J. Plasma Fusion Res. SERIES, Vol.1 (1998) 187–190

Phase Control of Magnetic Islands in a Heliac Vacuum Field

HUDSON Stuart R.* and DEWAR Robert L.[1]
Fusion Plasma Research, J.A.E.R.I., Naka, Ibaraki 311-0193, Japan
[1]Department of Theoretical Physics, RSPhysSE, A.N.U. A.C.T. 0200, Australia

(Received: 30 September 1997/Accepted: 24 December 1997)

Abstract

A technique by which magnetic islands in the vacuum field of heliacs may be controlled is presented. The method is based upon the construction of quadratic flux minimizing surfaces, which are defined as surfaces that minimize a suitable quadratic flux functional. The technique for manipulating the magnetic islands is based upon a quickly computed measure of the island width and phase. A major island chain in the standard configuration of the heliac in operation at the Australian National University, H-1NF, has been eliminated using the method. It was shown that variation of the vertical coils only enables subtle control of the island content.

Keywords:

magnetic islands, quadratic-flux minimizing surfaces, width and phase of islands, H-1NF Heliac, nearly integrable magnetic fields, resonant perturbation

1. Introduction – Magnetic Islands

For non-axisymmetric devices such as stellarators, magnetic islands will naturally occur even in the vacuum configuration. It is clear that magnetic islands with large width will deteriorate plasma confinement. Also important for plasma confinement is the phase of magnetic island chains. The phase of the island chain was important with respect to the possibility of self-healing of magnetic islands [1]. Also, for the reversed shear tokamak configurations, the relative phase of the double island chains associated with rational surfaces may be important with respect to the reconnection of magnetic field lines and the flattening of the rotational transform profile [2].

Generally, one may calculate the width of magnetic islands from the magnitude of the resonant perturbation and the magnitude of the shear at the rational surface. This calculation exploits the Hamiltonian character of toroidal magnetic fields. This procedure requires the given magnetic field to be known as a small perturbing field superimposed on an integrable field. To determine the phase of the island chain, it is required to know the sign of the perturbation and the sign of the shear.

With these ideas implicit in the following discussion, this paper will present a technique by which the magnitude and phase of magnetic islands in stellarator vacuum fields may be controlled. Results from the heliac H1-NF in operation at the Australian National University indicate that subtle control of the islands is possible using by small variations of the vertical field current coils. This technique is an alternative to the method of Cary and Hanson [3] for eliminating island chains from vacuum configurations.

The basis of the method is the definition of quadratic flux minimizing surfaces. These are surfaces which generalize the concept of flux surfaces. During the

Corresponding author's e-mail: stuart@ptl01.naka.jaeri.go.jp

construction of these surfaces, a natural efficient parameter that indicates island width and phase is determined.

In section 2, quadratic flux minimizing surfaces are defined. An important quantity that arises is the action gradient function defined on rational rotational transform surfaces. In section 3, it is shown how the action gradient function may be used to obtain a quickly computed parameter that indicates island width and phase. This parameter may be used in an optimization routine that determines preferred vacuum magnetic field configurations. Some concluding remarks are made in section 4, and some indications of further applications of quadratic flux minimizing surfaces are given.

2. Quadratic Flux Minimizing surfaces

Introduced by Dewar *et al.* [4], quadratic flux-minimizing surfaces are a natural generalization of flux surfaces. Quadratic-flux minimizing surfaces are surfaces that minimize the quadratic-flux functional. Given field B in toroidal coordinates (ρ, θ, ϕ), with the contravariant components B^{ρ}, B^{θ} and B^{ϕ}, the quadratic flux functional is:

$$\varphi_2 = \int_{\Gamma} \frac{B_n^2}{2C_n} \mathrm{d}\sigma \tag{1}$$

where $A_n = \boldsymbol{A} \cdot \boldsymbol{n}$, with $\boldsymbol{n}$ the unit normal to the trial surface Γ, and where $\boldsymbol{C}$ is an auxiliary divergence free field everywhere transverse to the surface Γ.

The choice of $\boldsymbol{C}$ is arbitrary, but typically we use $\boldsymbol{C} = \nabla\theta \times \nabla\phi$. With this choice $\boldsymbol{C}$ is both parallel to the radial basis vector $e_{\rho} \equiv \partial_{\rho} r$ and related to the action gradient defined on periodic curves [5].

