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Beta Studies in Quasi-Symmetric Configurations

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Abstract

Quasi-symmetrical configurations – quasi-helically symmetric (qH), quasi-axisymmetric (qA) and quasi-isodynamic (qI) – are investigated with respect to their MHD stability properties. QH configurations with aspect ratio $\mathcal{O}(10)$ are found which appear to be MHD stable at $<\beta> \approx 0.06$, if no net toroidal current is present. QA tokamaks with two periods, aspect ratio ~ 4 and $\iota \sim \frac{1}{2}$ are found by combined symmetry and stability optimization which appear to be stable at $<\beta> \approx 0.03$ for a ratio of externally to current-created rotational transform of about 2. As for qI stellarators simultaneous qI and magnetic-well optimization at vanishing and finite β shows that very good collisionless particle confinement and a magnetic well in the vacuum field configuration are compatible.

Keywords:

quasi-symmetries, quasi-helical symmetry, quasi-axisymmetry, quasi-isodynamicity, configuration study, local and global MHD-stability

1. Introduction

Stellarators may exhibit large neo-classical transport in the long-mean-free-path regime and poor α-particle confinement so that these issues generally deserve critical attention in configurational studies. Quasi-symmetrical configurations – quasi-helically (qH) symmetric [1], quasi-axisymmetric (qA) [2,3] and quasi-isodynamic (qI) [4] – alleviate these problems, so that their MHD behavior is an important remaining area of study. As far as high-beta equilibria are concerned probably the most challenging situation is the high-beta qA tokamak, while for qH symmetric and qI (or W7-X type) configurations high-beta equilibria pose less problems. In the area of MHD stability several specific problems have been identified. High-beta qH symmetric configurations with rotational transform increasing towards the plasma boundary have been found which are stable to Mercier and resistive interchange modes but unstable to local as well as non-local ballooning modes, so that the relationship of the latter two types of modes can readily be investigated. Ballooning modes also appear to be one important obstacle to high-beta qA configurations if the externally created rotational transform is strong. As for qI configurations a more elementary problem is their compatibility with a vacuum magnetic well. In the remainder of this paper case studies of these problems are presented.

2. Ballooning Modes in a Quasi-Helically Symmetric Configuration

The qH symmetric stellarator was the first example of a true three-dimensional configuration which defeats the traditional stellarator plagues of large neo-classical transport and poor α-particle confinement. At moderate aspect ratio $[A \sim \mathcal{O}(10)]$ these configurations are genuinely three-dimensional in plasma shape, *i.e.* not close to a helically symmetric shape of their cross-sections. They are capable of very high equilibrium-β values since they do not exhibit a Shafranov shift. They also show quite large stability-β values with respect to

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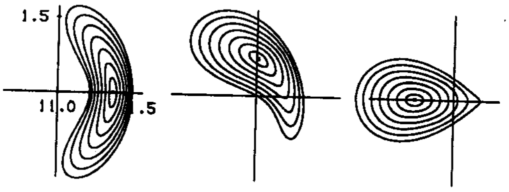

Fig. 1 $<\beta> \approx 0.1$ flux-surface cross-sections at the beginning, a quarter, and half of a field-period of an $N_p = 6$ qH configuration with aspect ratio $A \approx 11$ and vacuum magnetic well depth ≈ 0.02. For $<\beta> \approx 0.1$ the rotational transform varies between $\iota \approx 6/5$ and $12/9$.

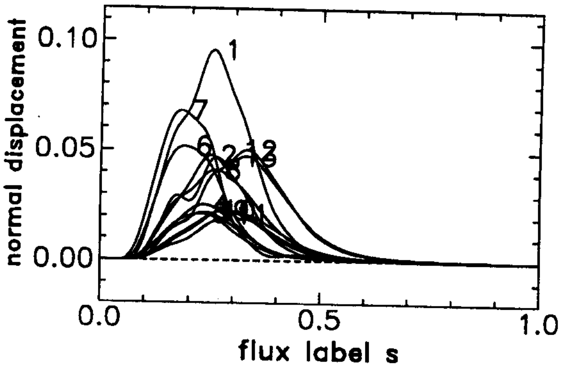

Fig. 2 Result from the CAS3D stability code: An unstable non-local ballooning-type mode (poloidal node-number $M \sim 20$) detected in the equilibrium shown in Fig. 1. A more detailed description is given in the main text. Note that this equilibrium is stable with respect to the Mercier criterion. Computation parameters: radial grid size $N_s = 192$, ~ 100 perturbation Fourier components, CAS3D2MN [12]. In a sequence of equilibria with lower $<\beta>$ the marginal point given by such medium-node-number perturbations is obtained for $<\beta> \approx 0.06$.

