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Toroidally Symmetric Stellarators

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Abstract

Extensive three-dimensional computations have led to the discovery of stellarators with just two field periods and quite low aspect ratio that have a remarkable toroidal symmetry in their magnetic structure. Studies of equilibrium, stability and transport show that this new configuration is a plausible candidate for a fusion reactor.

Keywords:
stellarator, two-dimensional symmetry, magnetic spectrum, confinement, stability

1. Introduction

Fast three-dimensional computer codes have led to the design of modern stellarators that at least in theory compare well with tokamaks [1-3]. The helias discovered in 1984 by Nuehrenberg and Zille [4] was the first of these configurations, and another is the Modular Helias-like Heliac (MHH) developed for reactor studies. An MHH2 with just two field periods and the exceptionally low aspect ratio $A=3.5$ has been found that has an axial symmetry property of its magnetic field structure providing good confinement of hot particles [5]. With a rotational transform in the range $0.25 \geq \iota \geq 0.20$ the cross sections of the MHH2 become only moderately elongated, and the external magnetic field can be generated by 16 adequately spaced modular coils. However, vacuum field calculations of the location of the coils should be corrected at finite β to account for a significant toroidal shift of the plasma, and additional coils are recommended to control ι and provide flexibility.

Computational methods have been applied to show that the MHH2 stellarator has good equilibrium, stability and transport properties. Because of the toroidal symmetry, the displacement of the magnetic axis and the growth of the bootstrap current behave as β increases like those in a comparable tokamak. The average β limit depends very much on the size of the derivative $\mathrm{d}p/\mathrm{d}s$ of the pressure with respect to the toroidal flux. It lies between 4% and 5% for a broad pressure profile, but for peakier profiles it may drop below 3%. The novel features and small aspect ratio of the MHH2 configuration make it an attractive candidate for a proof of principle experiment testing alternate concepts.

2. Estimation of β Limits

The NSTAB equilibrium code provides a numerical implementation of the variational principle of magnetohydrodynamics [3]. It is spectral in the toroidal and poloidal angles, but uses a finite difference scheme in the radial direction. This is so accurate that small magnetic islands can be captured on a crude grid. The solution emerges in a form convenient for evaluation of the flux functions s, θ and ϕ that occur in the Clebsch representations

$$\boldsymbol{B} = \nabla s \times \nabla \theta = \nabla \phi + \zeta \nabla s$$

of the magnetic field. The efficiency of the code facilitates parameter studies of the kind required to design a practical stellarator. However, convergence is asymptotic in the sense that when too many harmonics are included in the solution erroneous results may be

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obtained because of resonances at rational surfaces [6].

After s, θ and ϕ have been renormalized to become an invariant system of radial and angular coordinates, the magnetic field strength can be expanded in a Fourier series of the form

$$\frac{1}{B^2} = \sum B_{mn} \cos(m\theta - [n - \iota m]\phi).$$

The coefficients B_{mn}, which are calculated in a robust fashion by the NSTAB code, are known as the spectrum of the magnetic field. The property of toroidal symmetry for the MHH2 stellarator simply means that B_{mn}/B_{00} is small for all $m \neq 0$.

Theory shows that there is a related expansion

$$\frac{\boldsymbol{J} \cdot \boldsymbol{B}}{B^2} = p' \sum \frac{mB_{mn}}{n - \iota m} \cos(m\theta - [n - \iota m]\phi)$$

for the parallel current [7]. The appearance of small denominators at the rational surfaces where $\iota = n/m$ explains why difficulties should be expected in calculating smooth equilibria without two-dimensional symmetry. The nonexistence of differentiable solutions, which is a consequence of the KAM theorem, can reappear in linearized or local stability analyses as an erroneous prediction of instability [6]. This phenomenon is consistent with the observation that stellarator experiments have exceeded β limits derived from the Mercier criterion [8].