On allowing the surface Γ to vary, using the calculus of variations, Dewar *et al.* [4] obtain the Euler – Lagrange equation required to make the first order variation of the quadratic flux functional equal to zero, $\delta_{\varphi 2} - 0$.

$$\boldsymbol{B}_{\nu} \cdot \nabla \nu = 0, \tag{2}$$

where $\boldsymbol{B}_{\nu} \equiv \boldsymbol{B} - \nu\, \mathbf{C}$ and $\nu \equiv B_n / C_n$. We call $\boldsymbol{B}_{\nu}$ the pseudo-field and ν the action gradient for surface s.

We make a few comments about the quadratic flux functional and the minimizing surfaces. We only consider rational rotational transform quadratic flux minimizing surfaces. If the trial surface Γ is actually a flux surface, then $\boldsymbol{B}_n = 0$ and the quadratic flux functional is also zero. In this case, the quadratic flux minimizing surface is simply the flux surface. In such a case, the quadratic flux minimizing surface is comprised of a family of periodic field lines. Also, the action gradient function ν is everywhere zero on the surface.

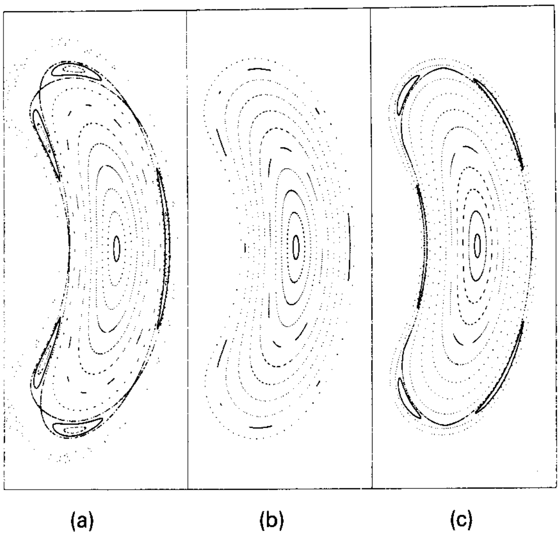

Fig. 1 Manipulation of the (5,3) magnetic island in the vacuum field of H-1. On the left is the standard configuration labeled Fig. 1(a). The middle plot, labeled Fig. 1(b), and the right plot, labeled Fig. 1(c), are obtained by variation of the currents in the vertical field coils.

More generally, Eq. (2) shows that ν is constant along a pseudo field line and that the solution surfaces, with rational rotational transform, are comprised of a family of periodic pseudo-field lines. Pseudo orbits are defined as the integral curves of the pseudo magnetic field $\boldsymbol{B}_{\nu}$. To locate the periodic pseudo-orbits we specify θ and to search in the two dimensions (ρ, ν) for a fixed point of the m-th iteration of the pseudo Poincar'e return map, P^m. The pseudo Poincaré return map is computed by following the pseudo magnetic field lines around toroidally for one toroidal period. If the action gradient vanishes on all good flux surfaces, the two dimensional search is reduced to a one dimensional search [5].

It is the properties of the action gradient function ν which are of primary interest for the present discussion. As examples, three vacuum magnetic field configurations of H-1NF are presented in the following sections of this paper. These are shown in Fig. 1(a), Fig. 1(b) and Fig. 1(c). A quadratic flux minimizing surface of periodicity type $(m, n) = (5, 3)$ was constructed for each of these configurations. Shown in Fig. 2 is the behavior of ν for each of the surfaces. Consistent with the existence of $2m$ X or O points are the $2m$ zeros of ν. Stellarator symmetry [6] implies that the angular location of max $(|\nu|)$, cannot rotate, though it may vary

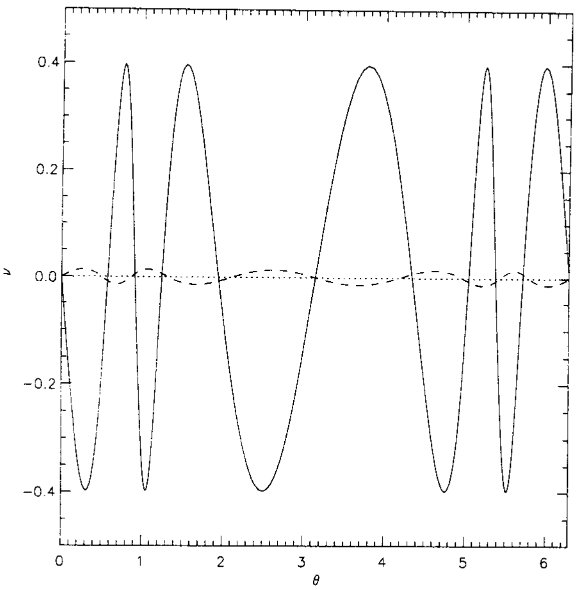

Fig. 2 Behavior of ν with θ for the (5,3) surface for the various current configurations. The solid line is the ν obtained for the configuration displayed in Fig. 1(a), the dotted line corresponds to the configuration shown in Fig. 1(b) and the dashed line corresponds to the configuration shown in Fig. 1(c). The phase locking of ν is clearly indicated as all three lines pass through zero for $\theta = 0$.

slightly. We define θ^* as the angle at which $|\nu|$ achieves its maximum. Once θ^* has been located for a particular current configuration, we may consider it fixed as we make small variations of the currents. For each configuration, $\nu(0) = 0$, which confirms the phase locking of ν, and $\theta^* \approx 0.31$.

