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Transport Optimization and MHD Stability of a Small Aspect Ratio Toroidal Hybrid Stellarator

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Abstract

A new class of stellarators is found that does not rely on quasi-symmetrization to achieve good confinement. These systems depart from canonical stellarators by allowing a small net plasma current. We have developed an optimization procedure with bounce-averaged omnigeneity and other desirable physical properties as target criteria. This method has been applied to show the existence of a compact plasma device having a small aspect ratio A, high β (ratio of thermal energy to magnetic field energy), and low plasma current. The added degree of design flexibility afforded by the plasma current leads to a potentially attractive low A hybrid device which is stable to magnetohydrodynamic (MHD) ballooning modes for $<\beta> < 6\%$.

Keywords:

optimization, compact stellarator, neoclassical transport, onmigeneity, plasma equilibrium, MHD ballooning stability, quasi-symmetry

The tokamak, a toroidally symmetric plasma trap that uses a large plasma current to produce a confining poloidal magnetic field, has been the most successful plasma confinement device to date, simultaneously achieving high temperature ($T_i \geq 10$ keV) and high $\beta < 10\%$ plasmas. However, the difficulty and expense of driving a large steady-state current, along with the complexity of protecting against current disruptions, is a disadvantage in a fusion reactor. Low aspect ratio ($A < 3$) stellarators [1-5] offer the attractive feature of a compact steady-state fusion power system with high volume utilization and reduced current-drive requirements. Stellarators, which are nonsymmetric plasma traps relying on external coils to produce the internal transform needed for confinement and stability, also have less current disruption potential compared with tokamaks.

Compact (low A) stellarators have been previously considered unattractive for several reasons. They suffered from a combination of poor neoclassical (collisional) transport due to lack of symmetry, low stability β limits due to localized helical wells in bad curvature regions of the plasma, and fragility of magnetic surfaces due to low order resonances and consequent chaotic surface destruction. Recently, progress has been made in substantially improving their collisional confinement by designing systems with "quasi-symmetry." (A quasi-symmetric confinement system is one in which the $|B|$ Fourier spectra, in Boozer magnetic coordinates [6], has spatial symmetry, but in which the metric tensor is generally *not* symmetric. Of the two quasi-symmetric approaches considered, only quasi-toroidal optimizations have been successful [7] at low A, but at unattractively low values [8] of $\beta < 2\%$. The quasi-helical approach [9] is expected to be applicable only at higher aspect ratios[10].

The rather large rotational transform (ι) values

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($\iota \geq 0.5$) associated with quasi-symmetric configurations also make them necessarily low-shear devices, which can be susceptible to magnetic surface breakup. Low edge transform values, $\iota \approx 0.1$, reduce the fragility of the magnetic configuration. We have found that one way to maintain this low pedestal value for ι as the pressure increases is to add a small, net toroidal current. This current also increases the design space for optimization of stability and transport, as discussed below.

Quasi-symmetry restricts the nonzero components of the $|B|$ spectra to be multiples of a fixed helicity (m/n value, where m is the poloidal, and n is the toroidal, Fourier-mode number). Since the pressure in stellarators is limited by helically localized regions of bad curvature associated with $|B|$, it may be possible to increase the allowable β of compact stellarators by easing the quasi-symmetry constraint. We have therefore considered an approach which uses the alignment of contours of the approximate second adiabatic invariant [11, 12] $J = \oint v_{\|} \mathrm{d}l$ with magnetic flux surfaces $\psi =$ const (ψ denotes the enclosed toroidal flux.) The component of the bounce-averaged particle drift, normal to a magnetic surface, satisfies $\langle V_{\mathrm{D}} \cdot \nabla\psi\rangle \propto \partial J/\partial\vartheta$. A configuration which satisfies $J = J(\psi)$ leads to confinement improvement over the entire trapped particle population, and also reduces the number of transitional particles. This criterion is a generalization to arbitrary $|B|$ spectra of the optimization previously proposed [13] for a simple model spectrum consisting of only two Fourier components. This bounce-averaged *omnigeneity* has recently been interpreted [14-16] in terms of equal spacing of $|B|$ contours on a magnetic flux surface.

