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Physics of Collapses in Toroidal Helical Plasmas

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Abstract

Theoretical model for the collapse events in toroidal helical plasmas with magnetic hill is presented. There exists a turbulent-turbulent transition at a critical pressure gradient, leading to a sudden increase of the anomalous transport. When the magnetic shear is low, the nonlinear excitation of the global mode is possible. This model explains an abrupt growth of the perturbations, *i.e.*, the trigger phenomena. Achievable limit of the plasma beta value is discussed.

Keywords:
collapse, transition of turbulence, high beta plasma

1. Introduction

The collapse phenomena in toroidal plasmas are an essential subject. Reports have been made on the beta-limiting phenomena in Heliotron-E [1,2]. Possible mechanisms for the onset of the collapses have been considered to be the MHD instabilities [3].

The pressure driven instabilities have been analyzed in conjunction with the beta-limiting phenomena. The usual analysis on the condition of the onset is based on the linear stability. Although there have been reports on coincidence between the linear stability boundary and the experimental condition for the onset of perturbations, there is a fundamental problem: The collapses are caused by such the activities of which growth rate changes abruptly. (See, *e.g.*, [2].) The abrupt jump cannot be explained by the conventional approach to the MHD instabilities. Such a sudden change of the growth rate of the symmetry-breaking perturbations has also been observed on tokamaks [4], and the understanding on the occurrence of the collapse events would provide a fundamental elements in the physics of toroidal plasmas [5].

Here we present the theoretical model of catastrophic events in toroidal helical plasmas. The turbulence-turbulence transition has been predicted (M-mode transition) [6, 7]. The relation between the pressure gradient and the heat flux has the hysteresis characteristics. At the critical pressure gradient, the turbulent transport coefficient suddenly increases. The rapid decay of the pressure gradient is predicted to occur. The abrupt destabilization of the global mode is also discussed. The sudden excitation of the perturbations and the rapid decay of the pressure gradient are predicted to occur via two mechanisms. Achievable limit of the plasma beta value is discussed.

2. Model Equation

The change of the growth rate of the symmetry-breaking perturbations (either global or microscopic ones) has been known very fast in experiments: It usually ranges from 10 μs to 100 μs [5]. This fast change of the growth rate shows that the mode growth is not described by the linear theory, in which the change of the growth rate is predicted within a time scale of the global transport. The processes that have the short correlation time (10 μs to 100 μs, or much faster) should play an essential role. In order to analyze the problem, the picture of the self-sustained turbulence is applied to the beta-limiting phenomena.

The method of dressed test mode is applied to the beta-limiting phenomena. The reduced set of equations is employed. The interaction with the back-ground turbulence is renormalized as nonlinear transfer rates, which have the form of diffusion operator. (See [8] for the detail of the theoretical procedure.) In the system with the magnetic shear and magnetic hill, the eigenvalue equation for the stationary state (*i.e.*, the nonlinear critical condition) is given as

$$k_\theta^2 s^2 \frac{\mathrm{d}}{\mathrm{d}k} \frac{1}{\eta + \lambda k_\perp^2} \frac{\mathrm{d}}{\mathrm{d}k} \tilde{\phi}(k) + \mu k_\perp^4 \tilde{\phi}(k) - \frac{k_\theta^2}{k_\perp^2}\left(\frac{G_{0\mathrm{e}}}{\chi_\mathrm{e}} + \frac{G_{0\mathrm{i}}}{\chi_\mathrm{i}}\right)\tilde{\phi}(k) = 0 \tag{1}$$

where, $k_\perp^2 = k_\theta^2 + k^2$, G_0 denotes the equilibrium pressure gradient coupled to the magnetic curvature,

$$G_{0\mathrm{e,j}} = -\Omega'(\mathrm{d}p_{0\mathrm{e,j}}/\mathrm{d}r), \tag{2}$$

