also has the relation with the chaotic properties of field line (especially with the existence of magnetic islands). But when we determine OMS, they easily suffer from bad effect of data finiteness.

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