We use the VMEC code, a three-dimensional magnetohydrodynamic (MHD) equilibrium solver [17] which is based on nested magnetic surfaces, as the inner physics evaluation loop of a Levenberg-Marquardt optimizer [18]. This method is used to minimize a positive-definite functional (χ^2) composed of a sum of squares of σ-weighted differences between physics-based *target* values and instantaneous *configuration* values (as computed numerically from VMEC). The configuration space is defined by the control (independent) variables, which are the Fourier harmonics of R and Z describing the shape of the outermost magnetic flux surface. In the case of stellarator-tokamak hybrids, plasma current is also a control variable. This method is similar to that originally used to design a large-A quasi-helical configuration [9]. It differs in the physics optimization targets comprising χ^2, which here consist of the following: (a) alignment of J (and specifically, $B_{\min}$ for deeply trapped particles and $B_{\max}$ to reduce transitional orbit losses) with magnetic flux surfaces, leading to terms of the form $\chi^2 = \langle[\partial J(\psi, \theta, \lambda)/\partial\theta]^2\rangle/\sigma\langle J^2\rangle + \ldots$ (angle brackets denote a flux surface average, the elipsis indicates similar contributions arising from $B_{\min}$ and $B_{\max}$, σ is the standard derivation, and $\lambda = \epsilon/\mu$ is the pitch; (b) matching the rotational transform $\iota(\psi)$ to a specific radial profile; (c) maintenance of a magnetic well, $V'' < 0$, needed for interchange stability, over most of the plasma cross section; and (d) $A \equiv R_0/a \approx 3$. The alignment of the $B_{\min}$, $B_{\max}$, and trapped J contours with ψ is performed at three or more radial flux surfaces and, for J, at four values of λ. In contrast to quasi-symmetric optimizations, the present method does *not* directly target the magnetic spectrum $\{B_{mn}\}$.

The optimization technique has been applied to a hybrid stellarator device (with current) with $N = 8$ field periods (number of identical toroidal sections), $A \approx 3$, $\langle\beta\rangle = 2\%$, and a net toroidal plasma current of 60 kA and a mean on-axis magnetic field of 1 T. The current is small compared with ≈ 1 MA in a tokamak of similar size and magnetic field. We will compare the initial unoptimized device (with $\chi^2 = 100$), whose outer flux surface was determined by a set of $N(=8)$ identical modular, tilted coils [2], with an optimized configuration based on the alignment of J with ψ (with $\chi^2 = 10$). Figure 1 shows the outer flux surfaces for the two configurations with shading to indicate the constant $|B|$ contours. The unoptimized configuration has $\iota\ (\psi = 0) = 0.25$, $\iota\ (\psi = \psi_{\mathrm{edge}}) = 0.15$, and a central region of reversed shear, while the optimized case has a monotonically decreasing rotational transform profile, which is tokamak-like (*i.e.*, decreasing toward the plasma edge): $\iota = 0.3 - 0.2(\psi/\psi_{\mathrm{edge}})$. The optimized case also has a lower magnetic ripple, by a factor of 2, over most of the plasma cross section. The $B_{\min}$ contours, depicting the orbits of deeply trapped particles, are shown in Fig. 2(a)-(b) for these two cases. They are presented in Boozer coordinate space in which the magnetic surfaces are concentric circles. The unoptimized configuration (a) has completely open $B_{\min}$ contours (*i.e.*, all deeply trapped particles are lost), while the optimized configuration (b) has a large area of closed $B_{\min}$ contours.