The iterative scheme exploited by the NSTAB code to compute magnetohydrodynamic equilibria involves an accelerated method of steepest descent based on the variational principle. This can be converted into a satisfactory test of stability by introducing a perturbation in the equations that is associated with some dangerous resonance and then tracking the decay of the corresponding mode in the solution after the perturbation has been removed. If the solution returns to its original state the mode is considered to be stable, but if it grows without limit or converges to a new bifurcated equilibrium then the configuration is unstable. Convenient forms of the perturbation that only require calculation over one field period break the stellarator symmetry causing exclusively cosine terms to appear in the Fourier expansion of $1/B^2$. The code can resolve modes of this kind that are dominated by trigonometric terms of degree as high as 7 or 8.

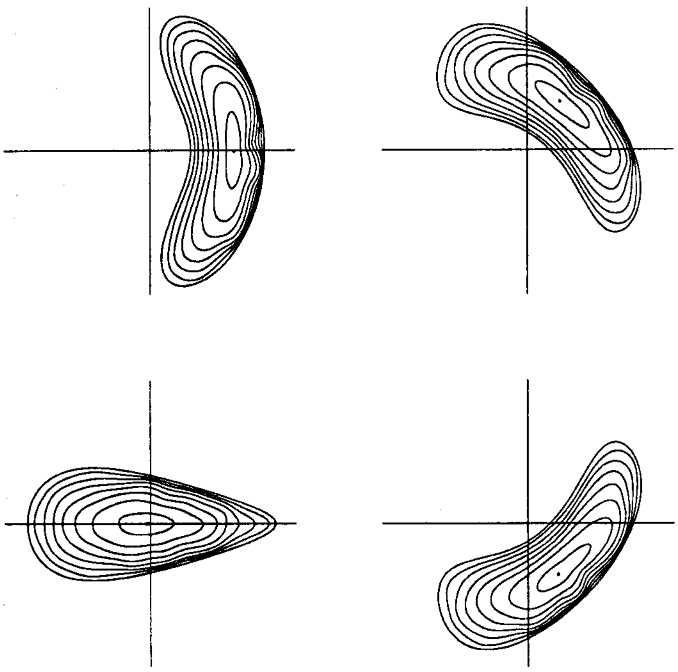

Fig. 1. Poincaré sections of the magnetic surfaces over one field period of a bifurcated MHH2 equilibrium with bootstrap current at an average β of 5.5 percent. The solution exhibits ripple without the stellarator symmetry of the plasma boundary. This establishes instability of a mode localized in the region of bad curvature, but extending globally along the radius.

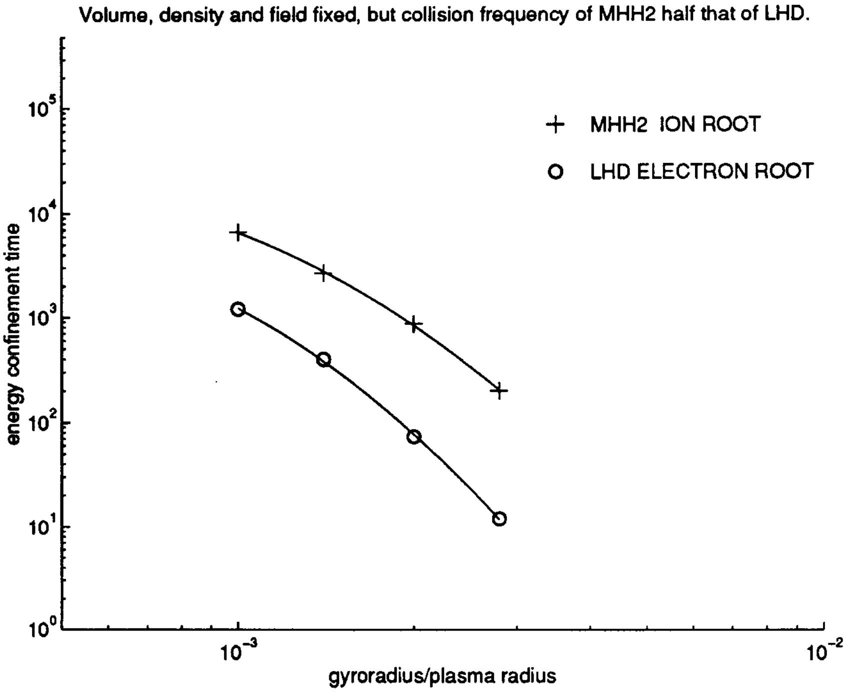

Fig. 2. Monte Carlo calculation of the energy confinement time in conventional and helias stellarators at reactor conditions using quasineutrality to determine the electric potential.

