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The Local Stability in Heliacs with Different Types of Quasisymmetry

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Abstract

The β limits with respect to ballooning and Mercier stability have been calculated for 4-period Heliac-like stellarators using the TERPSICHORE code. We compare the results for Heliacs with two types of the magnetic field symmetry: 1. the magnetic field strength has quasihelically symmetric structure; 2. the magnetic field strength depends mainly on the toroidal coordinate on each magnetic surface corresponding to a quasi-mirror symmetric system. The Mercier criterion is stable in the both cases at the level of 4%, but the β limit with respect to ballooning modes in the quasi-mirror case is 3 times higher than in the quasihelically symmetric case and equals to 3%.

Keywords:

quasisymmetrical stellarator, local stability, quasimirror configurations

1. Introduction

In this work, we explore the local stability limit which is defined by the value of $\beta=2<p>/<B^2>$, where $<p>$ and $<B^2>$ correspond to the volume averaged plasma pressure and volume averaged magnetic energy density, respectively. We look for the configurations which are stable with respect to the Mercier and ballooning modes at the maximal value of β.

Our main tools will be the three dimensional (3D) ideal magnetohydrodynamic (MHD) codes VMEC [1] and TERPSICHORE [2]. Here the magnetic configurations are defined via the shape of the boundary magnetic surface. We try to reduce the spectrum of the Fourier modes defining our boundary surface in the cylindrical coordinates to obtain a compact model for the optimization tasks. This spectrum is used as input to the VMEC equilibrium code (fixed boundary version) and then the equilibrium configuration is submitted to the TERPSICHORE stability code. Using the TERPSICHORE code, we can see for which value of β the Mercier criterion is stable (positive) and also too for which β the sign of the ballooning eigenvalue becomes unstable (positive). The TERPSICHORE code performs the transition to the special Boozer magnetic coordinates. The information about the spectrum of the magnetic field strength in Boozer coordinates can predict the behaviour of a given configuration from the point of view of the neoclassical losses [3].

Here we are interested in 4-period systems with nonplanar magnetic axes. This allows us the possibility to compare the results and predictions with those of HSX [4], TJ-II [5], HHHS [6]. Among the different 4-period stellarators we select Heliac-like cases.

We mean by the term "Heliac" the configuration where the magnetic surface cross-sections rotates simultaneously with the principal normal with respect to the magnetic axis. In Heliac systems, the flux surface cross sectional shape is vertically elongated at the beginning (the point of the maximal curvature of the magnetic

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axis, $\zeta=0$) and at the middle of the system period.

The "Helias" magnetic configuration corresponds to the system in which the magnetic surface cross-sections lags behind the principal normal with respect to the magnetic axis. In Helias systems, the flux surface cross sectional shape is vertically elongated at the beginning of the system period and horizontally elongated at the midperiod position. Such definitions of Heliac and Helias differ from the definitions used in [7-9].

The accuracy of the quasi-symmetry condition in different systems can be characterised by the ratio X of the dominant helical Fourier component of the magnetic field strength in Boozer coordinates to the maximal Fourier component that violates the symmetry evaluated at the plasma boundary. In our work we explore the Heliac-like configurations with two different directions of quasisymmetry. In the quasihelically symmetric configuration (QHS), the dominant mode of the magnetic field strengh spectrum per period is (1,1). In the quasi-mirror symmetric stellarator configuration (QMS), which is a quasisymmetrical analogue of the linked-mirror stellarator, the dominant mode of the magnetic spectrum per period is (0,1). The configurations which are close to one of the two types mentioned above were previously discussed in many articles (for QMS see, *e.g* [10-13], for QHS- [4,14]).

Ballooning modes that are strongly localised impose the most restrictive limits in Heliac-like configurations. The ballooning modes with such localised structure have been found in 3D configurations both for ordinary stellarators [15] and advanced stellarators with spatial magnetic axes. For the quasi-symmetric stellarators HSX and MHH2, the β value limit imposed by ballooning mode stability is around 1% [4,16]. In this article, we attempt to increase the β value by optimising the plasma boundary shape. We have found that this optimisation leads to a change in the direction of lines $B=$const on magnetic surfaces.

Our article consists of two main sections where we describe the local stability for QHS (Sec.2) and for QMS (Sec.3) systems.