To assess the effect of optimization on the thermal confinement properties, we have followed the Monte Carlo evolution [19] of 256 particles started at a single interior radial location with a random distribution in pitch, poloidal, and toroidal angles, and a Maxwellian distribution in energy. In Figure 3, we show the particle loss rates versus time for the original configuration ($<\beta>=2\%$) and several J-optimized cases ($<\beta>=2$, 4, and 6%) along with an equivalent tokamak case (obtained

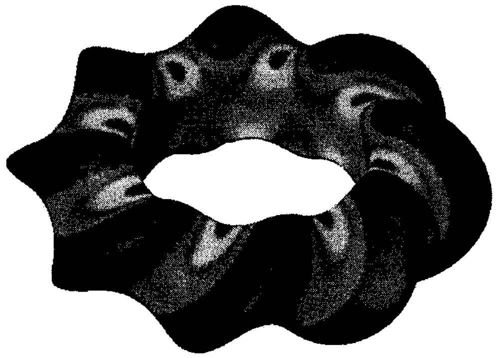

Fig. 1(a) Outer magnetic flux surface of the unoptimized configuration.

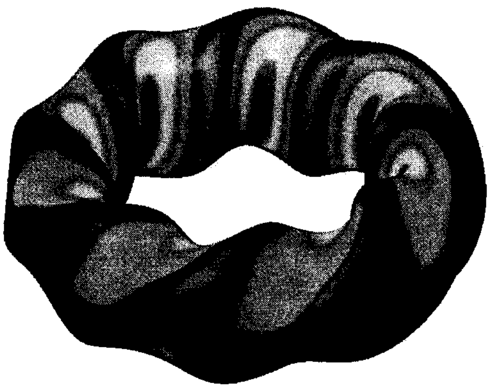

Fig. 1(b) Outer magnetic flux surface of optimized configuration.

by retaining only the axisymmetric $n = 0$ harmonics in the $|B|$ spectrum of the J-optimized cases). These simulations clearly demonstrate that the optimization procedure can substantially reduce loss rates, leading to roughly a factor of 10 confinement improvement for all pressures compared with the initial unoptimized configuration. The best J-optimized case has a loss rate within a factor of 4 of the equivalent tokamak.

The confinement of collisionless energetic particles is one of the primary motivations for the optimizations discussed here since the thermal particle confinement can also be improved by the ambipolar radial electric field. We followed an ensemble of orbits at 40 keV that initially pass through the magnetic axis and have a range of pitch angles. We find that although the unoptimized configuration has a significant loss cone over the range $-0.2 < (v_\parallel / v) < 0.4$, the J-optimized configuration has completely healed the loss cone and confines all of the orbits considered. Furthermore, calculations show that even at energies approaching 400 keV, the optimized configuration confines all orbits.

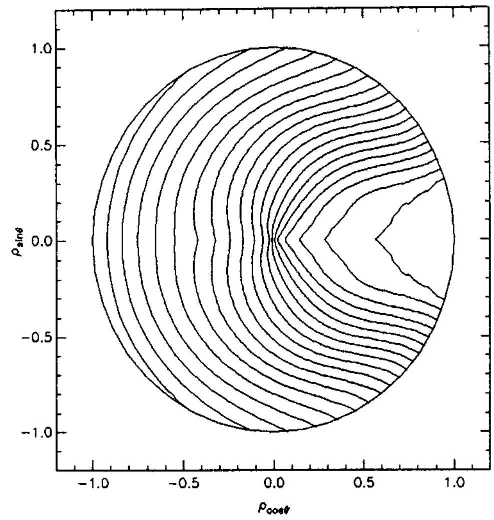

Fig. 2(a) B_{min} contours for unoptimized case.

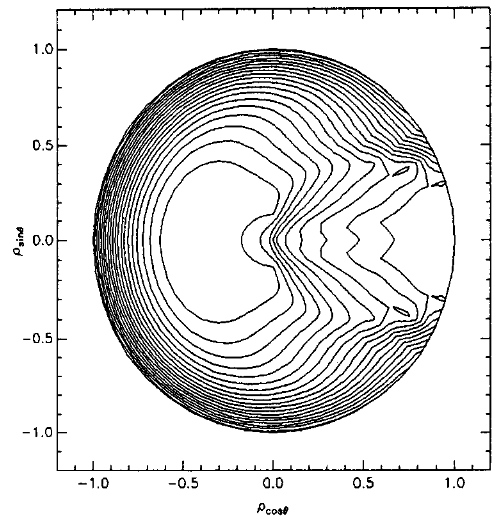

Fig. 2(b) B_{min} contours for optimized case.