Figure 1 displays four cross sections of a bifurcated equilibrium for the MHH2 configuration at an average β of 5.5% when enough bootstrap current is included in the NSTAB calculations to raise ι above 0.25. The asymmetric ripple of the magnetic surfaces is seen to characterize a mode that is localized in a region of bad curvature, but extends globally across the small radius of the plasma. If the net current is removed the instability disappears, but the equilibrium deteriorates because of a substantial outward shift of the magnetic axis, accompanied by resonant distortions of the flux surfaces. This treatment of β limits seems to be better correlated with experimental measurements than linearized theory or numerical computations of local criteria [6]. A basic conclusion is that equilibrium presents more of a problem for stellarators than stability at high β.

3. Confinement

Neoclassical transport in stellarators can be calculated by a Monte Carlo method that is described by the drift kinetic equation

$$f_t + \rho_\| [\boldsymbol{B} + \nabla_x \times (\rho_\| \boldsymbol{B})] \cdot \nabla_x f$$
$$= \nabla_v \cdot \nu [\nabla_v f + M(\boldsymbol{v} - \boldsymbol{u}) f / T],$$

where f can be the distribution function of either the ions or the electrons, $\rho_\|$ is the parallel gyroradius, and ν is a renormalized collision frequency introduced by Boozer and Kuo-Petravic [7]. The collision operator on the right, where differentiations are performed in velocity space, need not conserve momentum provided that results of the computation are interpreted as a measure of the time a test particle stays in the plasma while the electromagnetic and velocity fields $\boldsymbol{B}$ and $\boldsymbol{u}$ of the background remain fixed. The drift velocity on the left, where differentiations are performed in physical space, leads to a system of ordinary differential equations for the flux coordinates of guiding center orbits that just depend on the magnetic spectrum B_{mn}, together with profiles of the temperature T and the rotational transform. The invariant properties of a trapped particle in this formulation of the problem explain why transport is expected to be good for toroidally symmetric stellarators.

Kuhl [2] has demonstrated that neglecting the displacement current in Maxwell's equations is a process of singular perturbation suggesting that the quasineutrality requirement $n_i = n_e$ is preferable to Ohm's law as a rule to determine the electrostatic potential $\boldsymbol{\Phi}$. Let us introduce Fourier expansions for both

$$\boldsymbol{\Phi} = \sum P_{\mathrm{mn}} \cos(m\theta - [n - \iota m]\phi)$$

and the charge separation

$$n_{\mathrm{e}} - n_{\mathrm{i}} = \sum C_{\mathrm{mn}} \cos(m\theta - [n - \iota m]\phi)$$

like the one for $1/B^2$. Our transport calculations include an iterative scheme of the form

$$P_{\mathrm{mn}}^{l+1} = P_{\mathrm{mn}}^{l} + \varepsilon C_{\mathrm{mn}}^{l}$$

designed to drive C_{mn} toward zero by varying the corresponding coefficient P_{mn}. We have found that when independently computed particle confinement times τ_{i} and τ_{e} of the ions and the electrons are made to coincide in this way, then the corresponding result for the energy confinement time τ_{E} agrees well with experimental measurements, at least in cases of low collisionality. The conclusion holds for tokamaks if appropriate bifurcated equilibria without two-dimensional symmetry are employed to evaluate the magnetic field [2].

In Fig. 2 we compare well converged calculations of the energy confinement time in conventional and helias stellarators at reactor conditions. Mathematical models of the LHD and the MHH2 have been used in the computations, with $\nu=1$ for the LHD but $\nu=1/2$ for the MHH2, and with an average magnetic field of 5 tesla for both in the plasma [3]. The specifications have been arranged to exhibit desirable features of both configurations. Because of its toroidal symmetry the MHH2 turns out to have better confinement. Its physical properties are competitive with those of standard tokamaks, and no trouble with disruptions is anticipated.

Acknowledgments

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