which is the origin of interchange instability, s the shear parameter $r(\mathrm{d}q/\mathrm{d}r)q^{-2}$, Ω' the average curvature of the magnetic field, μ the viscosity, χ the thermal diffusivity, η the resistivity and λ the current diffusivity. We employed the normalization for resistive MHD modes as: $(\varepsilon v_\mathrm{A}/a)t \to t$, $r/a \to r$, $z/R \to z$, $\phi/(\varepsilon a v_\mathrm{A} B_0) \to \phi$, $J(a\mu_0/\varepsilon B_0) \to J$, $p(2\mu_0/\varepsilon B_0^2) \to p$, $\eta(\tau_{\mathrm{Ap}}/\mu_0 a^2) \to \eta$, $\mu(\tau_{\mathrm{Ap}}/a^2) \to \mu$, $\lambda(\tau_{\mathrm{Ap}}/\mu_0 a^4) \to \lambda$, $\chi(\tau_{\mathrm{Ap}}/a^2) \to \chi$, where ε is the inverse aspect ratio, a/R, v_A is the Alfven velocity, B_0 is the main magnetic field, and $\tau_{\mathrm{Ap}} = R/v_\mathrm{A}$. It is noted that the finite electron mass and convective nonlinearity on the current were kept in the Ohm's law, in order to discuss properly the electron dynamics.

The transport coefficients (μ, λ, χ, η) are the sum of the collisional transport and turbulent transport. One has an approximate estimate for high beta plasmas as [9]

$$\mu_{\mathrm{i,N}} \sim \chi_{\mathrm{i,N}} \sim \eta_\mathrm{N},\ \mu_{\mathrm{e,N}} \sim \chi_{\mathrm{e,N}} \ \text{ and } \ \lambda_{\mathrm{e,N}} \sim (c/\omega_\mathrm{p} a)^2 \mu_{\mathrm{e,N}}, \tag{3}$$

where suffix N indicates the turbulent transport . When the gradient is weak, the $E \times B$ convection dominates the turbulent transport, and we have

$$\mu_{\mathrm{e,N}} \sim \mu_{\mathrm{i,N}} \tag{4}$$

If the gradient is large, and magnetic braiding takes place, then the selective loss of electrons is induced as

$$\mu_{\mathrm{e,N}} \sim \sqrt{\frac{m_\mathrm{i} T_\mathrm{e}}{m_\mathrm{e} T_\mathrm{i}}}\, M \mu_{\mathrm{i,N}} \tag{5}$$

where the coefficient M is determined by the competition between the perpendicular decorrelation and the parallel decorrelation.

For the short wave length mode, the current diffusivity is the dominant instability mechanism [10]. On the contrary, the global mode is destabilized by the resistivity.

3. M-mode Transition

When shear is strong, $s^2 > G_0$, the short wave length mode is excited [10]. If the pressure gradient reaches the critical gradient

$$G_{0\mathrm{e}} + G_{0\mathrm{i}} > G_\mathrm{c} \simeq s \tag{6}$$

the overlapping of the micro-island takes place [6]. The turbulent transport coefficient becomes enhanced

$$\chi_\mathrm{e} = \left(\frac{G_{0\mathrm{i}} + G_{0\mathrm{e}}\sqrt{m_\mathrm{e} T_\mathrm{i}/m_\mathrm{i} T_\mathrm{e}}}{G_{0\mathrm{i}} + G_{0\mathrm{e}}}\right)^{3/2} \left(\frac{m_\mathrm{i} T_\mathrm{e}}{m_\mathrm{e} T_\mathrm{i}} M^2\right) \chi_\mathrm{L} \tag{7}$$

where χ_L is the turbulent transport coefficient in the L-mode $\chi_\mathrm{L} \simeq (G_{0\mathrm{i}} + G_{0\mathrm{e}})^{3/2} s^{-2} (c/a\omega_\mathrm{p})^2$, and the factor M is approximately given as $M \simeq \sqrt{G_0 \beta_\mathrm{i}}$. The back transition occurs at the lower gradient G_1.

$$\left(G_{0\mathrm{i}} + G_{0\mathrm{e}} \sqrt{\frac{m_\mathrm{e} T_\mathrm{i}}{m_\mathrm{i} T_\mathrm{e}}}\right) \geq G_1 \sim 9\beta_\mathrm{i} \tag{8}$$

The gradient-flux relation has the cusp-type catastrophe, as is illustrated in Fig.1. When the gradient reaches the criterion, the transport suddenly becomes large. (The time scale of the growth is given as $G_0^{-1/2}$.) Due to the enhanced transport, the pressure gradient starts to decay. Owing to the hysteresis nature of the gradient-flux relation, the crash continues until the gradient reaches to the lower critical point. This transition is called as M-mode transition [6]. When the transport catastrophe occurs at a certain magnetic surface, there appears' an avalanche of the crashes across the magnetic surfaces [11]. The propagation velocity of the avalanche was obtained. Owing to the avalanche, the pressure gradient in the wide region is rapidly depleted.