3. Control off Magnetic Islands

Starting with the standard configuration shown in Fig. 1(a) we define a measure, ν^*, of the size of the (m, n) island and its phase to be the value of ν at this angle, $\nu^* = \nu(\theta^*)$. Note that the scalar parameter ν^* contains information regarding the size of the island chain from the magnitude, $|\nu^*|$. We use the fact that the (m, n) island has vanished if ν^* is zero, and the phase of the island chain is related to $\mathrm{sgn}(\nu^*)$. An intuitive understanding of the parameter ν^* is that it is related to the amplitude and phase of the resonant perturbation responsible for the island chain. We will show that magnetic islands may be manipulated by variation of the vertical field coil currents. We consider the function

$$\Lambda = (\nu^* - \nu_0^*)^2 \tag{3}$$

where ν_0^* is to be set, indicating the desired island size and phase (magnitude and sign of ν_0^* respectively).

To firstly remove this island chain, we set $\nu_0^* = 0$. The configuration that minimizes Λ, and thus minimizes the amplitude of the resonance harmonic producing the island, is determined by a numerical search in the two-dimensional, vertical-field coil current space. By varying the vertical fields, it was possible to reduce Λ by several orders of magnitude. A Poincaré plot of the resulting magnetic field is shown in Fig. 1(b) and the full quadratic-flux minimizing surface was constructed. Here we see the (5,3) island chain has become negligible in width. The action gradient (shown in Fig. 2 as the dotted curve) indicates that the family of periodic pseudo orbits is degenerate in action and the quadratic- flux minimizing surface has reduced to the periodic flux surface.

The island chain may be recreated, but with the X and O points having swapped their location, by setting $\nu_0^* = 0.4$ and again minimizing Λ. A Poincaré section of the optimized configuration is shown in Fig 1(c) where we see the (5,3) island chain has changed phase by 180°. The behavior of ν is shown as the dashed curve in Fig. 2.

We observe that for this configuration region, the vertical field coils provide a means to 'fine-tune' the configuration. Such results indicate the flexibility of the stellarator design.

4. Conclusion

From the construction of quadratic flux minimizing surfaces, a natural parameter that quantifies the size and phase of magnetic islands emerges. This parameter, ν^*, is convenient for use in computational optimization routines as it is efficiently calculated given the magnetic field in toroidal coordinates. A numerical search quickly locates the optimal current settings for H-1NF to remove a major island chain. As the phase information of the island chain is contained in the action gradient, the search for optimal settings may be extended to locate configurations with non-zero width islands of a desired phase. This is important in context of the self-healing phenomena results recently reported.

Further work using quadratic flux minimizing surfaces is presently underway at developing a robust magnetic coordinate algorithm. As quadratic flux minimizing surfaces automatically reduce to flux surfaces if the flux surface exists, coordinates constructed from quadratic flux minimizing coordinates may be designed to agree with straight field line coordinates. Also, quadratic flux minimizing surfaces form a convenient set of surfaces to construct a nearby integrable magnetic field to any given non-integrable magnetic field [7].

References

[1] T. Hayashi, T. Sato, H.J. Gardner and J.D. Meiss, Physics of Plasmas **2**, 752 (1995).

[2] S.R. Hudson, S. Tokuda and A. Monticello. *Double Magnetic Island Chains in Reversed Shear Equilibria*, draft, 1997.

[3] J.R. Cary and J.D. Hanson, Physics of Fluids **29**, 2464 (1986).

[4] R.L. Dewar, S.R. Hudson and P. Price, Physics Letters A **194**, 49 (1994).

[5] S.R. Hudson and R.L. Dewar, Journal of Plasma Physics **56**, 361 (1996).

[6] R.L. Dewar and S.R. Hudson, Physica D **112**, 275 (1997).

[7] S.R. Hudson and R.L. Dewar, *Construction of an Integrable Case Close to Any Non-Integrable Toroidal Magnetic Field, submitted to* Physics Letters A (1997).