The J optimization process leads to configurations that are neither quasi-toroidal nor quasi-helical. This is shown in Fig. 4, where the $\{B_{mn}\}$ spectra is plotted versus a radial coordinate for the first few dominant Fourier modes. There is a mixture of different helicities. The dominant modes (m, n) are (0, 1), (1, 1) and (1, 0), corresponding (in Boozer space) to toroidal bumpiness, helical axis, and an axisymmetric $1/R$ contribution, respectively (where n is in field period units).

The present configuration was optimized at a relatively low value of $<\beta> = 2\%$. To test its high-pressure ballooning stability properties, we have raised the plasma pressure to $<\beta> = 6\%$, keeping the ι-profile fixed (flux-conserving) and therefore, increasing the net

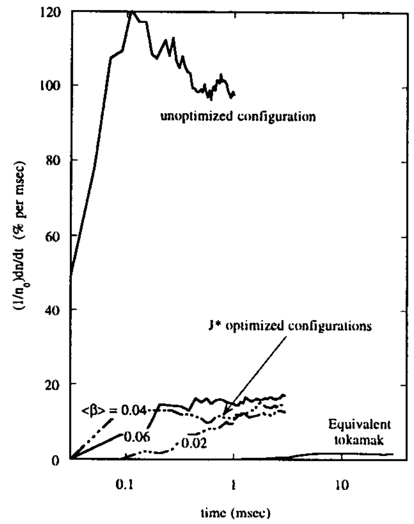

Fig. 3 Comparison of Monte Carlo loss rates of unoptimized and J-optimized configurations for various values of $<\beta>$.

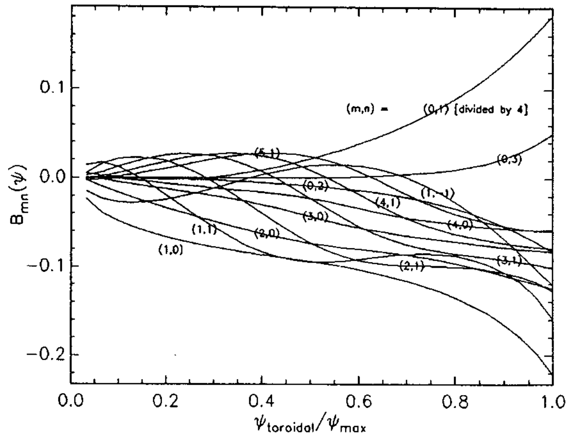

Fig. 4 Fourier coefficients of $|B|$ *vs.* normalized toroidal flux for the unoptimized and optimized case.

plasma current. In this way, it is found that the plasma is stable to ballooning modes over the inner 80% of its cross section. Flattening the pressure near the edge completely stabilizes this configuration. This configuration has improved stability compared with the unoptimized one, which was unstable to ballooning modes over most of its cross section for $<\beta> \geq 2.5\%$. The plasma current required to maintain the ι profile (while keeping the plasma-bounding surface fixed) increased modestly, by less than a factor of 2, over this range of pressure, and peaked toward the edge of the plasma as the pressure was raised. Although the current density profile obtained by this flux-conserving optimization had a narrow reversal region at the plasma edge, further optimization seems possible with respect to the current by relaxing the constraint on the iota profile. In addition, self-consistency with respect to the high-β-driven (bootstrap) current warrants further investigation. The reason for the improved stability of the optimized configuration is presently unknown but may be related to its tokamak-like ι profile and the edge-localized current [20]. Further study to analyze current-driven modes is required.

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