4. Destabilization of Low-m Mode

The low-m mode is destabilized by the anomalous resistivity. In the weak shear case, $s^2 < G_0$, the destabilization of the global mode (through the nonlinear interactions with the back ground turbulence) takes place. Neglecting the $\lambda k_\perp^2$ term in comparison with η in Eq.(1), the criteria is obtained as [12]

$$\frac{G_0 \eta}{s^2 \chi} = C_r \equiv \pi^2 [4 \ln\{2(G_0 \mu_i^{-1} \chi_i^{-1} m^{-4})^{1/6}\}]^{-2} \qquad (9)$$

The coefficient C_r is weakly varying, but shows that the lowest-m mode is the least stable for the anomalous-resistivity driven modes. The critical gradient is determined by the turbulence as

$$G_0 = C_r \frac{\chi_N + \chi_c}{\eta_N + \eta_c} s^2 . \qquad (10)$$

Figure 2 illustrates the stability boundary. The crossing of two lines, $\chi_L(G_0, \cdot s \cdot \cdot \cdot)$ and that in Fig.2, determines the condition where the excitation of the global mode takes place. It should be noted that the nonlinear acceleration of the mode growth is possible. When the low-m mode appears, the cascade to shorter wave-length modes induces the increment of the transport coefficients.

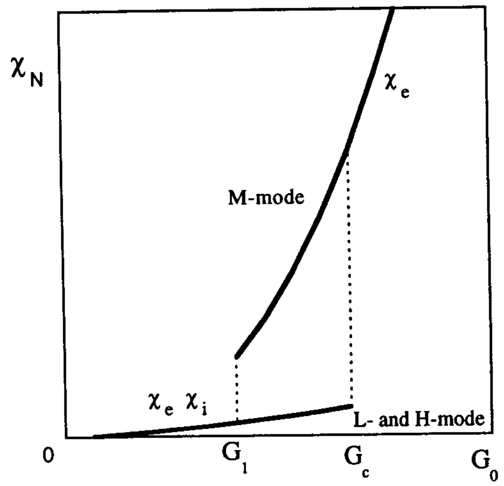

Fig. 1 Turbulent transport coefficient as a function of the gradient. Transition is predicted at critical gradient.

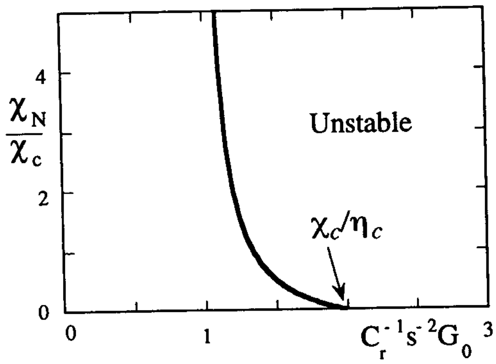

Fig. 2 Stability criterion of the global mode in the presence of back-ground turbulence.

5. Summary and Discussion

The theory of the self-sustained turbulence is applied to investigate the collapse phenomena in toroidal helical plasmas. The theoretical model for the sudden onset of collapse is made. Two processes are proposed as mechanisms of the crash events. One is the transport catastrophe in the high shear case, and the other is the excitation of the global mode by the turbulence in the low shear case. The criterion for the onset of crash is given as

$$|R\beta'| > (1/R\Omega')s \quad \text{for } s > 1 \qquad (11)$$

and

$$|R\beta'| > (C/R\Omega')s^2 \quad \text{for } s < 1. \qquad (12)$$

The criteria for the onset of the collapse, which are presented here, have a similarity to the one which has been obtained in the linear and ideal MHD stability theory. This finding also explains the coincidence that some linear theories have shown.

Acknowledgements

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