2. Local Stability of the Quasihelically Symmetric Heliac

Figure 1 displays three magnetic surface cross-sections of the QHS Heliac at the beginning, at the quarter and at the middle of the system period.

The magnetic field spectrum has the ratio X equal to 3. The Mercier criterion calculations with the TERPSICHORE code were first presented in [17]. The $\beta=4\%$ limit with respect to Mercier modes is quite high compared with other 4-periods system. High Mercier stability for Heliac-like stellarators was predicted from the paraxial estimations.

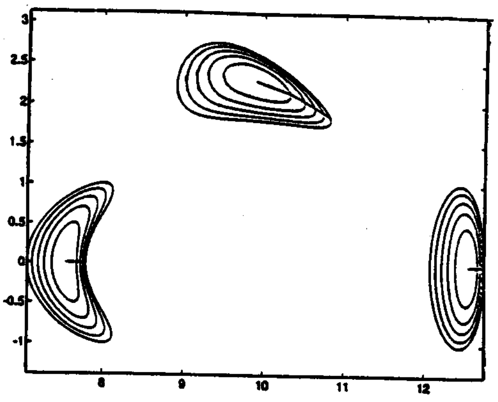

Fig. 1 Flux surface cross-sections for the optimised 4-period quasihelically symmetric Heliac with β=4% (φ=0 - beginning of the period, $\varphi=\pi/8$ - one quarter of the period, $\varphi=\pi/4$ - middle of the period).

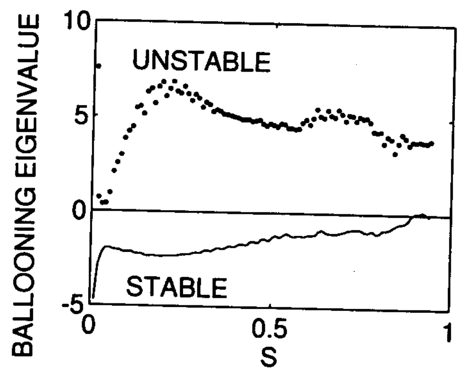

Fig. 2 The ballooning eigenvalues profiles as a function of the radial variable s for a 4-period quasihelically symmetric Heliac. The cases with volume average β of 0.986% (dotted line) and 2.98% (solid line) are shown.

Here we show also the ballooning eigenvalue as a function of the plasma radius (Fig. 2) corresponding to the most unstable field line on each flux surface. These lines pass through the outer edge of each surface ($\theta=\zeta=0$). The β limit with respect to ballooning modes equals to 1%. Therefore, the β limit with respect to Mercier modes is 4 times higher than with respect to ballooning modes for the QHS Heliac. The ballooning instability is driven by the large normal curvature of the magnetic force field lines, which is a characteristic feature of the Heliac-like systems.

3. Local stability of the Quasi-Mirror Heliac

Starting with the boundary obtained from the paraxial approximation, we could get a QHS Heliac with rather high Mercier stability. Now we continue the optimisation of Heliac-like stellarators with respect to the β limit imposed by ballooning modes. We use as an initial point of our optimisation the boundary of the QHS Heliac. We choose to vary the VMEC input boundary modes that describe the plasma shape as the means for increasing the ballooning stability limit. The conservation of the quasihelical symmetry in our configuration does not permit an increase in the ballooning plasma pressure limit. Thus, we do not impose constraints or restrictions on the magnetic field strength as we vary the plasma shaping.

As a result of this optimisation, we obtain a Heliac-like magnetic configuration with large triangularity, elongation and bumpiness (Fig. 3). The shear is larger than in the QHS configuration. The rotational transforms ι profile for the QHS case is almost constant, $\iota = 1.60$. For the new case, ι changes from 1.45 on the axis to 1.70 on the boundary.

The ballooning mode limit for the new configuration approaches the level of $\beta \simeq 3\%$, almost 3 times higher than for the QHS Heliac (Fig. 4). The calculations were performed with a prescribed parabolic plasma pressure profile. The variation of the plasma pressure profile does not seem to significantly alter the value of the ballooning mode limit.