Mercier and resistive interchange modes, see Fig. 1 for a case with six field periods, *i.e.* $N_p = 6$, so that their ballooning stability properties are interesting. Results concerning this question are shown in Fig. 2, in which the Fourier harmonics of the scalar component of the ideal MHD displacement vector normal to flux surfaces are plotted versus the normalized flux label (which is $s = 0$ on the magnetic axis and $s = 1$ on the plasma boundary). For this specific perturbation the value of the mode resonant rotational transform is $\iota \approx 1.25 = 25/20$. Out of 100 perturbation Fourier harmonics 15 are above the plotting threshold. The dominant harmonic (curve 1 in Fig. 2) has poloidal Fourier index $m = 20$ and toroidal Fourier index $n = -25$. Curves 2 and 8 represent tokamak-like couplings (same toroidal index as principal harmonic, *i.e.* $n = -25$, $m = 19$ and $m = 21$), curves 6, 7 ($n = -19$, $m = 16$, 15) and curves 12, 13 ($n = -31$, $m = 24$, 25) the dominant helical couplings. Non-local ballooning modes occur at $<\beta>$ values above 0.06, while local ballooning instability (obtained from the ballooning equation along a field line [5]) prevails for $<\beta>$ values above 0.05. The case considered here exhibits a rotational transform profile increasing towards the plasma edge. A discrepancy between non-local and local ballooning was observed in W7-X type cases, too, as opposed to the standard tokamak situation characterized by a rotational transform decreasing towards the plasma edge [6].

3. Ballooning Modes in a Quasi-Axisymmetric Tokamak

QA configurations have been studied with tokamak [2] as well as stellarator emphasis [3,7,8,9]. Here, a case intermediate to these extremes with a rotational transform (without net current) of $\iota_{\mathrm{edge}} \lesssim 2/5$ with $N_p = 2$ and aspect ratio $A \approx 4$ is studied. This configuration also served to investigate the problem of control of the quasi-axisymmetry for free-boundary equilibria with toroidal current by external coil currents (see Ref. [10]).

The configuration shown in Fig. 3 was obtained by simultaneous optimization of quasi-axisymmetry and resistive-interchange stability at vanishing toroidal current and fixed $<\beta> = 0.02$. Non-local ballooning stability investigations have been started for cases with toroidal current, here simply by fixing the rotational transform profile in the equilibrium calculations. For two cases with slightly increasing and slightly decreasing profiles ($0.51 < \iota < 0.56$, $0.56 > \iota > 0.51$) marginal stability is found for $<\beta> \approx 0.026$ and $<\beta> \approx 0.022$, respectively, compare Fig. 4.

These results are preliminary, because simultaneous consideration of a ballooning optimization [11] still needs to be done.

4. Quasi-Isodynamic Configurations with a Magnetic Well

Excellent orbit confinement in a 3-D structure of $B(s, \theta, \phi)$ of a vacuum field configuration was demonstrated recently [4]. Here, combined optimization of

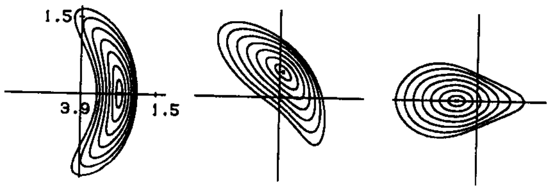

Fig. 3 Same as Fig. 1, but for a qa tokamak with $N_p = 2$, aspect ratio $A \approx 4$ at $<\beta> = 0.03$. Here, $0.51 < \iota < 0.56$.

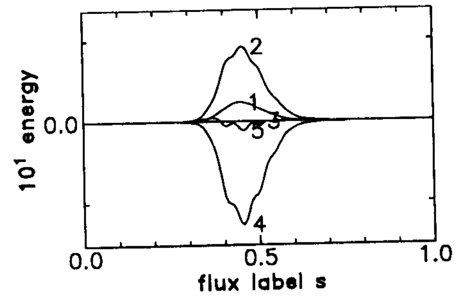

Fig. 4 Result from the CAS3D stability code: Flux-surface averaged energy terms for an unstable perturbation in the qA case of Fig. 3. The various contributions are driven by the field line bending (label 1), local shear and parallel current density (2), the field compression (3), curvature terms (4), and sum up to $\delta^2 W$ (5). Computation parameters: poloidal node-number ~60, radial grid size $N_s=192$, ~80 perturbation Fourier components, CAS3D2MN. Extrapolation of $<\beta>=0.03$ and 0.027 results leads to a marginal value of 0.026; local ballooning is found to be unstable for $<\beta>=0.017$. The corresponding case with outwardly decreasing rotational transform leads to a marginal value of $<\beta>\approx 0.022$.