The magnetic field spectrum displays a new type of symmetry. The dominant component in the spectrum is the (0,1) correponding to that of a linked mirror system (Fig. 5). Therefore, this new configuration can be

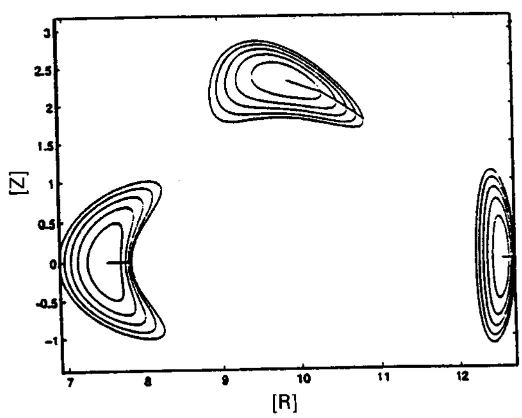

Fig. 3 Flux surface cross-sections for the optimised 4-period quasimirror Heliac with β=3.02% (φ=0 - beginning of the period, $\varphi=\pi/8$ - one quarter of the period, $\varphi=\pi/4$ - middle of the period).

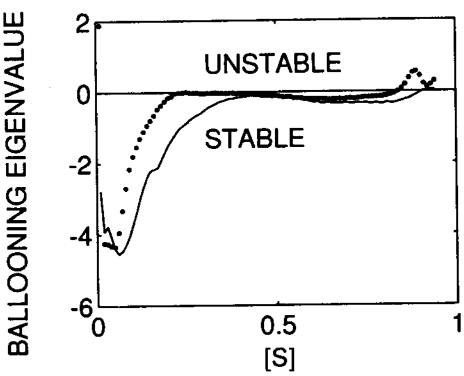

Fig. 4 The ballooning eigenvalue profiles as a function of the radial variable *s* in a 4-period quasimirror Heliac for the cases with volume average β of 3.02% (solid line), 3.34% (dotted line).

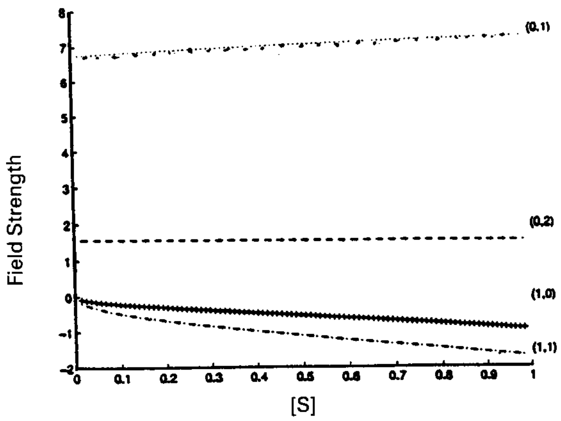

Fig. 5 The magnetic field spectrum in Boozer coordinates for a 4-period quasimirror Heliac. Only the 4 leading components of field strength (except the (0,0) component) versus the radial variable are shown. These components are: (1,1), (1,0), (0,2), (0,1). The ratio X equals 4.5. In this case β=3.02%. The spectrum depends weakly on the β value for the boundary chosen to realise the magnetic configuration.

referred to as a quasi-mirror stellarator (QMS). The ratio X for the QMS equals to 4.5 and it is possible to hope for reduced neoclassical losses in this system too. It is very interesting that the same character of the magnetic field spectrum was obtained recently in a Helias-like 5-period stellarator [18]. It demonstrates that stellarators with very different geometry of the magnetic surface shapes can have almost the same spectra of the magnetic field strength. The Mercier criterion for the QMS Heliac is shown in Fig. 6. For Mercier modes, the β limit equals to 4% is determined by Mercier criterion value near the edge. The spikes are not taken into account.

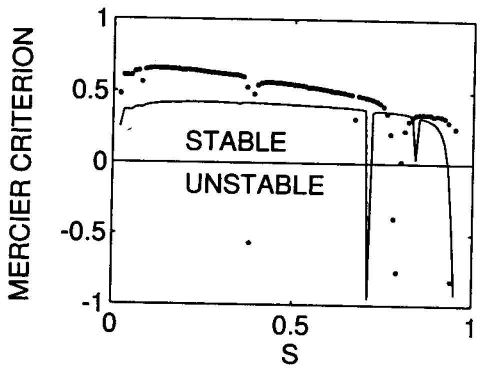

Fig. 6 The Mercier criterion as a function of the radial variable s for a 4-period quasimirror Heliac. The cases with volume average β of 3.02% (dotted line) and 4.16% (solid line) are shown.

4. Conclusions

We have investigated the local ideal MHD stability properties of a 4-period quasisymmetric Heliac-like stellarator with the TERPSICHORE code. Our aim was to numerically optimise the β limit with respect to Mercier and ballooning stability. We have also tried to control the different types of quasisymmetry conditions with moderate levels of accuracy.

As a result, two quasisymmetric Heliac-like configurations were found. The Heliac with quasihelical symmetry in the magnetic field strength spectrum is stable with respect to Mercier modes up to β=4%. Ballooning modes impose a more restrictive limit, $\beta\simeq$ 1%, for this Mercier stability optimised configuration.

To improve the ballooning stability in the Heliac, we needed to relax the constraint of quasihelical symmetry. As a result of the optimisation, a Heliac with linked-mirror symmetry, the quasimirror configuration (QMS), was found which is stable with respect to the ballooning modes up to β=3%. The Mercier criterion for the QMS is stable (positive) at a level of β=4%.

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References

[1] S.P. Hirshman and O. Betancourt, J. Comput. Phys. **96**, 99 (1991).

[2] D.V. Anderson, W.A. Cooper, R. Gruber, S. Merazzi and U. Schwenn, *Scientific Computing on Supercomputers II*, Plenum Press, New York, 1990, p.159; W.A. Cooper, Plasma Phys. Control. Fusion **34**, 1011 (1992).

[3] A. Boozer, Phys Fluids. **25**, 496 (1983).

[4] J.N. Talmadge and W.A. Cooper, Phys. Plasmas **3**, 3713 (1996).

[5] J.A. Jiménez, A. Varias, A.L. Fraguas, C. Alejaldre *et al.*, Nuclear Fusion **37**, 467 (1997); Jiménez J.A. *et al.*, *Joint Varenna-Lausanne Int. Workshop on Theory of Fusion Plasmas*, edited by J.W. Connor, E. Sindoni, J. Vaclavik (Societa Italiana di Fisica-Editrice Compositori, Bologna, 1996), p.343.

[6] M. Yokoyama, Y. Nakamura and M. Wakatani, Transactions of Fus. Technology (*Proc. of 6th Int. Toki Conf.*, Japan, 1994) **27**, 186 (1995).

[7] J. Nührenberg and R. Zille, Phys. Lett. A **114**, 129 (1986).

[8] J. Nührenberg and R. Zille, *Joint Varenna-Lausanne Int. Workshop on Theory of Fusion Plasmas*, edited by J. Vaclavic, F. Troyon, and E. Sindoni (Societa Italiana di Fisica-Editrice Compositori, Bologna, 1989), p.3.

[9] A. Boozer *et al.*, *Proc. 9th Int. Conf. on Plasma Phys. and Contr. Nucl. Fus. Res.* (Baltimore, 1982), IAEA, Vienna, 1983, **3**, 129.

[10] J.R. Cary and S.G. Shasharina, Plasma Phys. Rep. **23**, 509 (1997).

[11] A. Boozer, Plasma Phys. Rep. **23**, 449 (1997).

[12] S. Gori, W. Lotz, Nührenberg J., *Joint Varenna-Lausanne Int. Workshop on Theory of Fusion Plasmas*, edited by J.W. Connor, E. Sindoni, J. Vaclavik (Societa Italiana di Fisica-Editrice Compositori, Bologna, 1996), p.335.

[13] M.I. Mikhailov, V.D. Shafranov, A.A. Subbotin and D. Sunder, *Proc. 24th EPS Conf. on Control. Fusion and Plasma Phys.*, Berchtesgaden, 1997.

[14] J. Nührenberg and R. Zille, Phys. Lett. A **129**, 113 (1988).

[15] W.A. Cooper, S.P. Hirshman and D.K. Lee, Nucl. Fusion **29**, 617 (1989).

[16] A.H. Reiman, L.P. Ku, D.A. Monticello, C. Nührenberg and W.A. Cooper, Plasma Phys. Rep. **23**, 472 (1997).

[17] M.Yu. Isaev, W.A. Cooper, S.Yu. Medvedev, M.I. Mikhailov, V.D. Shafranov and A.A. Subbotin, Nucl. Fusion **37**, 1431 (1997).

[18] W.A. Cooper, *private comm.* (1997).