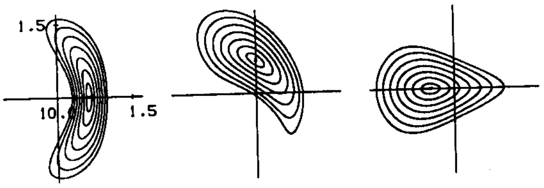

Fig. 5 Same as Fig. 1, but for the vacuum field of a qI stellarator with $N_p=5$, aspect ratio $A\approx 10$. The magnetic well is ≈ 0.01, ι_{axis} and $\iota_{edge}\approx 1$, and $\iota_{min}=0.93$.

the collisionless α-particle confinement (simultaneous at $<\beta>=0$ and 0.05) and the magnetic well property (necessary for MHD stability) was used to find a configuration with very good confinement of core α-particles (for details see Fig. 6) and a vacuum field magnetic well of ≈ 0.01. The results of Fig. 6 suggest that the qI condition cannot be satisfied at low aspect ratio within this class of configurations. The optimizations (in ≈ 200 variables) leading to the results of Fig. 6 became only possible using B and its partial derivatives on a 3-D grid (radially 90, poloidally 140, half a field period

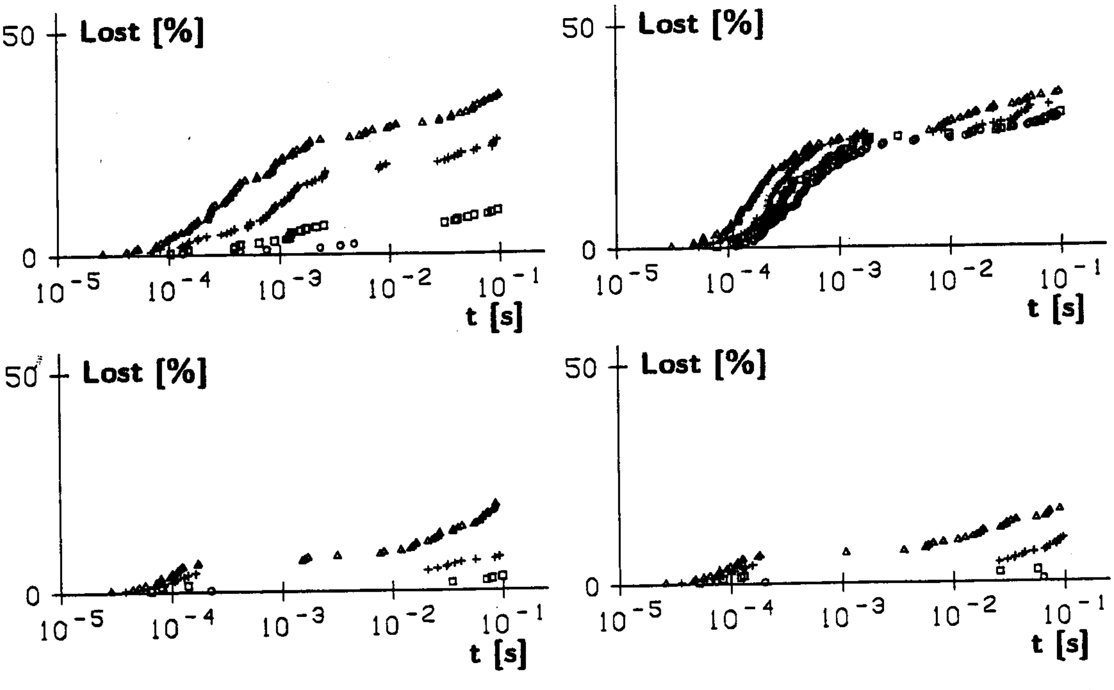

Fig. 6 Collisionless α-particle confinement, qI and W7-X stellarators. Normalizations used: $B=5$ T, average minor plasma radius $a=1.5$ m, kinetic energy of α-particles 3.6 MeV. Shown are the histories of losses of samples of particles – started at various surfaces with random distributions in the poloidal and toroidal variables and the pitch angle – up to approximately the slowing-down-time in a cumulative way, each symbol indicating the loss of a particle at the time of its loss. Symbols indicating start surfaces in normalized flux: ○: 0.08, □: 0.24, +: 0.4, △: 0.56. Left column for the qI case of Fig. 5, right column for W7-X. Top frames for $\beta=0$, bottom frames for $<\beta>=0.05$.

toroidally 80 exploiting stellarator symmetry) and achieving ≈ 0.02 TFlops on 256 processors of the CRAY T3E of the Rechenzentrum Garching. Evaluation of the local ballooning criterion for such qI configurations indicates stability for $<\beta> \approx 0.05$, while the global-mode stability limit at $<\beta> \approx 0.058$ as given by the CAS3D stability code [12] cas is slightly less stringent. These results are preliminary, since simultaneous optimization of the rotational transform profile still needs to be done.

5. Conclusion

QH configurations should be investigated with respect to the transition to very large β values as $N_p \propto A$ increase and to the influence of a selfconsistent bootstrap current on these results. QA configurations should be optimized — in addition — against ballooning. The results for qI configurations show that the qI condition can probably not be satisfied at low aspect ratio in vacuum field configurations in the configurational neighborhood of W7-X.

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